Comparison of ray–matrix and finite–difference methods in a simple 1-D model

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Summary

A simple 1-D model, consisting of a low velocity layer covering a homogeneous halfspace, is used to compare the ray–matrix synthetic seismograms with the finite–difference synthetic seismograms. The computation has been carried out according to the history file which contains both the input data and the instructions on program execution. The structure of the history file is discussed in detail, including the comments on the input data.

Keywords

Ray methods, matrix methods, Green function, synthetic seismograms, history files.

1 Introduction

Ray methods, especially those designed to calculate synthetic wavefields, require velocity models composed of several smooth blocks separated by interfaces covered by smooth surfaces. The smaller is the number of blocks and surfaces, the better. Such models miss possible fine layering in the vicinities of the source and receivers or along structural interfaces. Since the frequency–dependent response of the stacks of fine layers may considerably influence synthetic wavefields, the ray–theory synthetic seismograms are too distorted if the stacks of fine layers are neglected.

This drawback of the ray methods can be approximately corrected if the "thin" stacks of fine layers are locally approximated by the 1-D stacks, the wave is locally approximated by the plane wave and a matrix method is used to calculate the local response of each stack. The ray–theory elementary Green functions are then combined with the frequency–dependent matrix–method responses. For more details refer, e.g., to the papers by Červený (1994) and Jílek & Červený (1994, 1995, 1996a, 1996b).

Thomson (1998a, 1998b) developed program package RMATRIX for the calculation of the frequency–dependent responses of the stacks of fine isotropic or anisotropic layers. One of the versions of program package RMATRIX is included on compact disk SW3D-CD-2. Program `greenss.for` of package CRT has been updated in version 5.20 to optionally call the subroutines of package RMATRIX calculating the response of the transmission through a stack of fine horizontal layers at each receiver location. Note that the option is conditioned by a copyright (see package RMATRIX) and may be enabled by editing several comment lines in file `greenss.for`. Responses of fine layers at the source or along structural interfaces may be included in the future.

A simple 1-D model, named RM, has been designed to debug the software and to compare the ray–matrix synthetic seismograms with the finite–difference synthetic seismograms. The test calculations will be discussed in this paper.

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Version 5.20 of package CRT includes also a new possibility to run the programs according to the "history files". This considerably simplifies and clarifies the preparation of the jobs requiring several programs to run, especially if the programs read input data from "SEP (Stanford Exploration Project) parameter files". The history files are the SEP parameter files with instructions when and how to run the programs. Programs reading the SEP parameter files can share the same input data and may be run several times with various modifications of the input data. The history files are designed to be computer–independent job descriptions.

The additional aim of this paper is thus to demonstrate the preparation and usage of the history files and to illustrate new input data of some programs. Especially the programs of packages FORMS, CRT and FD, calculating or plotting synthetic seismograms, have been revised in version 5.20 to read the data from the SEP parameter files. History file `rm-fd.h` ("`rm`" stands for the model, "`fd`" for comparison with finite differences which are not a part of package CRT), prepared to compare the ray–matrix synthetic seismograms with the finite–difference ones, will thus be discussed here line by line.

2 Model, source, receivers

A very simple 1-D model consists of a low velocity layer covering a homogeneous halfspace. Vertical axis x_3 points downwards, free Earth surface is situated at the depth of $x_3 = 0.000\,\mathrm{km}$. The thickness of the low velocity layer is 0.010 km, with material properties $v_\mathrm{P} = 1.0\,\mathrm{km\,s^{-1}}$, $v_\mathrm{S} = 0.5\,\mathrm{km\,s^{-1}}$, $\rho = 1.6\,\mathrm{kg\,dm^{-1}}$. The velocities in real shallow sediments might be smaller but would imply denser grid for finite differences. The homogeneous halfspace has material properties $v_\mathrm{P} = 3.2\,\mathrm{km\,s^{-1}}$, $v_\mathrm{S} = 2.0\,\mathrm{km\,s^{-1}}$, $\rho = 2.5\,\mathrm{kg\,dm^{-1}}$.

The point explosive source is located at the depth of $x_3 = 0.200\,\mathrm{km}$, below the origin of the Cartesian coordinates. The profile of 19 receivers with spacing 0.020 km at the earth surface is centred above the source.

