Properties of weak contrast PP reflection/transmission coefficients for weakly anisotropic elastic media [1]

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Abstract

Approximate formulae for PP reflection/transmission (R/T) displacement coefficients for weak contrast interfaces separating weakly anisotropic media of an arbitrary symmetry are presented. The coefficients have a form of a sum of a well-known approximate PP reflection/transmission coefficient for weak contrast interfaces separating two background isotropic halfspaces and a correction due to weak anisotropy. The correction is controlled, linearly, by the so-called weak anisotropy (WA) parameters. The coefficients are defined with respect to an arbitrary isotropic background. The coefficients can describe reflection and transmission not only in low symmetry weakly anisotropic media but also in media with higher symmetry (orthorhombic, hexagonal) but with arbitrarily oriented axes of symmetry. For anisotropies of higher symmetry, for which approximate formulae for R coefficients of other authors exist, we discuss the differences between these and our formulae and their effects on accuracy of approximate coefficients. Performance of the approximate formulae for reflection coefficients is tested on models with weak anisotropy and weak contrast interfaces as well as on models whose anisotropy and velocity contrast are in no way weak. Even in the latter case, the formulae yield satisfactory results. Accuracy of the approximate R_{PP} coefficient in dependence on the choice of the background is then investigated. Presented formulae are convenient for solving an inverse problem: determination of contrasts of parameters of anisotropic media surrounding an interface. Sensitivity of the approximate R_{PP} coefficient to basic weak anisotropy (WA) parameters is presented.

1. Introduction

Study of relation of the reflection (R) or, possibly, transmission (T) coefficients to parameters of the media surrounding the reflector is a basic part of the amplitude-versus-offset (AVO) or amplitude-versus-azimuth (AVA) analysis. In case of a reflector surrounded by anisotropic media, formulae for the R/T coefficients are rather complicated if their analytic form exists at all. They can be simplified substantially if considered anisotropy and the

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velocity contrast across the reflector are weak. Since this is a case in many practical situations, derivation of simplified and transparent formulae for the R/T coefficients attracted attention of many seismologists. For a brief history of the problem see, e.g., Rüger and Tsvankin (1997), Vavryčuk and Pšenčík (1998).

Vavryčuk and Pšenčík (1998) and Pšenčík and Vavryčuk (1998) derived approximate formulae for PP R/T coefficients for weak contrast interfaces separating weakly anisotropic media of arbitrary symmetry. In final formulae, they considered a medium with P and S wave velocities $\alpha^2 = A_{33}$ and $\beta^2 = A_{55}$ as an isotropic background in each of the halfspaces surrounding the reflector. Symbols $A_{\alpha\beta}$ denote density normalized elastic parameters in Voigt notation. The above assumption concerning the background reduces applicability of the derived formulae. The formulae, which we present in this contribution allow to use arbitrary values of α and β for the background media. Due to the arbitrary background, the formulae contain new WA parameters (WA parameters are generalizations of parameters introduced by Thomsen, 1986), important for characterization of the medium. The formulae can be used even in cases when parameters characterizing vertical propagation, A_{33}, A_{44} or A_{55} are not known or are known with low precision. In fact, the presented formulae can be used for retrieving these parameters from the R/T coefficients in the form of newly introduced WA parameters ϵ_z (related to A_{33}), γ_x (related to A_{55}) or γ_y (related to A_{44}). Introduction of an arbitrary background makes determination of a WA parameter related to shear wave propagation possible even in VTI media.

With exception of definition of two WA parameters, the presented formulae for PP reflection coefficients reduce to formulae derived by Rüger (1997, 1998) for symmetry planes of anisotropic media of VTI, HTI or orthorhombic symmetry. We discuss the role of the above mentioned two WA parameters different from Rüger's definition and show how a different definition can lead to a substantial decrease of accuracy of the approximate formulae. We illustrate this on two typical models proposed by Rüger (1997). We then calculate approximate reflection coefficients for angles of incidence from 0^o to 40^o and for all azimuths. By comparing them with exact coefficients, we show an excellent performance of the presented formulae. Anisotropy and velocity contrast are around 10% in this case. In order to show performance of the formulae in more complicated models with stronger anisotropy and velocity contrast, we calculate R_{PP} coefficients for angles of incidence between 10^o and 30^o and for all azimuths in a model consisting of a VTI overburden over an orthorhombic reflecting medium. Anisotropy is about 25% and a minimum velocity contrast 13% in this case. Even in such an extreme case, the approximate formula yields results of a reasonable accuracy within the considered interval of angles of incidence. Finally, we use again Rüger's models to investigate influence of the choice of the background on the accuracy of the approximate R_{PP} coefficients.

The accuracy and the form of presented formulae (sum of an approximate R/T coefficient for weak contrast background isotropic media and a term depending linearly on WA parameters) offer a straightforward application. The formulae can be directly used for an inversion of observed coefficients into WA parameters, see e.g. Vavryčuk (1999). For this purpose, it is good to know the sensitivity of the formulae to WA parameters. We present maps of sensitivity of the PP reflection coefficients to most important WA parameters or their combinations in order to indicate which parameters and with which reliability can be found by inverting an R_{PP} coefficient.

2. Weak contrast PP reflection coefficient for weak anisotropy of arbitraty symmetry

The approximate formula for the PP reflection coefficient for a weak contrast interface separating two homogeneous weakly anisotropic halfspaces of arbitrary symmetry, expressed with respect to an arbitrary background, has the following form

$$R_{PP}(\varphi,\theta) = R^{iso}_{PP}(\theta) + \frac{1}{2}\Delta\epsilon_z + \frac{1}{2}[(\Delta\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_x)\cos^2\varphi + (\Delta\delta_y - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_y)\sin^2\varphi$$
$$+2(\Delta\chi_z - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\epsilon_{45})\cos\varphi\sin\varphi - \Delta\epsilon_z]\sin^2\theta + \frac{1}{2}[\Delta\epsilon_x\cos^4\varphi + \Delta\epsilon_y\sin^4\varphi$$
$$+\Delta\delta_z\cos^2\varphi\sin^2\varphi + 2(\Delta\epsilon_{16}\cos^2\varphi + \Delta\epsilon_{26}\sin^2\varphi)\sin\varphi\cos\varphi]\sin^2\theta\tan^2\theta. \tag{1}$$

Here $R^{iso}_{PP}(\theta)$ denotes the weak contrast PP reflection coefficient for isotropic media

$$R^{iso}_{PP}(\theta) = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}[\frac{\Delta\alpha}{\bar{\alpha}} - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\frac{\Delta G}{\bar{G}}]\sin^2\theta + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\sin^2\theta\tan^2\theta, \tag{2}$$

where

$$Z = \rho\alpha, \quad G = \rho\beta^2. \tag{3}$$

The symbols Δ denote contrast across an interface, e.g. $\Delta Z = Z_2 - Z_1$. Bar over a symbol denotes an average, e.g. $\bar{Z} = \frac{1}{2}(Z_1 + Z_2)$. The index 1 corresponds to the upper halfspace, the index 2 to the lower halfspace. The symbol θ denotes an angle of incidence ($\theta = 0^o$ for a vertical incidence) and the symbol φ an azimuth ($\varphi = 0^o$ for direction along x axis). The reflection coefficient (1) depends on nine of the following fifteen P wave WA parameters, see Pšenčík and Gajewski (1998)

$$\delta_x = \frac{A_{13} + 2A_{55} - \alpha^2}{\alpha^2}, \quad \delta_y = \frac{A_{23} + 2A_{44} - \alpha^2}{\alpha^2}, \quad \delta_z = \frac{A_{12} + 2A_{66} - \alpha^2}{\alpha^2},$$

$$\chi_x = \frac{A_{14} + 2A_{56}}{\alpha^2}, \quad \chi_y = \frac{A_{25} + 2A_{46}}{\alpha^2}, \quad \chi_z = \frac{A_{36} + 2A_{45}}{\alpha^2},$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2},$$

$$\epsilon_x = \frac{A_{11} - \alpha^2}{2\alpha^2}, \quad \epsilon_y = \frac{A_{22} - \alpha^2}{2\alpha^2}, \quad \epsilon_z = \frac{A_{33} - \alpha^2}{2\alpha^2} \tag{4}$$

and on three S wave WA parameters

$$\gamma_x = \frac{A_{55} - \beta^2}{2\beta^2}, \quad \gamma_y = \frac{A_{44} - \beta^2}{2\beta^2}, \quad \epsilon_{45} = \frac{A_{45}}{\beta^2}. \tag{5}$$

Here α and β are P and S wave velocities of the background isotropic medium, which can be chosen arbitrarily (for example so that one or two of the WA parameters are made zero).

