Triangulation of structural interfaces

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Abstract

A triangulation of structural interfaces is very helpful to display the interfaces in models where the interfaces are defined by implicit functions. An algorithm for such triangulation is presented. The resolution of the method is prescribed a priori by a given 3-D regular rectangular grid of points. Another option of the presented algorithm is to generate triangulated 2-D sections through the models.

1 Introduction

There are basically two different approaches to the building of seismic heterogeneous models with structural interfaces. In the first approach, structural interfaces are defined explicitly, mostly by sets of triangles with coordinates of vertices given. This definition of triangulated interfaces is very convenient to display the models. For ray tracing in such models, some kind of smoothing is necessary, because most of ray–tracing methods require smoothly varying normal to structural interfaces. Simultaneously, checking for the position of a point and an interface is complex. In the second approach, interfaces are defined implicitly, mostly by implicit functions. This definition of smooth interfaces is suitable for computational purposes, but displaying of such surfaces is complicated.

The second approach was selected in the `MODEL` package. To be able to display the models, it is necessary to find an explicit definition of the structural interfaces. The representation of the interfaces by triangular meshes seems to be the most convenient for displaying purposes. Terms like *simple block* or *complex block* will be used in future text. Refer to Červený *et al* (1988) for explanation of these terms.

Before triangulation of an interface, some criteria for the triangulation must be defined. These may be for example the dimension of triangles, the maximum distance of the centers of the triangles from the interface, or some other conditions. One of the possibilities is to prescribe a regular rectangular 3-D grid of points, and to allow the interface to be approximated by polygons within individual gridcubes of the grid. The triangulation of the interfaces may be then done in the following steps: a) computation of indices of simple blocks and complex blocks in all the gridpoints, b) computation of intersections of gridlegs with the interfaces, c) connecting intersection points of the same kind at gridfaces and determination of intersections of gridfaces with edges at structural interfaces, d) connection of the intersection points in each gridcube into polygons approximating the interfaces, e) division of the polygons into triangles. All structural interfaces within the grid are triangulated after these five steps.

Note that as the three gridsteps of the grid may be different, term "grid rectangular parallelepiped" would be more exact. Nevertheless we use the term "gridcube".

In: Seismic Waves in Complex 3-D Structures, Report 8 (Department of Geophysics, Charles University, Prague, 1999), pp. 17–26

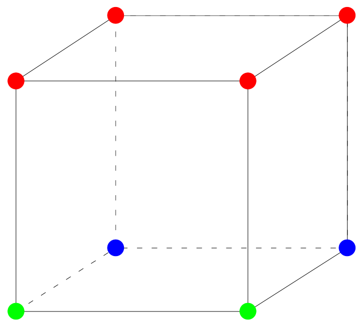

Figure 1a: Step a) of the algorithm: Indices of simple blocks are computed at the gridpoints of a gridcube. The gridpoints display different values of indices of simple blocks – marked by bullets of different colours.

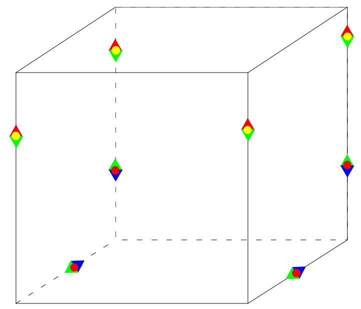

Figure 1b: Step b): Points of intersection of gridlegs with structural interfaces are computed. The intersection points are marked by diamonds coloured according to the values of the indices of simple blocks at both sides of the interfaces. Bullets in centers of the diamonds are coloured according to the indices of interfaces.

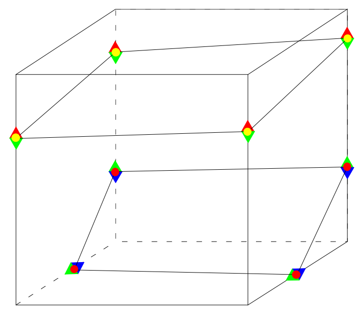

Figure 1c: Step c): At each gridface, the intersection points of the same kind are to be connected into couples. Here, all the intersection points at all gridfaces are coupled, the search for edges need not be done.