The sedimentary layer represents the fine local layered structures at the receiver locations and normally would not be contained within the macro model. Here we need the sedimentary layer in the model for the finite differences and use the same model also for ray tracing. The two–point rays are thus caught at the top of the homogeneous halfspace, exactly below the receivers. In the ray–matrix method, the plane–wave approximation is used to extrapolate the wavefield from the endpoint of the two–point ray to the bottom of the local layer stack and from the top of the layer stack to the receiver point.

History file `rm-fd.h` begins with some comments, which are preceded by "`#`", and the `PARAMETER=VALUE` couple saying that the value of parameter `MODEL` is string `'rm-mod.dat'`, i.e., saying that the file specifying the model is named `'rm-mod.dat'`.

```
# Comparing ray-matrix and FD seismograms in 1-D model RM
# =========================================================
# Input files required:
 #chk.pl: "crt/" "writall.dat"
 #chk.pl: "crt/" "sourcex.dat"

# Data specifying the model by means of the MODEL package
```

```
MODEL='rm-mod.dat'
```

Everything from hash mark "#" to the end of line is assumed to be a comment and is ignored. However, comments containing no equals "=" nor colons ":" need not be preceded by a hash mark, because only equals "=" and colons ":" are interpreted as commands in the history files. Each "=" (equal sign) immediately preceded by a string is interpreted as the PARAMETER=VALUE couple specifying the value of the parameter named PARAMETER. Each ":" (colon) immediately preceded by a string is interpreted as the instruction to execute a program. The string between the last space (or the beginning of the line if there are no spaces to the left of ":") and the colon is the name of the program. If the name contains substring "`.pl`", it is assumed to represent a Perl script. Otherwise, it is assumed to represent executable program, compiled from the Fortran source. Everything to the right from the colon is assumed to represent the data for the program read from the standard input (Fortran) or command line (Perl). For example, if the third line of the history file were not commented, it would imply the execution of program named `required` which does not exist. After removing the colon, the comment lines could be written without the hashes.

The list of files required by the history file contains just data files `writall.dat` and `sourcex.dat`, although several other input data files are necessary. Concerning the data files listed, there are at least three possibilities (a) no list, (b) collect the data specific to the computational job in a single directory and list just the data files which are located elswhere and may simply be forgotten, (c) list all input data. Here (b) is the case.

The list has the form of instructions `chk.pl:` to run Perl script `chk.pl`, commented by "#". The string preceding the data filename is the path to the root directory of the SW3D-CD-2 software (corresponding to subdirectory `unix/` or `dos/` of the compact disk) where the original of the data file is located. The instructions to run Perl script `chk.pl`, with input data like `"crt/" "writall.dat"` are disabled (commented) because they are not very useful in version 5.20 of the packages used. Perl script `chk.pl` checks the existence of a file (here `writall.dat`) in the current directory. If the path to the root directory of the SW3D-CD-2 software is specified and the file exists in its sudirectory (here `crt`), `chk.pl` copies the file into the current directory. If the file is not found `chk.pl` should stop the execution. However, specification of the path to the root directory of the SW3D-CD-2 software requires to manually edit file `chk.pl`. Moreover, stopping the execution often does not work in the current version (at least under MS-DOS).

Parameter `MODEL` is now understood just by programs `green.for`* and `modfd.for`, but will likely be used by more programs in the future. Also other parameters will be extended to more programs in the future.

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* Note that program `green.for` checks whether the model is anisotropic. If yes, it calculates the perturbation from the reference isotropic model to the anisotropic model and applies the coupling ray theory to S waves. However, version 5.20 cannot consider anisotropic models with structural interfaces. In our case of an isotropic model, program `green.for` does nothing with the model. If parameter `MODEL` is blank, the model is considered isotropic by `green.for`.

```
# Source and receivers
  SRC='rm-src.dat'  REC='rm-rec.dat'  SOURCE='sourcex.dat'

# Stacks of fine layers at the receiver sites (program GREENSS)
  RMATRIX='rm-rm.dat'
```