For orthorhombic media with symmetry axes coinciding with coordinate axes in both halfspaces, we have

$$\chi_x = \chi_y = \chi_z = \epsilon_{15} = \epsilon_{16} = \epsilon_{24} = \epsilon_{26} = \epsilon_{34} = \epsilon_{35} = \epsilon_{45} = 0. \tag{6}$$

For such a specification, Eq.(1) reduces to a simple form

$$R_{PP}(\varphi,\theta) = R_{PP}^{iso}(\theta) + \frac{1}{2}\Delta\epsilon_z + \frac{1}{2}[(\Delta\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_x)\cos^2\varphi + (\Delta\delta_y - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_y)\sin^2\varphi - \Delta\epsilon_z]\sin^2\theta$$

$$+\frac{1}{2}[\Delta\epsilon_x\cos^4\varphi + \Delta\epsilon_y\sin^4\varphi + \Delta\delta_z\cos^2\varphi\sin^2\varphi]\sin^2\theta\tan^2\theta. \tag{7}$$

If we specify the formula (7) for the plane (x,z), i.e. for $\varphi = 0^o$, and for the plane (y,z), i.e. for $\varphi = 90^o$, and if we set $\alpha^2 = A_{33}$ and $\beta^2 = A_{55}$, the results can be compared with corresponding formulae (22) and (24) of Rüger (1998). It can be easily shown that the formulae coincide, only definitions of the WA parameter δ differ. Rüger (1998) uses the definition of δ as introduced by Thomsen (1986)

$$\delta = \frac{(A_{13} + A_{55})^2 - (A_{33} - A_{55})^2}{2A_{33}(A_{33} - A_{55})}, \tag{8a}$$

while Eqs.(4) yield

$$\delta_x = \frac{A_{13} + 2A_{55} - A_{33}}{A_{33}}. \tag{8b}$$

As shown by Sayers (1994), the difference between the two δ's is of second order. Thomsen's δ can be approximately expressed in terms of δ_x in the following way

$$\delta \sim \delta_x + \frac{1}{2}\frac{\delta_x^2}{\alpha^2 - \beta^2}. \tag{9}$$

We can see that Thomsen's δ is always larger than δ_x. By neglecting the second order term in Eq.(9), we make an error of roughly $10\delta_x\%$ if we assume $\alpha^2 \sim 3\beta^2$ and $\beta^2 \sim 2.0$. Thus substitution of δ by δ_x for $\delta \sim \pm 0.1$ leads to differences of about $\pm 1\%$.

In his Eq.(23), Rüger (1998) expresses the approximate R_{PP} coefficient in the plane (x,z) in terms of the shear wave splitting parameter γ. His γ is defined as

$$\gamma = \frac{A_{44} - A_{55}}{2A_{55}}, \tag{10}$$

while our Eqs.(5) yield for Rüger's specification $\beta^2 = A_{44}$

$$\gamma_x = \frac{A_{55} - A_{44}}{2A_{44}}. \tag{11}$$

Rüger's γ expressed in terms of γ_x reads

$$\gamma \sim -\gamma_x + 2\gamma_x^2. \tag{12}$$

We can see, that neglecting the second order term in (12) can lead to an error of roughly $200\gamma_x\%$. For $\gamma_x \sim 0.1$, substitution of γ_x by γ leads to an error substantially larger than in substituting δ_x for Thomsen's δ. The use of γ defined in (10) in the approximate formula for PP wave reflection coefficient causes "reduced numerical accuracy" reported by Rüger (1998). Use of γ_x and γ_y defined in Eqs.(5) in Eq.(7) removes the above mentioned problem.

For an isotropic upper halfspace, which we consider in some of the following examples, the contrasts of the WA parameters reduce to the parameters of the lower halfspace: $\Delta\delta_x \to \delta_x^{(2)}$, $\Delta\epsilon_x \to \epsilon_x^{(2)}$, etc. If the lower halfspace is transversely isotropic with vertical axis of symmetry (VTI), the WA parameters have, in addition to (6), the following symmetries

$$\delta_y = \delta_x, \quad \delta_z = 2\epsilon_x, \quad \epsilon_y = \epsilon_x, \quad \gamma_y = \gamma_x. \tag{13}$$

In this case, Eq.(1) for the weak contrast reflection coefficient attains the form:

$$R_{PP}(\theta) = R_{PP}^{iso}(\theta) + \frac{1}{2}\epsilon_z + \frac{1}{2}[(\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\gamma_x) - \epsilon_z]\sin^2\theta + \frac{1}{2}\epsilon_x \sin^2\theta \tan^2\theta. \tag{14}$$

The symbol $R_{PP}^{iso}(\theta)$ denotes again the weak contrast PP reflection coefficient for isotropic media (2).

We can see that the coefficient (14) differs from the coefficient derived for the same model by Thomsen (1993). There are two WA parameters, ϵ_z and γ_x, which do not appear in Thomsen's formula. The reason for appearance of these parameters is that Thomsen (1993) considers background velocities specified as $\alpha^2 = A_{33}$, $\beta^2 = A_{55}$. This leads to $\epsilon_z = 0$ and $\gamma_x = 0$ and thus to reduction of (14) to Thomsen's formula. Another slight difference consists in a different definition of Thomsen's δ (see Eq.(8a)) and our δ_x (see Eq.(8b)), which has been discussed above. From comparison of formula (14) with Thomsen's (1993) formula, we can see advantage of the use of an arbitrary background. Eq.(14) allows, in principle, determination of ϵ_z, and thus of A_{33}, and of γ_x, and thus of A_{55}, even in VTI media.

For an isotropic halfspace over a halfspace with transversely isotropic medium with horizontal axis of symmetry (HTI), parallel to the axis x, the WA parameters of the lower halfspace satisfy, in addition to (6), the following relations

$$\delta_y = 2\epsilon_z, \quad \delta_z = \delta_x, \quad \epsilon_y = \epsilon_z. \tag{15}$$

Eq.(1) then yields

$$R_{PP}(\varphi,\theta) = R_{PP}^{iso}(\theta) + \frac{1}{2}\epsilon_z + \frac{1}{2}[(\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\gamma_x)\cos^2\varphi + (2\epsilon_z - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\gamma_y)\sin^2\varphi - \epsilon_z]\sin^2\theta$$

$$+\frac{1}{2}(\epsilon_x \cos^4\varphi + \epsilon_z \sin^4\varphi + \delta_x \sin^2\varphi\cos^2\varphi)\sin^2\theta\tan^2\theta. \tag{16}$$

Comparison of Eq.(16) with Eq.(5) of Rüger (1998) leads to a similar discussion as that after Eq.(7).

We have shown the previous specifications of Eq.(1) only for comparison with formulae derived for special situations by other authors. In applications, use of the formula (1) is preferable.

