Synthetic seismograms in 2-D model UNCONFORMITY

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Summary

The calculation of synthetic seismograms in 3-D models composed of heterogeneous viscoelastic blocks of arbitrary shape, using the results the `CRT` program, is briefly described and illustrated on a simple 2-D example.

Keywords

Ray theory, 3-D ray tracing, synthetic seismograms, single force and seismic moment, attenuation.

1 Introduction

Application of programs to calculate ray–theory synthetic seismograms is conditioned by reasonably reliable two–point ray tracing. Each missing two–point ray creates a gap in the synthetic seismograms and each erroneously doubled two–point ray doubles the amplitude of the corresponding arrival.

When Bulant (1996a,1996b) supplemented the `CRT` ray–tracing program with 3-D two–point ray tracing by the shooting method, the need for subsequent programs to calculate ray–theory synthetic seismograms in 3-D block models arose.

Hereinafter, the calculation of synthetic seismograms using the results of the `CRT` program is briefly described and illustrated on a simple example. The example is, however, chosen two–dimensional in order to have the possibility to compare the results of the `CRT` package with the results of older packages for debugging and testing purposes. On the other hand, the programs to calculate the synthetic seismograms, unlike ray tracing programs, do nearly the same job in 2-D and 3-D.

2 Programs to calculate ray–theory synthetic seismograms

The output of the `CRT` program may be step–by–step transformed into ray–theory synthetic seismograms stored in the GSE data exchange format using programs `GREEN`, `GREENSS` and `SS`. Seismograms in the GSE data exchange format may be plotted, e.g., using program `SP`.

Seismic Waves in Complex 3-D Structures, Report 4 (Department of Geophysics, Charles University, Prague 1996)

2.1 Program GREEN to calculate ray–theory Green functions

Program GREEN reads the quantities calculated by the CRT ray–tracing program, stored at the points of intersection of the two–point rays with the reference surface along which the receivers are located, and generates the file with the quantities describing the contributions of individual two–point rays to the ray–theory Green functions at the corresponding receivers.

The contributions are sorted according to the receivers. The contributions at each receiver are sorted according to increasing travel time.

Each contribution is identified by the string specifying the receiver and, if required, also by the string specifying the source. Each contribution is expressed in terms of the following 22 real–valued quantities:

1: real part of the travel time;
2: imaginary part of the travel time;
3–5: coordinates of the receiver;
6–8: coordinates of the initial point of the ray (source point);
9–11: derivatives of the real part of travel time with respect to the receiver;
12–14: derivatives of the real part of travel time with respect to the initial point of the ray;
15–32: amplitude of the Green function in the model coordinates (3×3 complex–valued matrix).

The derivatives of travel time are required when calculating synthetic seismograms for the seismic moment sources.

2.2 Program GREENSS to calculate response functions

Point source specified in terms of a single force or a seismic moment is considered. The force or the moment is assumed to be factorized, i.e. to be expressed as the product of a scalar complex–valued source time function with a complex–valued vector or a complex–valued matrix, respectively. The imaginary part of the source time function is assumed to be the Hilbert transform of the real part.

Program GREENSS reads the complex–valued amplitude of the seismic force (vector) or moment (3×3 matrix), and the ray–theory Green function written by program GREEN.

As the simplest option, it may also read the complex–valued source time function (for seismic source) or its time derivative (for seismic moment) in the GSE format (generated, e.g., by the SS program) and synthesize seismograms without attenuation directly in the time domain. This option not only throw away the imaginary part of the travel time, but also often requires source time function to be discretized with smaller time step then is required when seismograms are synthesized in the frequency domain. This is caused by the piece–wise linear interpolation of the source time function in the time domain. On the other hand, there are no problems with the alias in the time domain.

The other, more important option consists in the calculation of the frequency–domain response functions at individual receivers. The synthetic seismograms are then simply calculated using program SS by multiplying the Fourier image of the scalar source time function (for seismic force) or its derivative (for seismic moment) with the complex–valued vectorial response function (output of GREENSS) and transforming the product back to the time domain.

The contribution of a two–point ray to the frequency–domain response function $R_i(f)$ is approximated by

$$\Delta R_i(f) = G_{ij} F_j \exp[2\pi \mathrm{i} f \tau(f)] \tag{1}$$

for seismic force. Here G_{ij} is the complex–valued 3×3 matrix amplitude of the ray-theory Green function. In the case of a moment source, the complex–valued vectorial amplitude F_j of the seismic force is replaced by

$$F'_j = -M_{jk} \frac{\partial \mathrm{Re}\,[\tau(f_{\mathrm{ref}})]}{\partial s_k} \quad , \tag{2}$$

where M_{jk} is the complex–valued 3×3 matrix amplitude of the seismic moment and travel time τ is differentiated with respect to the source coordinates s_k. The multiplication factor of $-2\pi \mathrm{i} f$ has been removed from the right–hand side of (2) and replaced by differentiating the source time function inside program `SS` constructing the seismograms.