Source coordinates are stored in file `rm-src.dat` and the names and coordinates of the receivers are stored in file `rm-rec.dat`. File `sourcex.dat` contains the data describing the unit seismic moment of an explosive source, because finite difference program `fd2d.for` version 5.20 considers only the explosive source. File `rm-rm.dat` contains the descriptions of the fine local layered structures at the receiver locations. In this example, we specified a single structure for all receivers in order not to repeat the same structure for all receivers. If parameter `RMATRIX` is not included or is blank (default), the local responses are not applied.

```
# Source time function
  KSIG=3   SIGW=2  # Kuepper signal with 2 local maxima
  SIGF=20.         # reference frequency
  SIGA=1.          # maximum amplitude
  DER=0.5          # halfth derivative for the point source
```

We selected the Küpper (known also as Müller) signal with 2 local maxima because finite difference program `fd2d.for` version 5.20 does not support another source time function. The reference frequency of 20Hz corresponds to the P–wave wavelength of 0.050 km in the sedimentary layer, i.e., 5 times the thickness of the layer.

We wish to approximate the point source by the line source excited by the Küpper signal in 2-D program `fd2d.for`. We thus take the halfth derivative of the Küpper signal as the source time function describing the time dependence of the seismic moment of the explosive point source. Then the point and line sources will generate wavelets of the same form in the far–field approximation.

```
# Data to control seismogram plotting (program SP)
  KODESP=1  SPCHRH=0.30
  SPTMIN=0.050  SPTMAX=0.280  SPTLEN=23.0  SPTDIV=4.6 SPTSUB=5
  SPXMIN=-.200  SPXMAX=0.200  SPXLEN=14.0  SPXDIV=4   SPXSUB=1
  NORMSP=1  SPAMP=-4.00  # amplitude scaling for point source in 3-D
```

The seismograms will be plotted between times `SPTMIN`=0.050 and `SPTMAX`=0.280 (in seconds), with the length of the time axis of `SPTLEN`=23.0 (in cm), etc. The amplitude scale corresponds to a point source in 3-D.

To be able to plot also the 2-D FD seismograms, corresponding to the line source, in a comparable scale, we choose the reference distance `SPDIST` for the amplitude power scaling during the plotting equal to the hypocentral distance (in a homogeneous medium), at which the wavefields generated by the point and line sources have the same amplitudes.

```
# Hypocentral distance where the point and line sources have comparable
# amplitudes (for amplitude power scaling roughly converting line
# source to point source)
  SPDIST=0.049736  # SPDIST=1/(2*pi*VP), VP=3.2
```

When plotting, we will use no amplitude power scaling for the point source but the power scaling, proportional to the hypocentral distance powered to $-\frac{1}{2}$, for the line source. In this way, seismograms corresponding to the point and line sources will coincide in the neighbourhood of the source point. They, of course, may considerably differ at large distances in a heterogeneous medium.

3 Ray–matrix method

The data specific to the ray–matrix method do not require many comments.

```
# Ray-matrix synthetic seismograms (point source)
# ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
# Data describing the filtration of the source time function
  FMIN=0.  FLOW=2.  FHIGH=110.  FMAX=120. # (cosine filter)

# Time step and time interval for the Fast Fourier Transform
  DT=0.00035  NFFT=2048

# GSE file with synthetic seismograms
  SS='rm-ss.gse'

# Data to control optional plotting of program SS
  SIGPLOT='rm-sig.ps'  MPTS=400
```

Parameters FMIN, FLOW, FHIGH, FMAX specify the cosine window to filter the source time function in order to remove low or negative frequencies for the ray method or to speed up the matrix method. Since finite difference program fd2d.for version 5.20 does not filter the signal, the filtering should not distort the signal. Maybe that the default values would be sufficient. The time step and time interval for the Fast Fourier Transform should always be specified. However, DT could be chosen by an order of magnitude greater in this case.

The GSE file with synthetic seismograms would have its default name of ss.gse if not specified. It should be specified when calculating several sets of seismograms like in this example. Parameters SIGPLOT and MPTS need not be specified unless you wish to plot the source time function by program ss.for.

```
# Programs to execute
  crt:     'rm-crt.dat' /      # two-point ray tracing
                               # (do not link 'crt.for' with 'scropc.for'!)
  green:   'rm-fd.h' /         # Green function
  greenss: 'rm-fd.h' /         # response function
  ss:      'rm-fd.h' /         # synthetic seismograms
```

Program `crt.for` version 5.20 does not read a SEP parameter file. All data are placed in file `rm-crt.dat` and in files referred from it. Programs `green.for`, `greenss.for` and `ss.for` take the data from history file `rm-fd.h`. When executing the history file, only data preceding these lines are submitted to the programs. Plotting of the seismograms is located at the end of the history file.