3 Weak contrast PP transmission coefficient for weak anisotropy of arbitrary symmetry

The approximate formula for the PP transmission coefficient for a weak contrast interface separating weakly anisotropic media of arbitrary symmetry, expressed with respect to an arbitrary background, has the following form

$$\begin{aligned}
T_{PP}(\varphi,\theta) = T^{iso}_{PP}(\theta) - \frac{1}{2}\Delta\epsilon_z + \frac{1}{2}[\Delta\delta_x\cos^2\varphi + \Delta\delta_y\sin^2\varphi + 2\Delta\chi_z\cos\varphi\sin\varphi - \Delta\epsilon_z]\sin^2\theta \\
+2(\Delta\epsilon_{35}\cos\varphi + \Delta\epsilon_{34}\sin\varphi - \Delta\chi_x\cos^2\varphi\sin\varphi - \Delta\chi_y\cos\varphi\sin^2\varphi \\
-\Delta\epsilon_{15}\cos^3\varphi - \Delta\epsilon_{24}\sin^3\varphi)\sin^3\theta\cos\theta + \frac{1}{2}[\Delta\epsilon_x\cos^4\varphi + \Delta\epsilon_y\sin^4\varphi \\
+\Delta\delta_z\cos^2\varphi\sin^2\varphi + 2(\Delta\epsilon_{16}\cos^2\varphi + \Delta\epsilon_{26}\sin^2\varphi)\sin\varphi\cos\varphi]\sin^2\theta\tan^2\theta \\
+[\Delta\epsilon_x\cos^4\varphi + \Delta\epsilon_y\sin^4\varphi + \Delta\delta_z\cos^2\varphi\sin^2\varphi + 2(\Delta\epsilon_{16}\cos^2\varphi + \Delta\epsilon_{26}\sin^2\varphi)\sin\varphi\cos\varphi \\
-\Delta\delta_x\cos^2\varphi - \Delta\delta_y\sin^2\varphi - 2\Delta\chi_z\cos\varphi\sin\varphi + \Delta\epsilon_z]\sin^4\theta.
\end{aligned} \tag{17}$$

Here $T^{iso}_{PP}(\theta)$ denotes the weak contrast PP transmission coefficient for isotropic media

$$T^{iso}_{PP}(\theta) = 1 - \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\tan^2\theta. \tag{18}$$

The approximate formula (17) for the transmission coefficient depends on all 15 P wave WA parameters. It, however, does depend on neither of parameters related to shear wave propagation, see Eqs.(5). Thus, from the transmission coefficient all the parameters necessary for the determination of the P wave phase velocity, see Eq.(17a) of Pšenčík and Gajewski (1998), can be, in principle, recovered. On the other hand, no estimates of parameters related to shear wave propagation can be made.

For orthorhombic media with symmetry axes coinciding with coordinate axes in both halfspaces, Eqs.(6) hold and Eq.(17) reduces to

$$\begin{aligned}
T_{PP}(\varphi,\theta) = T^{iso}_{PP}(\theta) - \frac{1}{2}\Delta\epsilon_z + \frac{1}{2}[\Delta\delta_x\cos^2\varphi + \Delta\delta_y\sin^2\varphi - \Delta\epsilon_z]\sin^2\theta \\
+\frac{1}{2}[\Delta\epsilon_x\cos^4\varphi + \Delta\epsilon_y\sin^4\varphi + \Delta\delta_z\cos^2\varphi\sin^2\varphi]\sin^2\theta\tan^2\theta \\
+[\Delta\epsilon_x\cos^4\varphi + \Delta\epsilon_y\sin^4\varphi + \Delta\delta_z\cos^2\varphi\sin^2\varphi - \Delta\delta_x\cos^2\varphi - \Delta\delta_y\sin^2\varphi + \Delta\epsilon_z]\sin^4\theta.
\end{aligned} \tag{19}$$

Again, all six WA parameters necessary for the determination of the P wave phase velocity from Eq.(19) of Pšenčík and Gajewski (1998) can be, in principle, recovered from Eq.(19).

For an isotropic upper halfspace and transversely isotropic lower halfspace with vertical axis of symmetry (VTI), i.e. for the WA parameters specified as in Eqs.(6) and (13), formula (17) reduces substantially. It reads

$$T_{PP}(\varphi,\theta)=T_{PP}^{iso}(\theta)-\frac{1}{2}\epsilon_z+\frac{1}{2}(\delta_x-\epsilon_z)\sin^2\theta+\frac{1}{2}\epsilon_x\sin^2\theta\tan^2\theta+(\epsilon_x+\epsilon_z-\delta_x)\sin^4\theta. \quad (20)$$

For an isotropic upper halspace over a halfspace with transversely isotropic medium with horizontal axis of symmetry, parallel to the axis x, i.e. for the WA parameters specified as in Eqs.(6) and (15), the formula (17) reduces to

$$T_{PP}(\varphi,\theta)=T_{PP}^{iso}(\theta)-\frac{1}{2}\epsilon_z+\frac{1}{2}(\delta_x\cos^2\varphi-\epsilon_z\cos 2\varphi)\sin^2\theta$$

$$+\frac{1}{2}(\epsilon_x\cos^4\varphi+\epsilon_z\sin^4\varphi+\delta_x\sin^2\varphi\cos^2\varphi)\sin^2\theta\tan^2\theta+(\epsilon_x+\epsilon_z-\delta_x)\cos^4\varphi\sin^4\theta. \quad (21)$$

4 Accuracy of approximate formulae

In this section, we illustrate accuracy of the formulae for approximate reflection coefficients presented in a previous section. For this purpose, we use models 6 and 11 of Rüger (1997). From the contrasts of P wave velocity, impedance and shear wave modulus and from Thomsen's parameters specified in Rüger (1997), we reconstructed elastic parameters of his models. The isotropic material in the upper halfspace of model 6 is specified by α =2.26 km/s, β =1.43 km/s and ρ =2.70 g/cm^3, in model 11 by α =2.76 km/s, β =1.58 km/s and ρ =2.70 g/cm^3. The lower halfspace is the same in both cases and it is specified

by the following matrix of density normalized elastic parameters (in km^2/s^2) of an HTI medium

$$\begin{pmatrix} 5.00 & 1.82 & 1.82 & 0.00 & 0.00 & 0.00 \\ & 6.25 & 1.75 & 0.00 & 0.00 & 0.00 \\ & & 6.25 & 0.00 & 0.00 & 0.00 \\ & & & 2.25 & 0.00 & 0.00 \\ & & & & 1.87 & 0.00 \\ & & & & & 1.87 \end{pmatrix}.$$

The axis of symmetry of the HTI medium is parallel to the x-axis and corresponds to azimuth of 0^o. P wave anisotropy is less than 10%. P wave velocity contrast is about 10% for vertical incidence. In the symmetry plane (plane (x,z)) for model 6, the contrast decreases with increasing angle of incidence, in model 11, the contrast increases. Thus we should expect better fit of approximate (Eq.(1)) and exact coefficients in the symmetry plane in model 6 than in model 11. The contrast remains constant in the isotropy planes (plane (y,z)).

In Figure 1 we show a comparison of exact and approximate reflection coefficients in the symmetry plane (0^o) and in the isotropy plane (90^o) for model 6 (upper plot) and model 11 (bottom plot). Since the approximation works only in the subcritical region, we deal with real-valued coefficients. The approximate coefficients are calculated using Eq.(1) with the WA parameter γ_x defined successively as in Eqs.(11) and (10).