Complex–valued frequency–dependent travel time $\tau(f)$ is determined from the complex–valued reference travel time $\tau(f_{\mathrm{ref}})$ calculated by the `CRT` program through the quasi–causal Futterman's (1962) dispersion equation

$$\mathrm{Re}\,[\tau(f)] = \mathrm{Re}\,[\tau(f_{\mathrm{ref}})] - \frac{2}{\pi} \ln\left(\frac{f}{f_{\mathrm{ref}}}\right) \mathrm{Im}\,[\tau(f_{\mathrm{ref}})] \ , \quad \mathrm{Im}\,[\tau(f)] = \mathrm{Im}\,[\tau(f_{\mathrm{ref}})] \ . \tag{3}$$

Here the seismic model is assumed to correspond to the reference frequency f_{ref}. Futterman's (1962) dispersion equation is selected because it is linear in the real and imaginary parts of the slowness, and thus may be applied also to the real and imaginary parts of the travel times. If the prevailing frequency in the seismograms is considerably different from the reference frequency f_{ref}, other dispersion equations may be approximated by carefully adjusting the reference frequency f_{ref} in (3), because all causal and quasi–causal dispersion equations are equivalent for all practical purposes if the propagation velocity at the prevailing frequency is correct (Klimeš & Musil 1986).

2.3 Program `SS` to calculate synthetic seismograms

Program `SS` reads or calculates the digitized scalar real–valued source time function and filters it with given noncausal frequency band–pass filter. Then it may
(a) output the filtered scalar source time function and its Hilbert transform as the complex–valued scalar source time function, or
(b) read the frequency–domain response functions at individual receivers and generate the synthetic seismograms.
The generated time functions are written in the GSE data exchange format in order to be readable by various kinds of seismic graphic software.

2.4 Program `SP` to plot synthetic seismograms

Program `SP` reads and plots the seismograms stored in the GSE data exchange format.