4 Finite differences

Finite differences are often limited by the memory requirements or by the computational costs. This is also the case of the following grid dimensions:

```
# FD synthetic seismograms (line source)
# ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
# Grid dimensions, FD time steps, snapshots
  N1=601     N3=344     NTFD=1601     N4=80
  D1=0.001   D3=0.001   DTFD=0.000175 D4=0.0035
  O1=-.300   O3=0.000                 O4=0.000
```

Dimension `MRAM` of array `RAM` in file `ram.inc` must be

$$\mathtt{MRAM} \geq 19\,N_1 N_3 + 10\,N_1 + 12\,N_3 + (2\,\mathtt{NTFD} + 5)\max(1, \mathtt{NREC}) \quad .$$

We have chosen $\mathtt{NTFD} = 1\,601$, $\mathtt{NREC} = 19$ and wish to keep the value of $\mathtt{MRAM} = 4\,000\,000$, which is set in the distributed version of `ram.inc` and should be suitable for 16MB of available RAM. Then

$$19\,N_1 N_3 + 10\,N_1 + 12\,N_3 \leq 4\,000\,000 - 60\,933 \quad ,$$

or

$$N_3 \leq (3\,939\,067 - 10\,N_1)/(19\,N_1 + 12) \quad .$$

For $N_1 = 601$, the maximum N_3 is $N_3 = 344$.

P-wave travel time along the ray reflected from the bottom of the computational box and arriving at the middle receiver is

$$\tau_0 = \frac{2(N_3 - 11) * D_3 - 0.190\,\mathrm{km}}{3.2\,\mathrm{km\,s^{-1}}} + \frac{0.010\,\mathrm{km}}{1.0\,\mathrm{km\,s^{-1}}} \quad .$$

For $D_3 = 0.001\,\mathrm{km}$ and $N_3 = 344$, $\tau_0 = 0.15875\,\mathrm{s}$.

P-wave travel time along the ray reflected from the side of the computational box and arriving at the outside receiver ($x_1 = \pm 0.180\,\mathrm{km}$) is

$$\tau_1 \approx \frac{\sqrt{[(N_1 - 1) * D_1 - 0.180\,\mathrm{km}]^2 + [0.190\,\mathrm{km}]^2}}{3.2\,\mathrm{km\,s^{-1}}} + \frac{0.010\,\mathrm{km}}{1.0\,\mathrm{km\,s^{-1}}} \quad .$$

For $D_1 = 0.001\,\mathrm{km}$ and $N_1 = 601$, $\tau_1 \approx 0.154\,\mathrm{s}$.

```
# GSE file with synthetic seismograms
  SS='rm-fd.gse'

# Form of files with effective elastic parameters and snapshots
  FORM='formatted'

# Files with snapshots (not generated by default)
 #SNAP1='rm-fd1.out'
 #SNAP3='rm-fd3.out'
```

If the names of the output files with snapshots are not specified, the snapshots are not generated. Uncomment these lines if you wish to store `N4` snapshots on your disk. The default form is `'formatted'`. Note that programs of package FORMS, processing the snapshots, work just with formatted files in version 5.20.

```
# Files with effective elastic parameters
  A11='a11.out'
  B11='b11.out'
  C11='c11.out'
  A13='a13.out'
  B13='b13.out'
  C13='c13.out'
  A31='a31.out'
  B31='b31.out'
  C31='c31.out'
  A33='a33.out'
  B33='b33.out'
  C33='c33.out'
  DEN='den.out'
```

The files with effective elastic parameters are written by program `modfd.for` and read by finite–difference program `fd2d.for`.

```
# Programs to execute (very time consuming!)
  modfd:   'rm-fd.h' /          # gridded elastic parameters
  fd2d:    'rm-fd.h' /          # FD synthetic seismograms
```