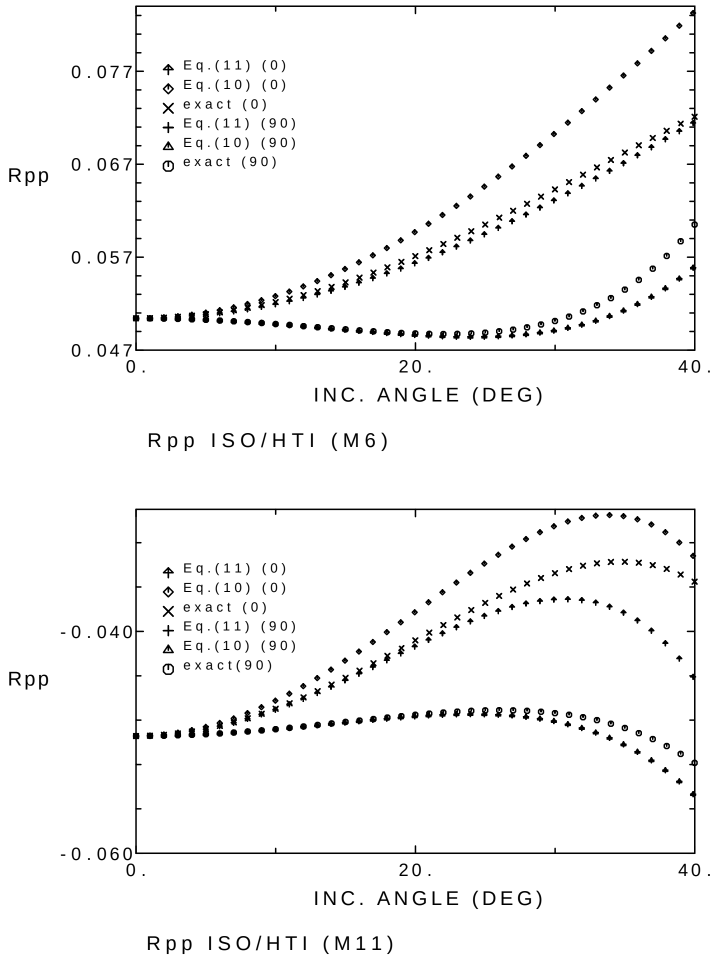

Figure 1. Comparison of exact and approximate R_{PP} coefficients in model 6 (upper plot) and model 11 (bottom plot), in the symmetry plane (0) and in the isotropy plane (90). The approximate results are obtained with γ parameters defined in Eqs.(10) and (11). Specification by Eq.(10) leads to considerable inaccuracies in the symmetry plane.

The former specification corresponds to the definition of the WA parameters introduced in this paper, the latter specification corresponds to Rüger's (1997, 1998) definition. Since in the isotropy plane, the approximate formula does not depend on the WA parameter γ_x, the curves calculated according to Eqs.(10) and (11) coincide. In the symmetry plane, however, we can observe considerable differences between curves calculated according to Eqs.(10) and (11). The curves with γ_x specified by Eq.(11) yield very good approximation

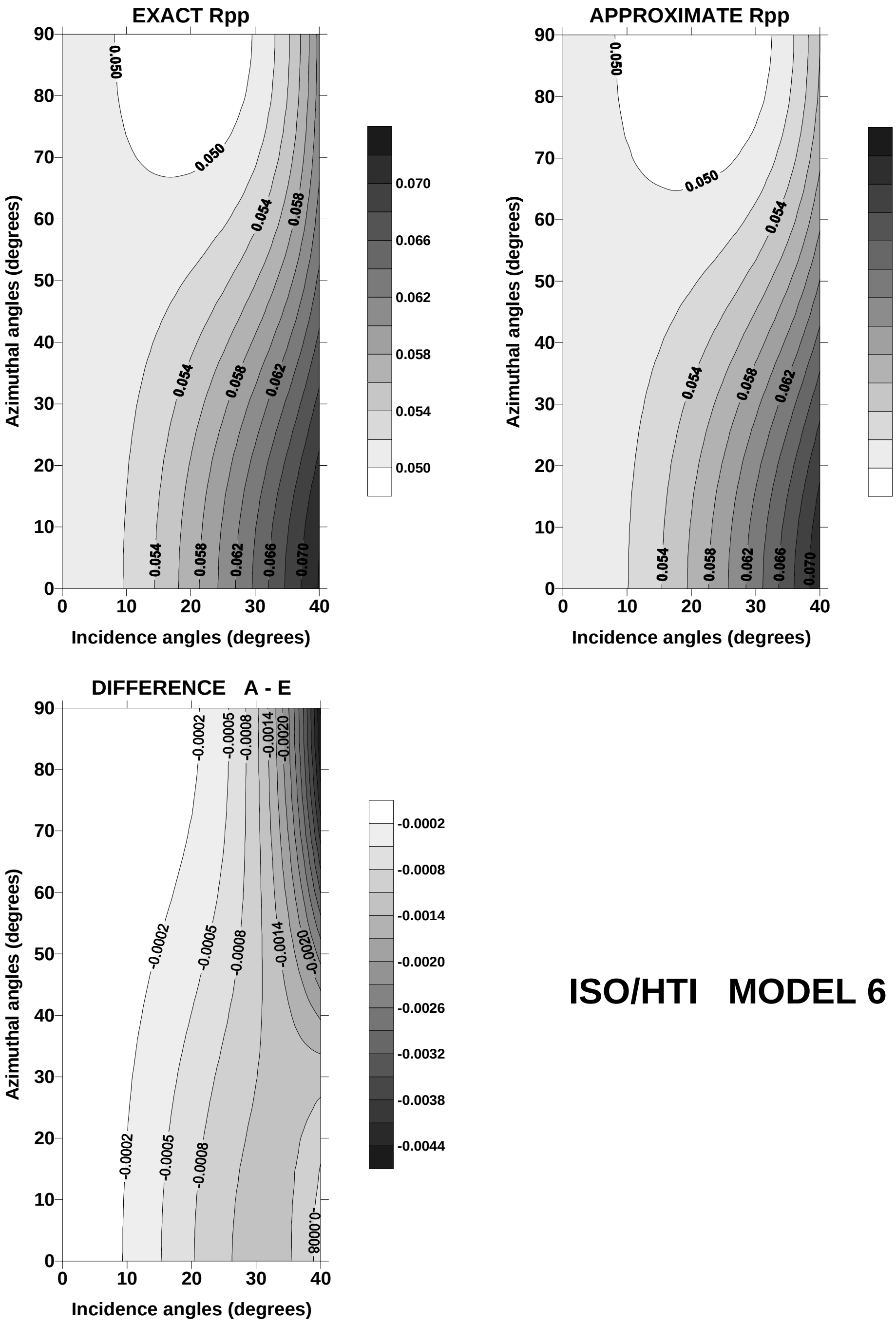

Figure 2. The maps of exact and approximate PP reflection coefficients (top) and of differences between approximate and exact coefficients (bottom) in model 6. Background: $\alpha^2 = A_{33}$, $\beta^2 = A_{44}$.