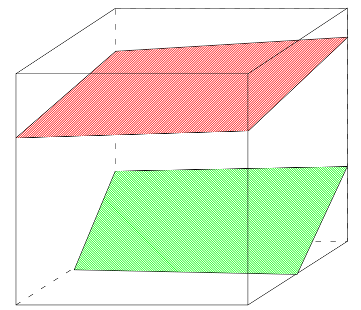

Figure 1d: Step d): Two polygons are created from the couples of intersection points.

2 Triangulation of interfaces

Triangulation of structural interfaces is done in five steps, each of the steps being done for all corresponding units (gridpoints, gridlegs, gridfaces, gridcubes, polygons). Good organization of indices of the units is thus necessary. As the values at positive and at negative sides of an interface are recorded at each point of intersection, the orientation of gridlegs and gridfaces is also very important.

a) Indices of blocks in gridpoints

Indices of simple and complex blocks are computed at all gridpoints in order to identify, which gridlegs may intersect structural interfaces.

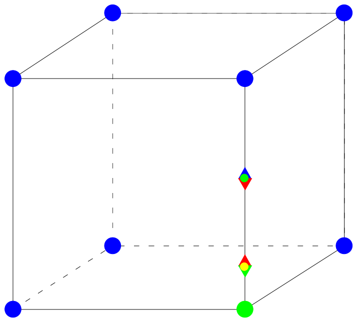

Figure 2a: Other gridcube than in figure 1. All the gridpoints are in the same complex block, but one of them is in different simple block then other gridpoints. After steps a) and b) of the algorithm, two points of intersection are found and are displayed again by diamonds coloured according to the indices of simple blocks. The bullets in the centers of the diamonds are coloured according to the indices of interfaces.

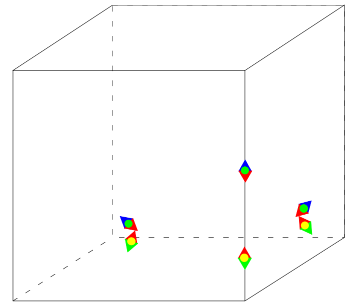

Figure 2b: Step c): As the two points of intersection are on different interfaces, they cannot be connected. The search for points of intersection of structural edges with the gridfaces is done at front and right-hand gridface. Two intersection points are recorded at each of the two gridfaces. See also figure 3.

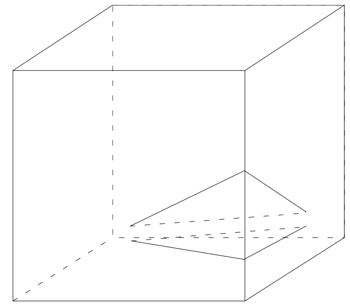

Figure 2c: Two couples of intersection points (solid lines) have been created in step c) of the algorithm at both front and right-hand gridface. Fifth and sixth couples across a gridface (dashed lines) are created in step d) of the algorithm.

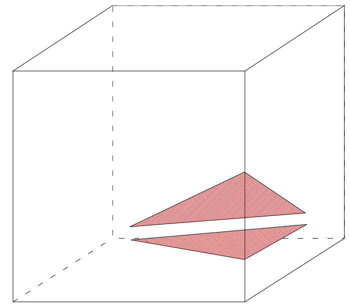

Figure 2d: Two polygons approximating the two structural interfaces inside the gridcube. Note that the two points approximating the edge at the same gridface are very close in practice, and that the belt between the polygons is thus much narrower in practice than in the figure.

b) Intersections of gridlegs with structural interfaces

Intersection of structural interfaces with each gridleg formed by gridpoints with different indices of simple blocks is checked for. The search for intersection points along the gridleg is same as the procedure used in ray tracing when a ray crosses an interface. All points of intersection along gridlegs are stored in memory, storing for each intersection point its coordinates, index of the crossed interface, and indices of simple blocks and complex blocks at both sides of the interface. Other values, such as values of material parameters in the points or normals to the interface, which will be used for displaying, may be computed in the points at the moment when the points are stored to the output file on a hard disk.

If the gridleg coincides with a planar interface, it may be considered to be located at either side of the interface. If the values of the indices of simple blocks are not the same at both the gridpoints of such a gridleg, the point of intersection of the gridleg with the interface may be considered to be, e.g., in the middle of the gridleg.

c1) Connecting the intersection points

At each gridface, the intersection points of the same kind must be connected into couples. The couples of intersection points at gridfaces will form the sides of future polygons approximating the interfaces. We couple the points, which have the same value of the index of the isosurface covering the interface, and the same values of indices of complex blocks at both sides of the interface.