Since the “nonreflecting” boundary conditions of finite differences may, to some extent, reflect the waves, it is highly recommended to change the finite–difference grid roughly by a quarter of the prevailing wavelength and to run the finite differences once again. The reflections from the boundaries are then shifted roughly by half the prevailing period and are clearly visible in the common plots of the seismograms from both the computations. The reference P–wave wavelength in the homogeneous halfspace is 160 grid intervals. We thus enlarge the grid by 40 grid intervals at the bottom and by 45 grid intervals at each side (increase of 40 to 45 accounts for the angle of reflection from the nonreflecting boundary, but 40 would be fine).

```
# FD synthetic seismograms on a larger grid to test boundary conditions
# ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
# Larger grid to test the boundary conditions:
  O1=-.345  N1=691  N3=384

# GSE file with synthetic seismograms
  SS='rm-fda.gse'

# No snapshots for this test:
  SNAP1=' '  SNAP3=' '
```

Parameters SNAP1 and SNAP3 are cleared to avoid the snapshot generation even if they were enabled for the previous FD computation.

```
# Programs to execute
 #modfd:   'rm-fd.h' /        # gridded elastic parameters
 #fd2d:    'rm-fd.h' /        # FD synthetic seismograms
# Do not forget to increase default value of MRAM in ram.inc!
```

Execution of the programs is disabled by commenting the lines because this calculation cannot be carried out with the distributed version 5.20 of `ram.inc`. You may increase `MRAM` in `ram.inc` accordingly, compile program `fd2d.for`, and enable this computation.

5 Comparison

```
# Plotting the seismograms
# ~~~~~~~~~~~~~~~~~~~~~~~~
# Plotting ray-matrix seismograms alone
  SS='rm-ss.gse'  SP1='rm-ss1.ps'  SP2=' '  SP3='rm-ss3.ps'
 #sp:      'rm-fd.h' /        # seismogram plotting

# Amplitude power scaling roughly converting line source to point source
  SPOWER=-.5

# Plotting FD seismograms alone
  SS='rm-fd.gse'  SP1='rm-fd1.ps'  SP2=' '  SP3='rm-fd3.ps'
 #sp:      'rm-fd.h' /        # seismogram plotting
```

The above lines are just examples how to plot the ray–matrix seismograms alone (in black) and how to plot the finite–difference seismograms alone (also in black). The execution is thus commented out. The above lines need not be present in the history file.

```
# Comparison of the seismograms
  SS=' '
  SS='rm-fd.gse'   KOLOR=4   SPOWER=-.5   # FD seismograms (blue)
# SS1='rm-fda.gse' KOLOR1=3  SPOWER1=-.5  # test FD seismograms (green)
  SS2='rm-ss.gse'  KOLOR2=2  SPOWER2=0.0  # ray-matrix seismograms (red)
  SP1='rm-1.ps'  SP2=' '  SP3='rm-3.ps'
  sp:      'rm-fd.h' /        # seismogram plotting
```

We plot the calculated synthetic seismograms together, in the colour PostScript plots of the horizontal in–plane (`rm-1.ps`) and vertical (`rm-3.ps`) components. The transversal component is zero. The colour indices correspond to the distributed version of file `calcops.rgb`. Parameter `SS` is cleared to be sure that commenting out the plotting of the first FD seismograms stored in file `rm-fd.gse` cannot cause problems. Plotting the second FD seismograms stored in file `rm-fda.gse` is disabled. Enable the plotting if you are calculating them.

The calculated seismograms are shown in Figures 1 and 2, where the reflections from the nonreflecting boundaries can clearly be identified. The blue reflections (the first FD calculation) come before the green ones (the second FD calculation, with the enlarged grid).

The agreement between the ray–matrix (red) and finite–difference seismograms is very good except for a small difference at the horizontal component around time $t = 0.15\,\mathrm{s}$, which might be worth to be explained in the future.

History file `rm-fd.h` requires programs of packages FORMS, MODEL, CRT and FD. It can be executed on systems supporting Perl scripting language (e.g., Unix or MS-DOS) by typing a command like "`perl go.pl rm-fd.h`". Perl program `go.pl` of package FORMS copies input history file `rm-fd.h` line by line to output history file `rm-fd.out`. When a "`:`" (colon) instruction to execute the program is encountered, the program is executed with the name of the output history file substituted for the name of the input history file in order to hide the PARAMETER=VALUE couples between the "`:`" (colon) instruction and end of file to the program. If you wish to name the output history file, e.g., `new.out`, the history file may be executed by command "`perl go.pl rm-fd.h new.out`".