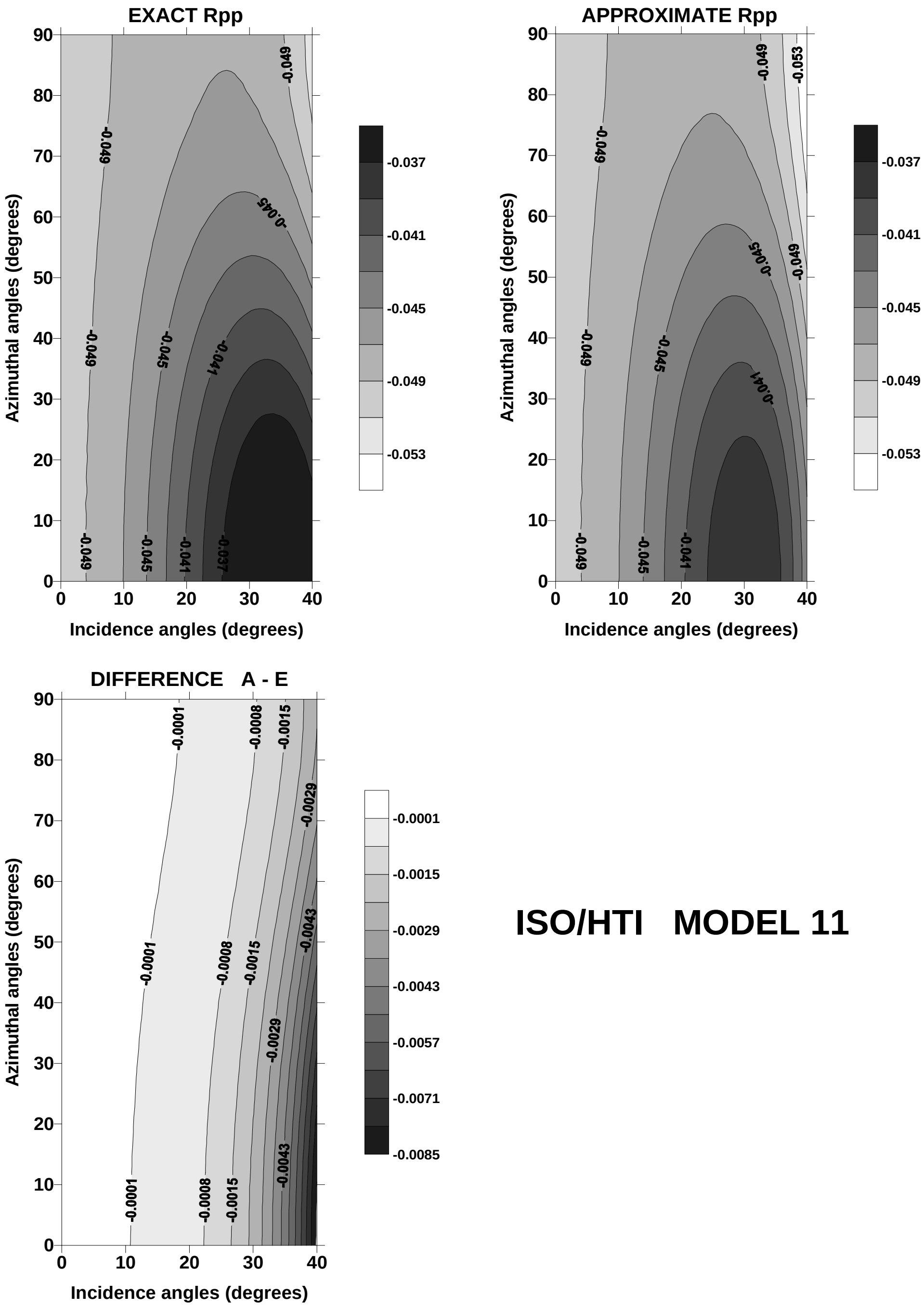

Figure 3. The maps of exact and approximate PP reflection coefficients (top) and of differences between approximate and exact coefficients in model 11. Background: $\alpha^2 = A_{33}$, $\beta^2 = A_{44}$.

of exact curves even for larger angles of incidence while the curves corresponding to γ_x specified by Eq.(10) start to deviate from exact ones already for small angles of incidence.

Figures 2 and 3 show that applicability of formula (1) for the R_{PP} coefficients is not limited to only planes of symmetry and isotropy. The figures show the coefficients for the same model as in Figure 1 as a function of angle of incidence (horizontal axes) and azimuth (vertical axes). As a background we have chosen $\alpha^2 = A_{33}$ and $\beta^2 = A_{44}$ as in Rüger (1997). This means that we did not choose the best background. The upper left figure shows exact results, the upper right figure shows results of the approximate formula. The lower left figure shows the difference between approximate and exact coefficients. The azimuths range from 0^o to 90^o and the angles of incidence from 0^o to 40^o. Because anisotropy of the models used is not too strong and the velocity contrast across the interface is mild, the figures show a very good performance of approximate formulae although the coefficients show relatively strong azimuthal variation.

In order to illustrate performance of the approximate formula (1) for a stronger anisotropy and stronger velocity contrast, we show comparison of exact and approximate calculations for a model proposed by Neves and de Hoop (1999) in Figure 4. Their model consists of two anisotropic halfspaces. The upper one is VTI and its matrix of density normalized elastic parameters (in km^2/s^2) has the form

$$\begin{pmatrix} 6.94 & 3.64 & 2.70 & 0.00 & 0.00 & 0.00 \\ & 6.94 & 2.70 & 0.00 & 0.00 & 0.00 \\ & & 4.28 & 0.00 & 0.00 & 0.00 \\ & & & 1.23 & 0.00 & 0.00 \\ & & & & 1.23 & 0.00 \\ & & & & & 1.65 \end{pmatrix} .$$

The density of the upper halfspace is 2.5 g/cm^3 and it represents a shale. The lower halfspace is orthorhombic, with the density of 2.2 g/cm^3 and with the matrix of density normalized elastic parameters of the form (in km^2/s^2)

$$\begin{pmatrix} 12.27 & 4.87 & 3.05 & 0.00 & 0.00 & 0.00 \\ & 13.44 & 3.31 & 0.00 & 0.00 & 0.00 \\ & & 8.10 & 0.00 & 0.00 & 0.00 \\ & & & 2.70 & 0.00 & 0.00 \\ & & & & 2.18 & 0.00 \\ & & & & & 2.97 \end{pmatrix} .$$

The lower halfspace represents a fractured sandstone. From both above matrices we can see that P wave anisotropy ($(v_{max} - v_{min})/v_{aver}$) in both halfspaces is rather strong (24% and 25%) so that none of the halfspaces can be called weakly anisotropic. The P wave velocity contrast is also rather strong (for vertical incidence, for which it is weakest, it reaches 13%). Indication of a strong velocity contrast is a rapid increase of values of the coefficient for angles of incidence close to 30^o, for azimuths greater than 60^o. It corresponds to a precritical rise of the coefficient. For such a model, we can expect only limited applica-