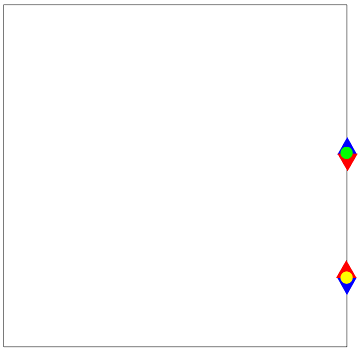

Figure 3a: The front gridface from figure 2. The points of intersection with the structural interfaces are now coloured according to the indices of complex blocks at the sides of the interfaces. The bullets in the centers of the diamonds are still coloured according to the indices of interfaces. As the two points of intersection are on different interfaces, they cannot be connected. The search for points of intersection of structural edges with the gridface has to be done. The search will be initiated, for instance, from the upper point.

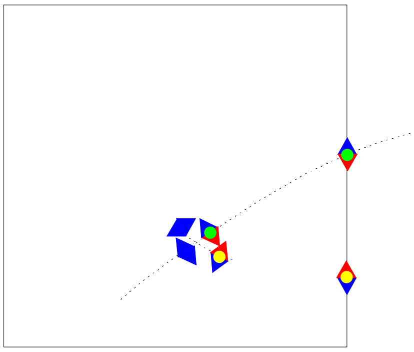

Figure 3b: Dotted lines represent parts of zero isolines of the implicit functions which describe the interfaces. The upper isoline had been followed from the upper point and its intersection with the lower isoline, the edge, has been found. Four points are at disposal to describe the edge, but in this case, only two of them lie at the structural interface and have different values of complex blocks. The remaining two points also lie at zero isosurfaces of the implicit functions, but they have the same value of indices of complex blocks at both sides of the surface. In other words, the upper structural interface ends at the edge, and the lower one starts there.

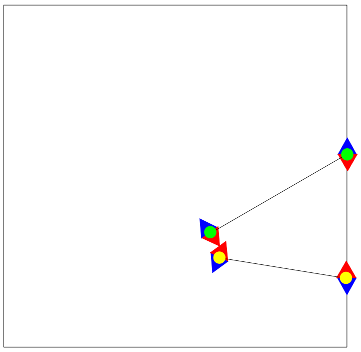

Figure 3c: Both the intersection points approximating the intersection of the structural edge with the gridface may be connected into couples with the intersection points at the gridlegs. Note that the two points approximating the edge are very close in practice, and not so far as in the figure.

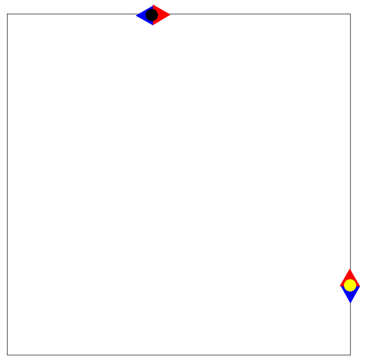

Figure 4a: Situation similar to that in figure 3a, again a gridface with two points of intersection at gridlegs. The points are again situated at different interfaces and thus cannot be connected into a couple.

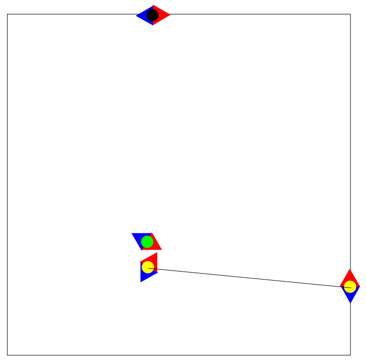

Figure 4b: The search for an edge has been started from the point at the right–hand gridleg. Two points approximating the intersection of the structural edge with the gridface are found. One of the points at the edge can be connected with the point at the right–hand gridleg, but the second one cannot be connected. The search for another edge must be performed, starting from one of the unconnected points.