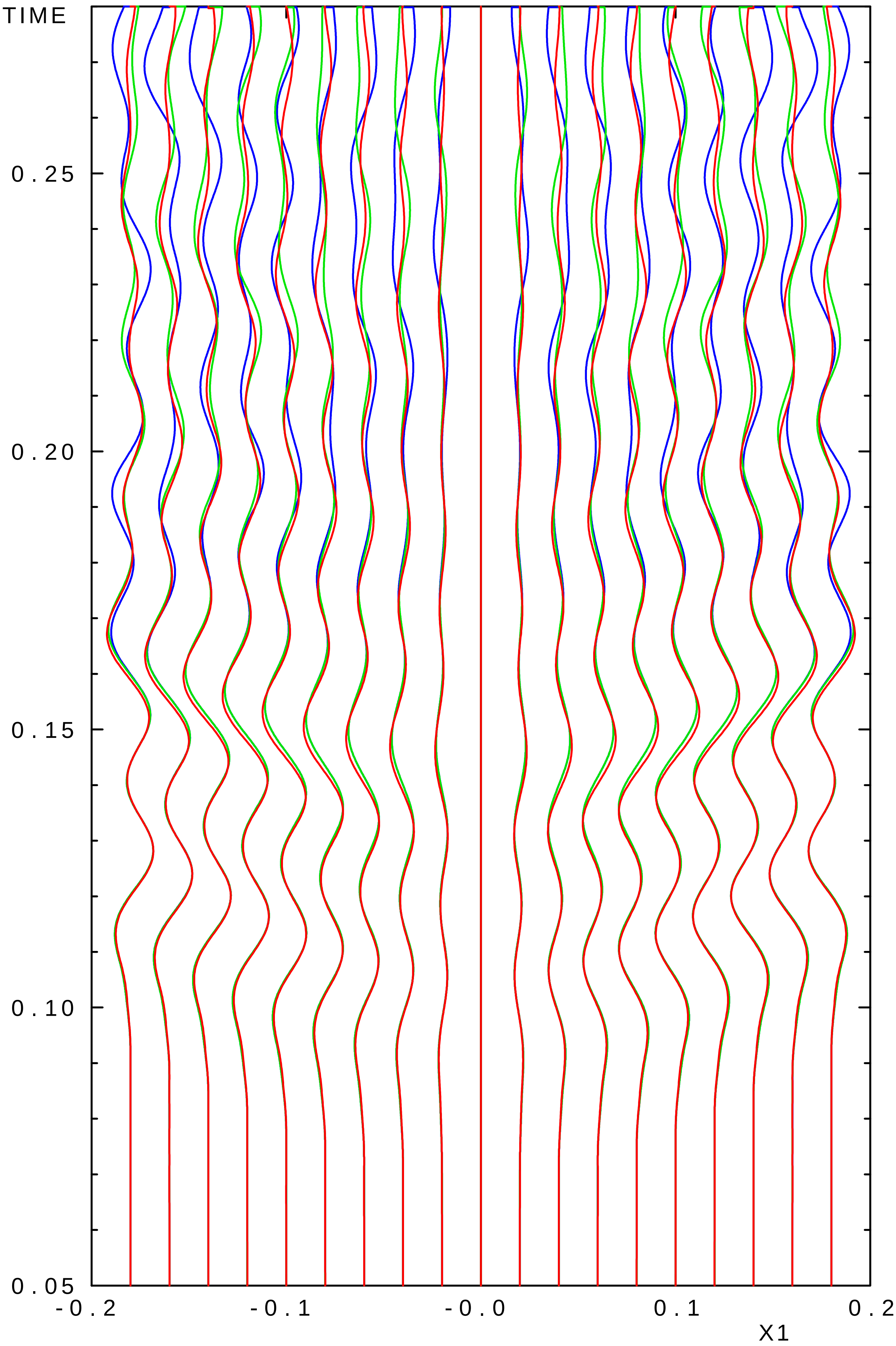

Fig. 1. Horizontal component. 2-D finite differences are *blue* and *green*, 3-D ray–matrix method is *red*.