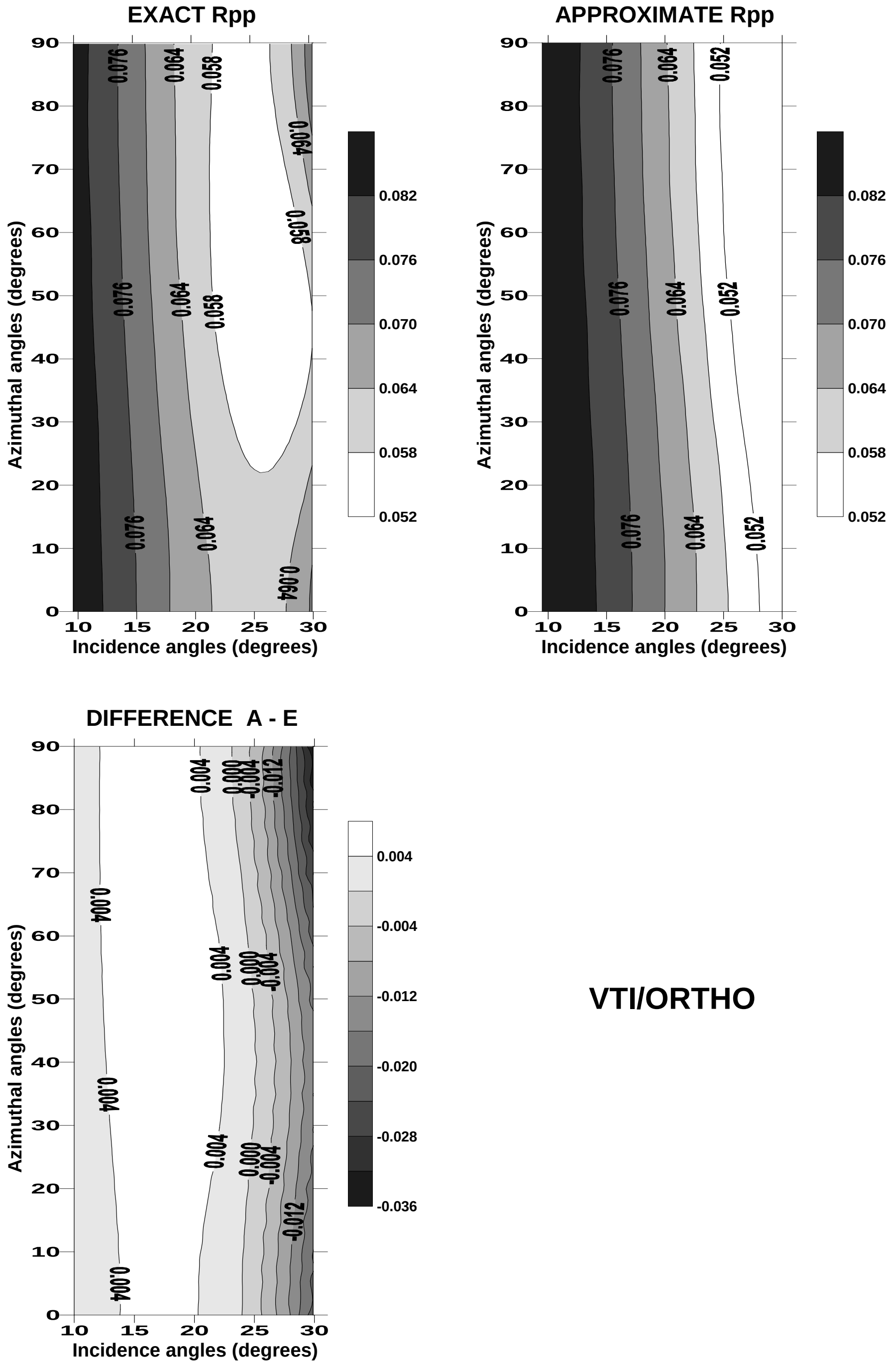

Figure 4. The maps of exact and approximate PP reflection coefficients (top) and of differences between approximate and exact coefficients (bottom) in model of a VTI halfspace over an orthorhombic halfspace. Background: $\alpha^2 = A_{33}$, $\beta^2 = A_{55}$.

bility of (1). The situation is complicated even more by a shear wave singularity in VTI medium for normal incidence, which complicates calculation of exact reflection coefficients.

Despite all these unfavorable effects, the approximate formula yields for small angles of incidence reasonably good approximation of exact results, see Figure 4. The azimuths range again from 0^o to 90^o and the angles of incidence from 10^o to 30^o. The range between 0^o and 10^o was omitted in order to avoid problems caused in calculation of exact coefficients in a vicinity of shear wave singularity.

5 Tests with various backgrounds

In this section, we illustrate influence of a choice of the isotropic background on the

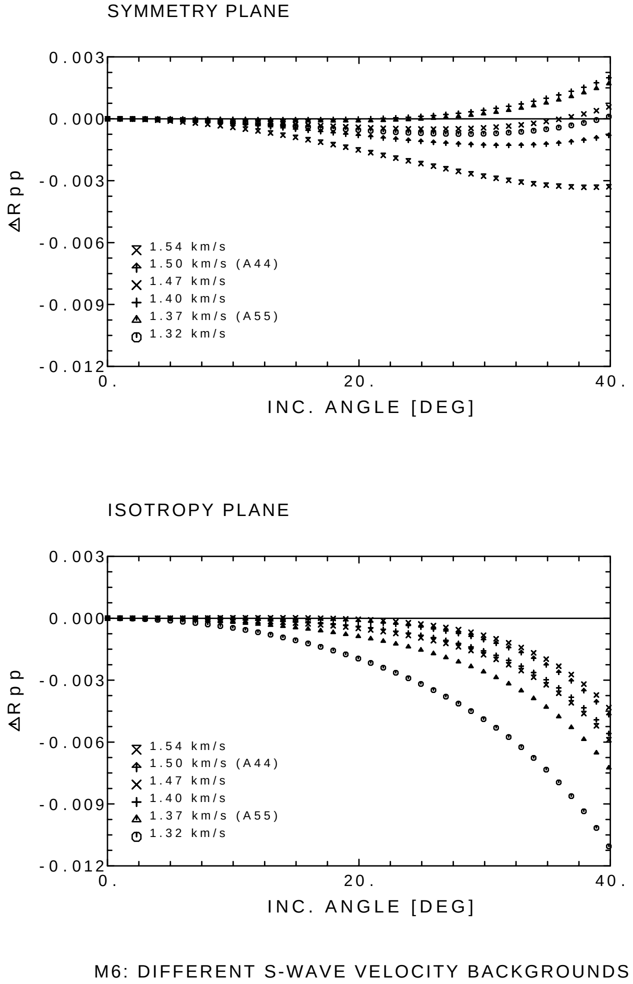

Figure 5. Deviations ΔR_{PP} of approximate R_{PP} coefficients from exact ones in model 6 in the symmetry plane (top) and in the isotropy plane (bottom). Individual curves correspond to different S wave velocities β of the background. P wave velocity is specified as $\alpha^2 = A_{33}$.

quality of the approximation of Eq.(1). For this purpose, we use again models 6 and 11 of

Rüger (1997). For both models, we calculated differences between approximate and exact R_{PP}coefficients for varying specification of background for angles of incidence between 0^o and 40^o, for the planes of symmetry and isotropy. In Figure 5 differences for the model 6 in the plane of symmetry (top) and in the plane of isotropy (bottom) are shown. Values of the R_{PP} coefficient vary from 0.05 to 0.075. The isotropic background in the reflecting medium was chosen as follows: $\alpha^2 = A_{33}$, i.e. $\alpha = 2.5$ km/s and β attained successively values of 1.32, 1.37, 1.40, 1.47, 1.50 and 1.54 km/s. The value of 1.37 km/s corresponds approximately to $(A_{55})^{1/2}$ and the value of 1.50 km/s to $(A_{44})^{1/2}$. We can see that the approximate formula works better in the symmetry plane than in the isotropy plane. For most backgrounds the errors remain below 10%. Neither A_{44} nor A_{55} yield the best approximation. Background A_{44} gives better results in the isotropy plane (A_{44} is connec-

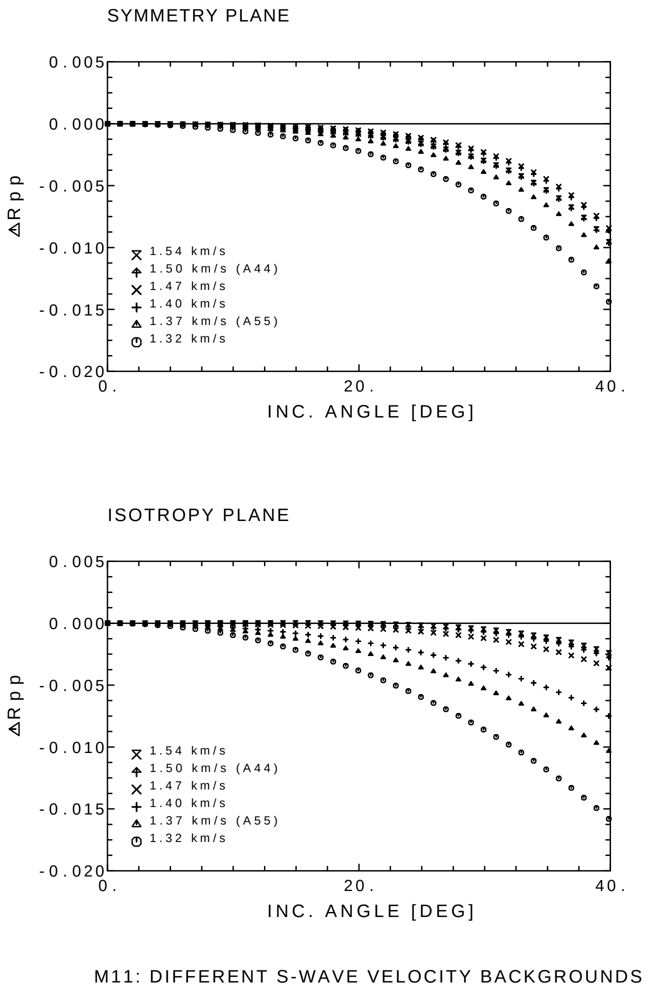

Figure 6. Deviations ΔR_{PP} of approximate R_{PP} coefficients from exact ones in model 11 in the symmetry plane (top) and in the isotropy plane (bottom). Individual curves correspond to different S wave velocities β of the background. P wave velocity is specified as $\alpha^2 = A_{33}$.

ted with SH wave propagation), background A_{55} in the symmetry plane (A_{55} is connected with SV wave propagation). The best approximation is obtained for values of β^2 between A_{44} and A_{55}.