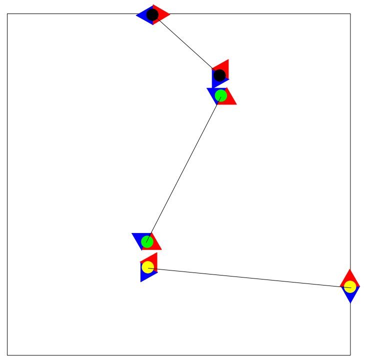

Figure 4c: The second edge has been found. All the intersections points are connected.

c2) Intersections of gridfaces with structural edges

An intersection point, which cannot be connected into a couple, indicates that a structural edge is intersecting the gridface. Point of intersection of the edge with the gridface has to be found. To find the point, we follow the line of intersection of the interface with the gridface, starting at the unconnected point of intersection of the gridleg with the interface, and continuing inside the gridface. We can follow the above mentioned line by numerical integration of the equation

$$\frac{d\vec{x}}{ds} = \vec{t} \quad , \tag{1}$$

where $\vec{x}$ is a point of the line and $\vec{t}$ is the unit tangent vector. In each step of the numerical integration, we check whether the point of intersection of the edge with the gridface has been crossed. If yes, we can find the point of intersection by the same procedure used in ray tracing for finding the intersection of a ray with an interface.

After identification of the intersection point on the edge, we connect the point with the point from which we started the search for the edge. Then we proceed to the point beyond the edge. If this point cannot be connected with other intersection point of the same kind at the gridface, we must continue along other line of intersection, see figures 3 and 4.

d) Polygons approximating structural interfaces within gridcubes

For each gridcube, whose gridfaces contain some couples of intersection points, the couples are connected into polygons approximating the interface within the gridcube. If there are no points of intersection of edges at structural interfaces with the gridfaces, the polygons made of the couples should be closed. If there are the points of intersection with the edges, the polygons are open and should be closed along the lines which connect the points on the edges, see figure 2c.

e) Division of polygons into triangles

There are at least two ways of dividing a polygon into triangles. We may consider all polygon diagonals, select one of them according to some criterion, divide the polygon into two smaller polygons, etc. In the simplified approach, we consider just the diagonals separating a triangle from the rest of the polygon. Although the first way looks much more general, any division of a polygon into triangles may be achieved also in the second way. Since the second approach considerably simplifies the control of the created triangles, we choose it.

First we must determine the handedness of the polygon and separate only right–handed triangles from a right–handed polygon or only left–handed triangles from a left–handed polygon. At each vertex $\mathbf{x}^B$ we calculate angle α^B by which the sides turn in the vertex with respect to the normal,

$$r\sin(\alpha^B) = \varepsilon_{ijk}(x_i^B - x_i^A)(x_j^C - x_j^B)n_k^B \ , \quad r\cos(\alpha^B) = (x_i^B - x_i^A)(x_i^C - x_i^B) \ , \tag{2}$$

where $r = |\mathbf{x}^B - \mathbf{x}^A||\mathbf{x}^C - \mathbf{x}^B|$ need not be calculated. Here ε_{ijk} is the Levi–Civita symbol. We separate only the triangles adjacent to vertices $\mathbf{x}^B$ at which α^B has the same sign as sum $\sum_B \alpha^B$ over all polygon vertices.

The current, but not good, criterion for the selection of the triangle to be separated is as follows: For each diagonal $\mathbf{x}^A\mathbf{x}^C$ separating triangle $\mathbf{x}^A\mathbf{x}^B\mathbf{x}^C$ from the polygon, we determine the plane given by the diagonal and the sum of normals $\mathbf{n}^A$ and $\mathbf{n}^C$. The normal to the plane is

$$\mathbf{z} = \frac{(\mathbf{x}^C - \mathbf{x}^A) \times (\mathbf{n}^C + \mathbf{n}^A)}{2|\mathbf{x}^C - \mathbf{x}^B|} \quad , \tag{3}$$

where $\times$ denotes the vector product. We then determine distances

$$d = z_i(x_i - x_i^A) \tag{4}$$

of polygon vertices from the plane. Among the acceptable diagonals, we select the diagonal for which the value of

$$\frac{|(n_i^C - n_i^A)(x_i^C - x_i^A)|}{\min(\max(-Hd), Hd^B)} \tag{5}$$

is minimal. Here $\max(-Hd)$ is taken over all polygon vertices $\mathbf{x}$ other than $\mathbf{x}^A$, $\mathbf{x}^B$, $\mathbf{x}^C$, $H = \mathrm{sgn}(\sum_B \alpha^B)$ is $+1$ for a right–handed polygon and -1 for a left–handed polygon.