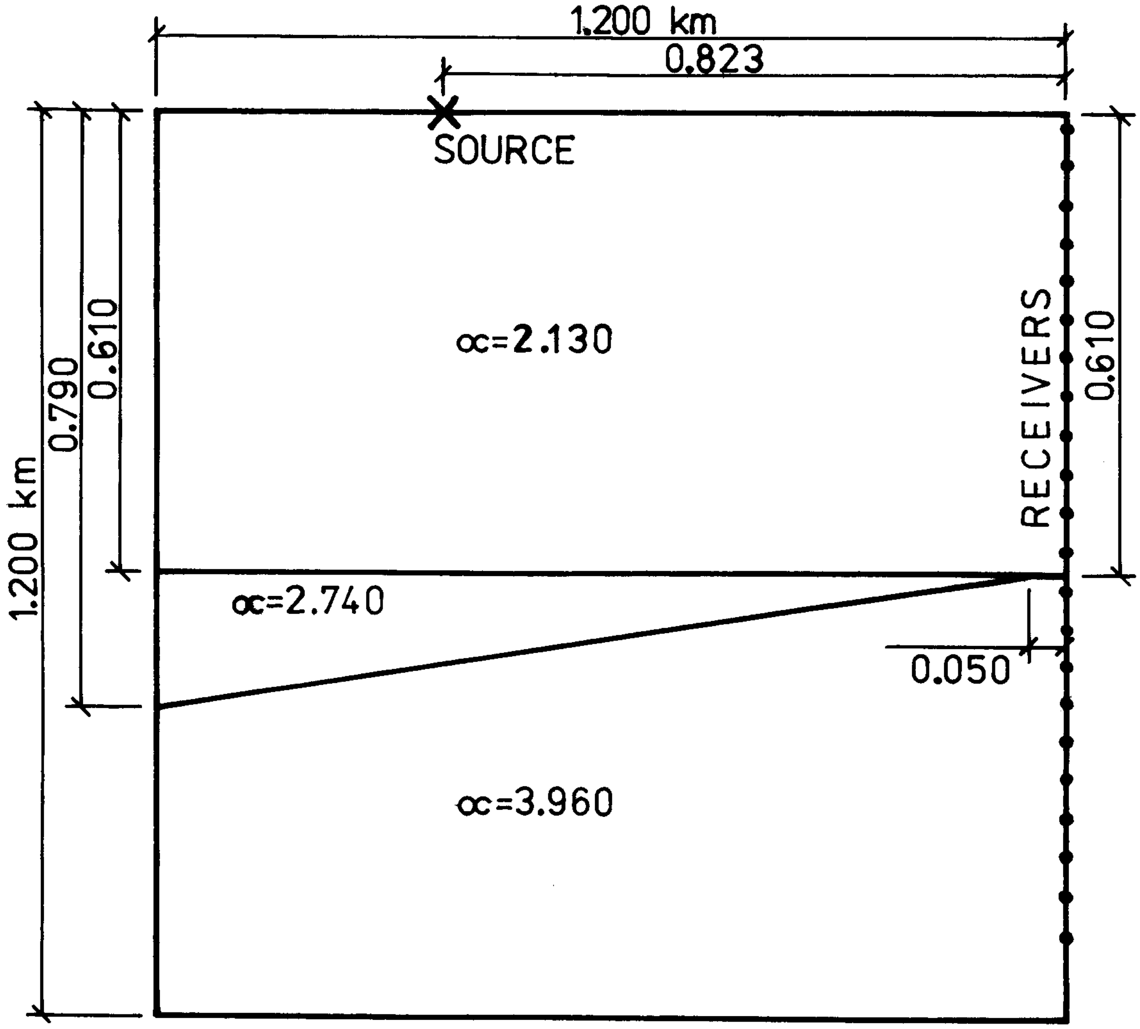

Figure 1. Two-dimensional model UNCONFORMITY by Cormier & Mellen (1984) composed of 3 homogeneous blocks. P-wave velocities are denoted by α.

3 Model UNCONFORMITY

The 2-D model UNCONFORMITY by Cormier & Mellen (1984), situated in a vertical square 1.2km×1.2km, is composed of 3 homogeneous blocks. The geometry of the model, P-wave velocities α and the positions of the point source and of 22 receivers are depicted in Figure 1. The origin of Cartesian coordinate system has been selected in the left top corner, with horizontal axis x_1 pointing to the right and vertical axis x_3 pointing upwards.

The same constant P-wave quality factor of

$$Q_\alpha = 200 \quad , \tag{4}$$

was chosen by the author in all 3 blocks. The density in all 3 blocks is given by

$$\frac{\varrho}{\text{g cm}^{-3}} = 1.7 + 0.2 \, \frac{\alpha}{\text{km s}^{-1}} \quad . \tag{5}$$

The S-wave velocities and S-wave quality factors are not specified in the input data file (named `u2d-mod.dat`) for the `MODEL` package. The default S-wave velocities β,

$$\beta = 0.57735 \, \alpha \quad , \tag{6}$$

and the default S-wave quality factors Q_β,

$$Q_\beta = \frac{4}{3}\left(\frac{\beta}{\alpha}\right)^2 Q_\alpha \quad , \tag{7}$$

corresponding to the viscoelastic media elastic at volume deformations (Anderson, Ben Menahem & Archambeau 1965), are thus assumed by the subroutines of source code file `parm.for` from the `MODEL` package in all 3 blocks.

4 Ray–theory synthetic seismograms

The selection of 22 elementary waves for ray tracing has been performed according to Cormier & Mellen (1984). The codes of the elementary waves used for 2-D two–point ray tracing are shown in Figure 2.

The ray–theory synthetic seismograms were calculated for the seismic moment source

$$\mathbf{M} = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix} \quad . \tag{8}$$

This is strongly nonphysical source, the combination of the explosive source and the rotation centre rotating along the x_2 axis. It has an isotropic radiation pattern in the x_1x_3 plane for both P waves (radiated by the explosive source) and S waves (radiated by the rotation centre).

The source time function is selected in the form of the indefinite integral of the Gabor signal

$$S(t) = A\exp\left[-\left(\frac{2\pi\, f_{\mathrm{ref}}\, t}{\gamma}\right)^2\right]\cos\left(2\pi\, f_{\mathrm{ref}}\, t\right) \quad . \tag{9}$$

Reference frequency $f_{\mathrm{ref}} = 100\,\mathrm{Hz}$, dumping factor $\gamma = 4$, and amplitude

$$A = 12\pi\rho_{\mathrm{source}}[\alpha_{\mathrm{source}}]^3 \approx 258.174 \tag{10}$$

were chosen. The time functions were digitized with time step of 0.001 s.

First, the ray–theory seismograms without attenuation were synthesized both in the time domain and in the frequency domain. Because of the selected time step, the maximum amplitudes of the seismograms synthesized in the time domain may be 0% to 5% lower than those calculated in the frequency domain. Within this tolerance, the seismograms are identical. The ray–theory seismograms without attenuation synthesized in the frequency domain are shown in Figures 3 and 4.