Figure 6 shows results for the model 11 in the same display as in Figure 5. Due to the increasing contrast with increasing angle of incidence in the symmetry plane, the accuracy of the approximate coeffcient rapidly decreases. Because of the small coefficient of the higher angle term in Eq.(2), Eq.(1) yields now better approximation in the isotropy plane than in model 6. Values of the R_{PP} coefficient are now between -0.05 and -0.035. For larger angles of incidence, the approximate formula yields results with a considerable error. Upto 20^o, however, the errors are well below 10% for all the backgrounds.

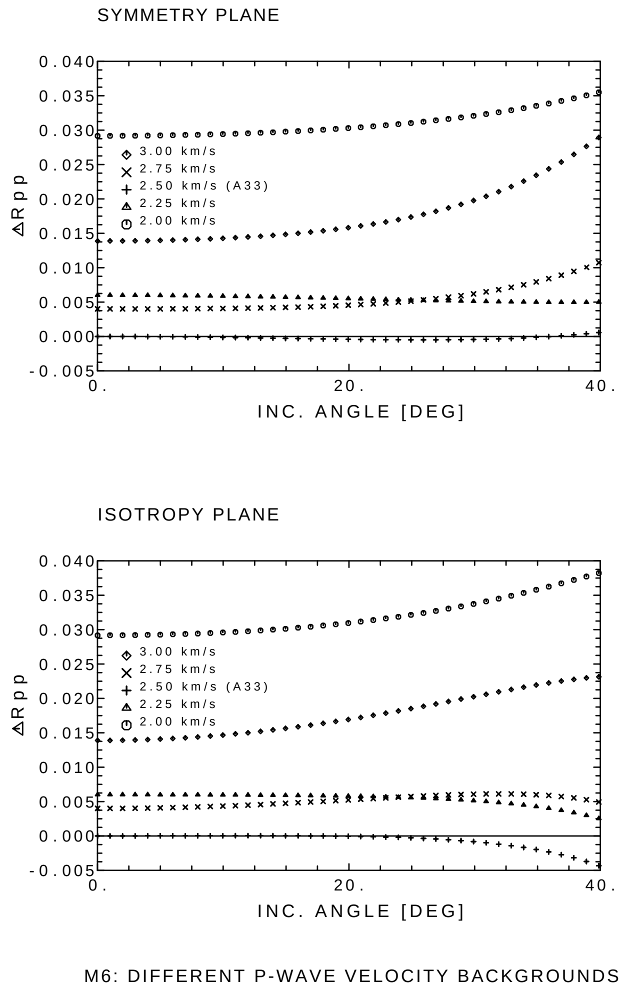

Figure 7. Deviations ΔR_{PP} of approximate R_{PP} coefficients from exact ones in model 6 in the symmetry plane (top) and in the isotropy plane (bottom). Individual curves correspond to different P wave velocities α of the background. S wave velocity is specified as $\beta = 1.47$ km/s.

The above two figures seem to indicate that the best choice of β^2 is somewhere between A_{44} and A_{55}, probably closer to A_{44}. Since β does not appear in the AVO intercept term in Eq.(1) and α^2 was specified as A_{33}, all the difference curves start from zero for the vertical incidence, which indicates that for this case the approximate coefficient coincides with the exact one. Let us now fix $\beta = 1.47$ km/s and vary α. We choose α successively as 2.00, 2.25, 2.50, 2.75 and 3.00 km/s. The results for the model 6 are shown in Figure 7, in the same form as in Figures 5 and 6. We can see that variation of α introduces a shift of the approximate coefficient with respect to the exact one even for vertical incidence. For the choices of the background not too different from $(A_{33})^{1/2}$, the shift of the coefficient varies only little, especially for small angles of incidence. We can see that a 10% difference of α^2 from A_{33} introduces approximately 10% shift of the approximate coefficient for practically all considered angles of incidence. As it could be expected, the best approximation is

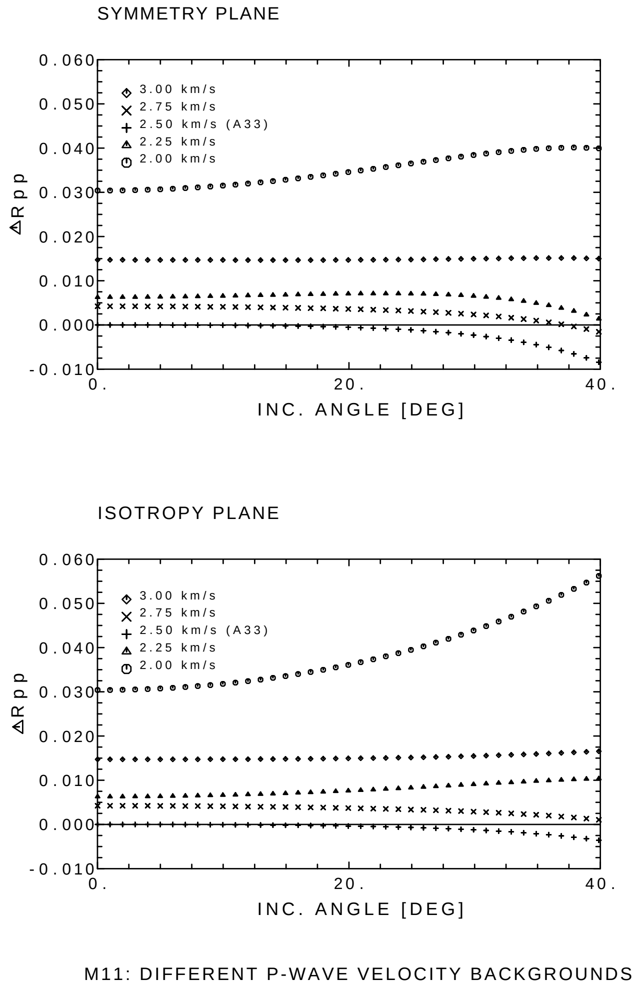

Figure 8. Deviations ΔR_{PP} of approximate R_{PP} coefficients from exact ones in model 11 in the symmetry plane (top) and in the isotropy plane (bottom). Individual curves correspond to different P wave velocities α of the background. S wave velocity is specified as $\beta = 1.47$ km/s.

obtained for the choice $\alpha^2 = A_{33}$, i.e. for the choice commonly used in seismic practice. It is of interest to check the curve corresponding to $\alpha = 2.25$ km/s. This curve corresponds roughly to a homogeneous continuous background (the P and S wave velocity contrasts