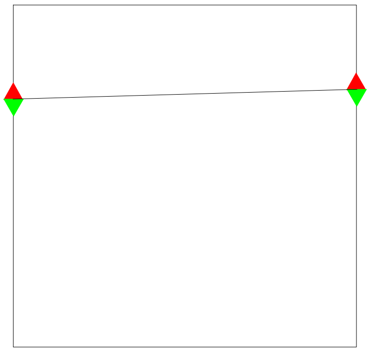

Figure 5a: A gridface with two points of intersection of gridlegs with a structural interface, the points are connected into a couple. Assume that a 2-D crossection is to be determined.

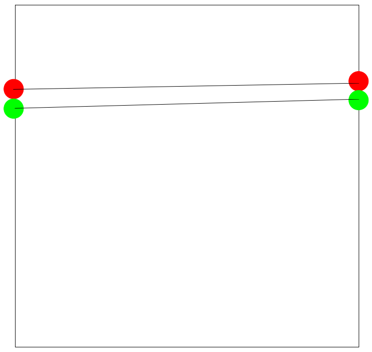

Figure 5b: Each of the two points at the interface is doubled into two points located at opposite sides of the interface. Analogously, the couple of intersection points is doubled into two couples of points approximating both sides of the interface.

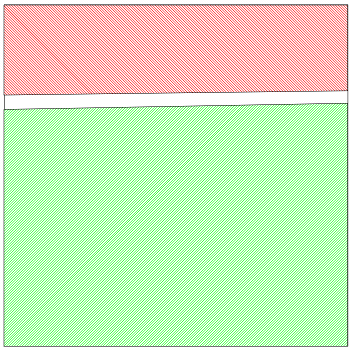

Figure 5c: The couples from figure 5b together with corresponding parts of the gridlegs form the sides of the two polygons, which approximate the 2-D crossection inside the gridface.

3 Triangulating 2-D sections

The above described algorithm for the triangulation of structural interfaces may be easily modified into the algorithm for triangulation of 2-D sections across the model. If the prescribed grid is two-dimensional, steps a) to c) are performed in the same way as described above. Instead of connecting the couples into polygons, different step d) is performed for each gridface containing couples of connected intersection points: Each couple situated at the interface is doubled into two couples, the first couple lying at one side and the second couple at the other side of the interface. Polygons approximating the 2-D section are then formed of the couples and of the relevant parts of the gridlegs, see figures 5 and 6. Final step e), decomposition of the polygons into triangles, is similar to the procedure described in section 2 subsection e). The polygons at 2-D crossections are planar, but need not be convex.

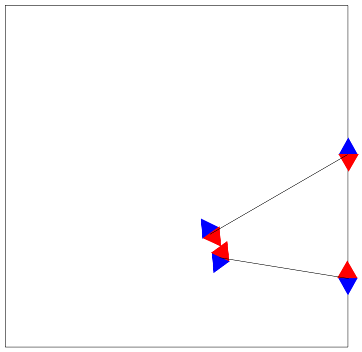

Figure 6a: A gridface with two couples of intersection points. Again, a 2-D crossection is to be determined.

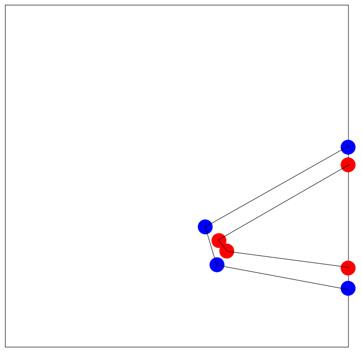

Figure 6b: Couples of intersection points are doubled into the couples approximating both sides of each of the two interfaces, as in figure 5b. Two more couples must be created at the edge in order to close the polygons approximating the 2-D crossection.

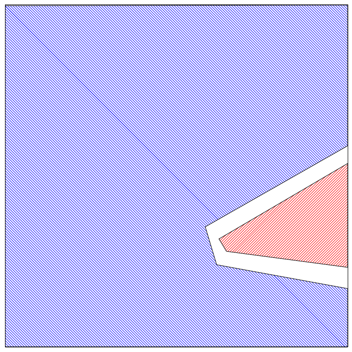

Figure 6c: Two polygons approximating the 2-D crossection inside the gridface. One of the polygons is not convex, which must be taken into account when dividing the polygon into triangles.