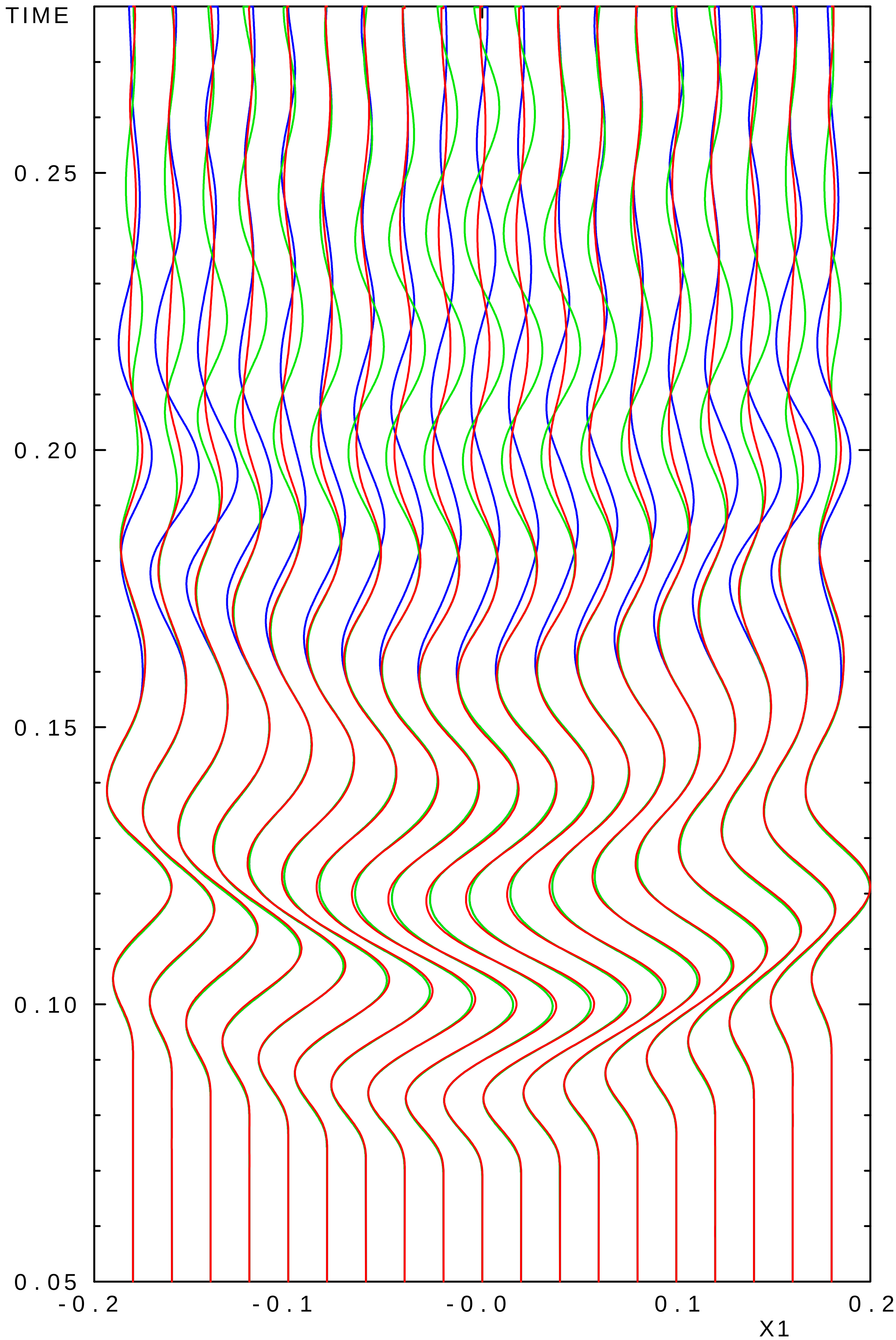

Fig. 2. Vertical component. 2-D finite differences are *blue* and *green*, 3-D ray–matrix method is *red*.

References

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