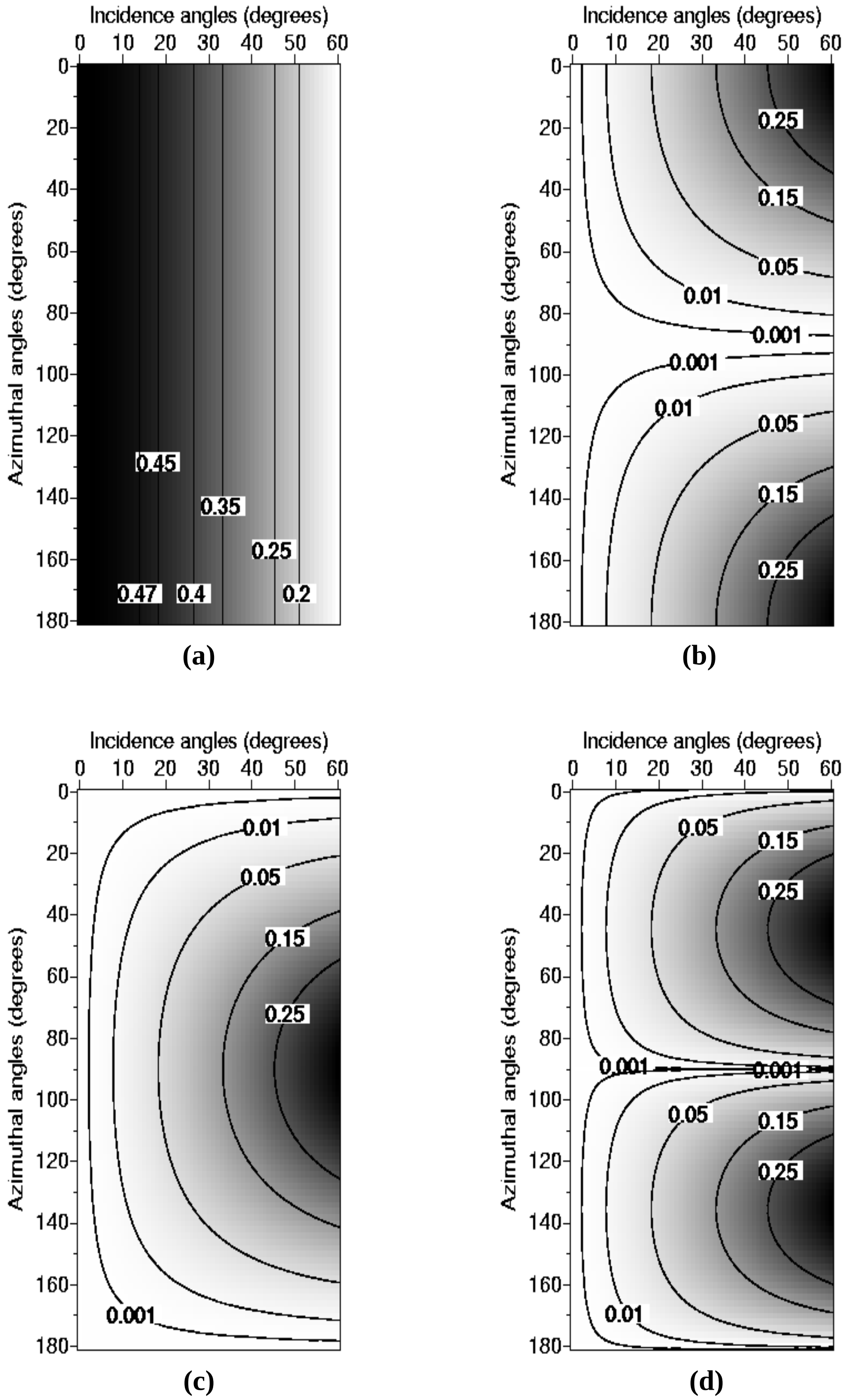

Figure 9. Sensitivity maps of the R_{PP} coefficient with respect to terms $\Delta\epsilon_z$ (a), $\Delta\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_x$ (b), $\Delta\delta_y - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_y$ (c) and $\Delta\chi_z - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\epsilon_{45}$ (d).

are negligible in this case). Similar features can be observed in Figure 8, which shows the same as Figure 7 but for model 11.

6 Sensitivity to WA parameters

Comparison of approximate formulae (1) for the PP reflection coefficient and (17) for the PP transmission coefficient implies that the formula for the transmission coefficient is more informative as to WA parameters. The transmission coefficient depends on all the P wave WA parameters. Thus it could be useful to use transmission coefficients in the AVO studies connected with VSP experiments.

Nevertheless, presently, the main source of information about the parameters of the medium in AVO/AVA studies are the reflection coefficients. It is generally accepted that determination of the multiplicative factor of $\sin^2\theta\tan^2\theta$ in the expression for the reflection coefficient is difficult (Rüger and Tsvankin, 1997). For practical purposes, only the remaining intercept and gradient terms can be used. These terms depend on only four combinations of WA parameters. Sensitivity of the approximate reflection coefficient formula to these four parameters is shown in Figure 9. The reflection coefficient is sensitive to $\Delta\epsilon_z$ equally well for all azimuths. Sensitivity to terms $\Delta\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_x$ and $\Delta\delta_y - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_y$ is similar. In both cases it increases with increasing angle of incidence. In the former case, it is large along the x-axis and zero along the y-axis; in the latter case, it is opposite. Sensitivity to $\Delta\chi_z - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\epsilon_{45}$ also increases with increasing angle of incidence but it is zero along the x- and y-axes and maximum for azimuths of 45^o and 135^o.

To separate the WA parameters in the above mentioned terms, an additional independent information is necessary. P wave moveout velocity can serve for this purpose (Rüger and Tsvankin, 1997). Equation (43) of Pšenčík and Gajewski (1998) yields such an idependent information on the WA parameters δ_x, δ_y and χ_z. With these parameters known, γ_x, γ_y and ϵ_{45} can be determined easily.

Use of the formulae (1) and (17) is preferable when nothing is known about studied structure. If an independent information exists about higher symmetry of anisotropy, simplified formulae, which can be deduced from (1) and (17) can be used.

If, for example, a reflecting structure under an isotropic oveburden is HTI, with unknown orientation of the axis of symmetry in the plane (x, y), a generalization of Eq.(16) can be used for the determination of the orientation of the axis of symmetry and of the terms ϵ_z, $\delta_x - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\gamma_x$ and $2\epsilon_z - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\gamma_y$. The generalization consists in substituting φ in Eq.(16) by $\varphi - \varphi_0$, where φ_0 denotes an unknown azimuth of the axis of symmetry. Measurements along three profiles, for example, $\varphi = 0^o$, 45^o and 90^o enable us to determine φ_0 and all the above mentioned parameters. Since ϵ_z can be determined from the intercept AVO term, γ_y can be found without difficulties. In order to find γ_x, parameter δ_x must be determined from moveout data, for example.

7 Conclusions

We generalized approximate formulae for PP reflection and transmission coefficients at a weak contrast interface separating two weakly anisotropic media derived by Vavryčuk and Pšenčík (1998) and Pšenčík and Vavryčuk (1998). Use of an arbitrary background introduces new WA parameters into the formulae for the R/T coefficients, which describe vertical propagation. Recovery of these parameters from observed R/T coefficients in the AVO studies can lead to a more precise determination of elastic parameters controlling vertical wave propagation.

Experiments with various backgrounds have shown that specification of the P wave velocity in the background as $\alpha^2 = A_{33}$ leads to best approximation while the determination of the optimum S wave velocity background is more complicated. The S wave velocity should be chosen so that β^2 is between A_{44} and A_{55}. By specifying the general formulae for anisotropic media with a higher symmetry, we found that reduced formulae differ from formulae derived for special situations by other authors by a different definition of the WA parameters. It was shown that some of these differences can lead to a substantial decrease of accuracy of approximate formulae (see Figure 1). On numerical examples we illustrated the accuracy of approximate formulae by comparing their results with exact coefficients. For models with anisotropy and velocity contrast about 10%, the error of approximate R_{PP} formulae was around 1% in most of the considered region. For a model with a lower symmetry and stronger anisotropy (about 25%) and velocity contrast larger than 13%, the approximate R_{PP} formula yielded results with approximately 10% error.

Presented formulae allow simple sensitivity studies. The approximate formula for the transmission coefficent seems to be more informative as to the P wave WA parameters than the approximate formula for the reflection coefficient. It could thus probably find applications in the processing of the VSP measurements. Because of the wide practical use of the reflection coefficients in the AVO studies, we presented maps of sensitivity to basic combinations of WA parameters for the R_{PP} coefficients.

The presented formulae for the approximate reflection/transmission coefficients can find applications not only in studying AVO/AVA in weakly anisotropic media with anisotropy of lower symmetry but also in higher symmetry media like orthorhombic or hexagonal media but with arbitrarily oriented axes of symmetry.

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