4 Examples

Triangulation of structural interfaces of the model with lenticular inclusion is shown in figure 7. Triangulation of 2-D crossections is shown in figure 8.

5 Conclusions

An algorithm for triangulation of structural interfaces defined by implicit functions has been proposed. The model volume is split into gridcubes corresponding to a given rectangular 3-D grid. The intersections of structural interfaces with the gridcubes are approximated by polygons, formed by points located at the gridlegs and at the gridfaces. The polygons are then divided into triangles.

Since the structural interfaces are approximated by polygons within the gridcubes, the proper density of the grid is required to provide the desired triangulation of the interfaces. Details smaller then the gridcubes remain hidden.

If the specified grid is planar, the triangulated two-dimensional crossection across the model is obtained.

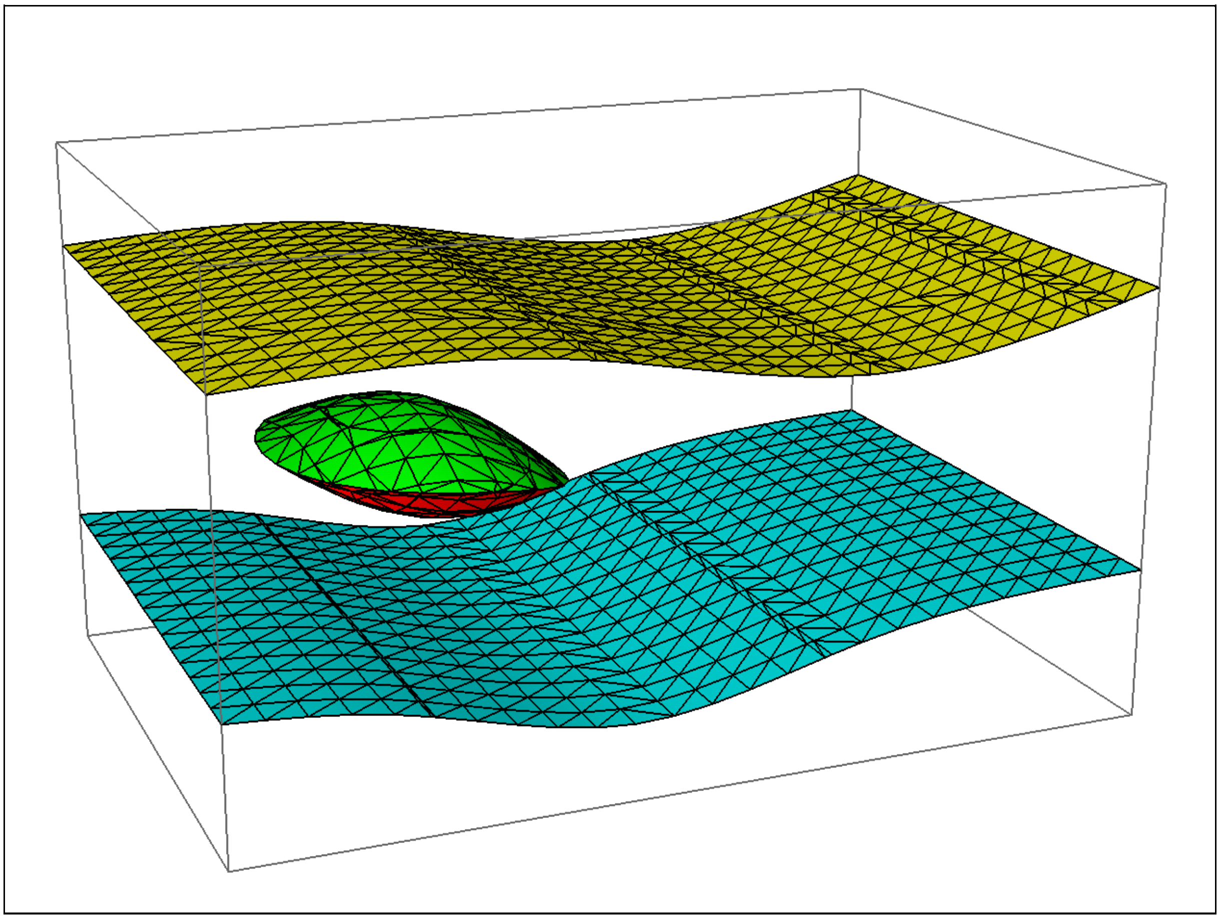

Figure 7: Triangulation of structural interfaces in the model with lenticular inclusion. The interfaces are coloured according to the indices of functions defining the interfaces.

References

Červený, V., Klimeš, L. & Pšenčík, I. (1988): Complete seismic–ray tracing in three–dimensional structures. In: Doornbos, D.J. (ed.), Seismological Algorithms, pp. 89–168, Academic Press, New York.

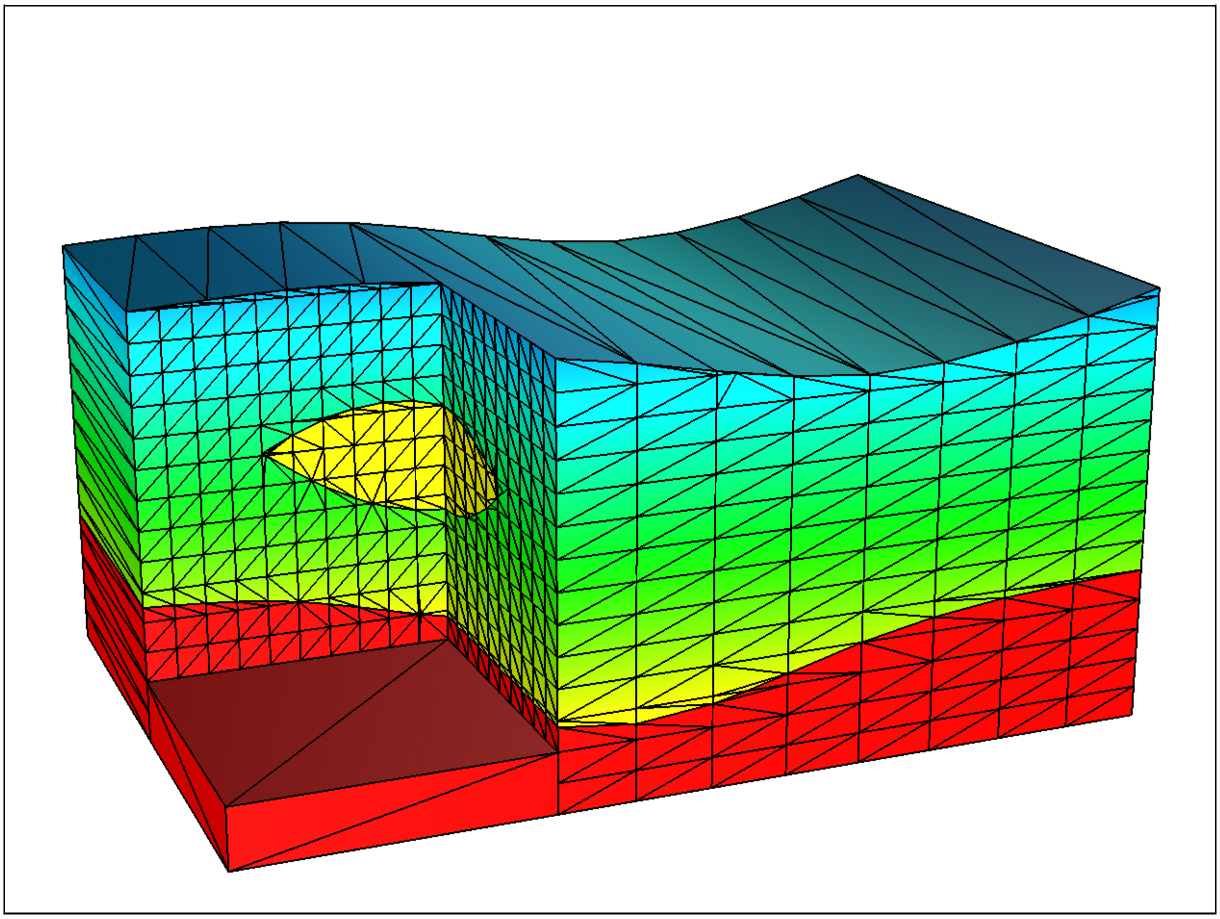

Figure 8: Triangulation of structural interfaces and 2-D crossections across the model with lenticular inclusion. The interfaces and the crossections are coloured according to the value of P-wave velocity.