The ray–theory seismograms with noncausal attenuation, i.e. with both real and imaginary parts of travel time frequency independent, have no physical meaning. They are displayed in Figures 5 and 6 just for comparison with the ray–theory seismograms corresponding to quasi–causal Futterman's (1962) attenuation, shown in Figures 7 and 8. The amplitude scaling of seismograms with attenuation is 4 times greater than the amplitude scaling of seismograms without attenuation.

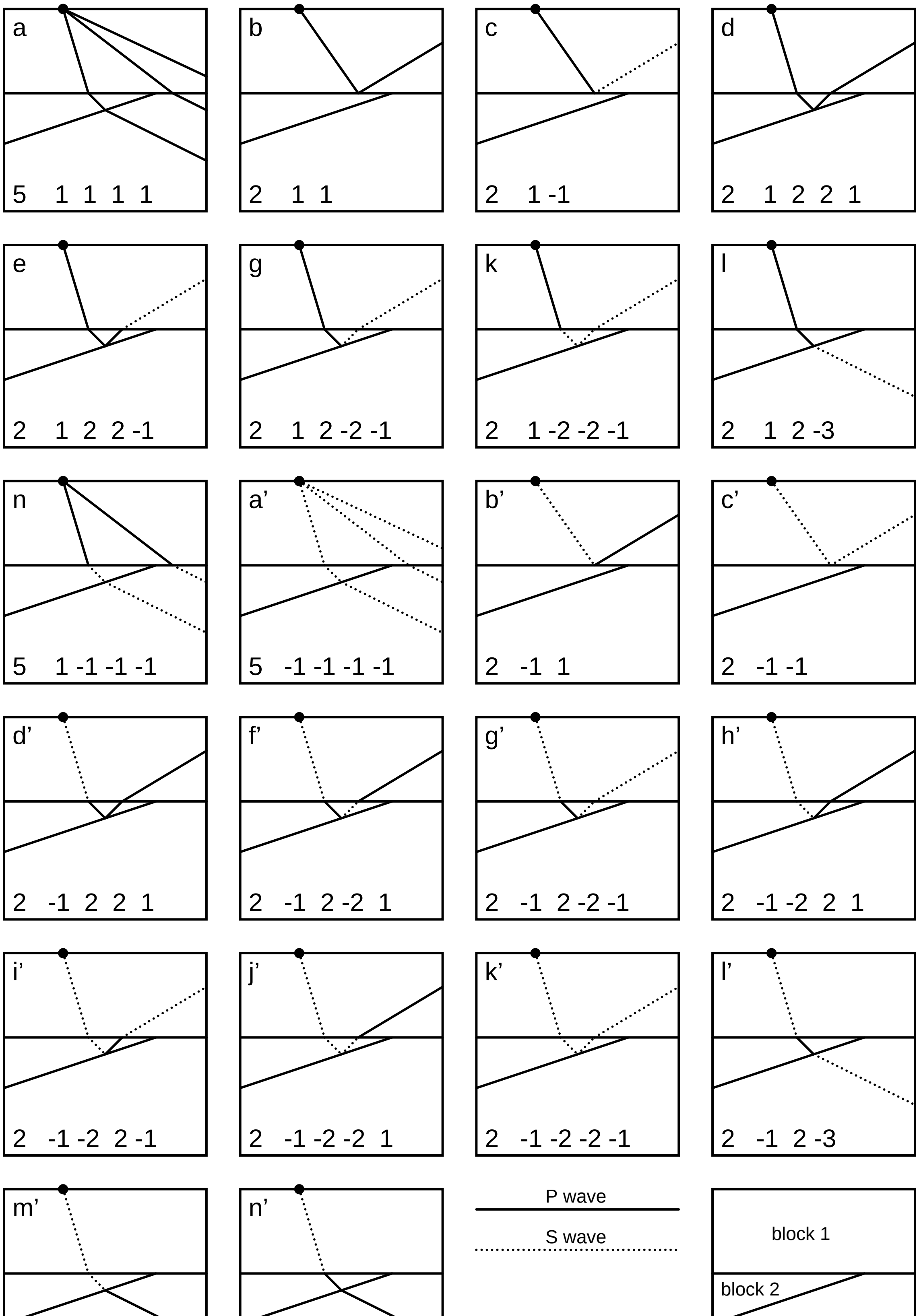

Figure 2. Sketch of the 22 elementary waves traced in the 2-D model UNCONFORMITY.

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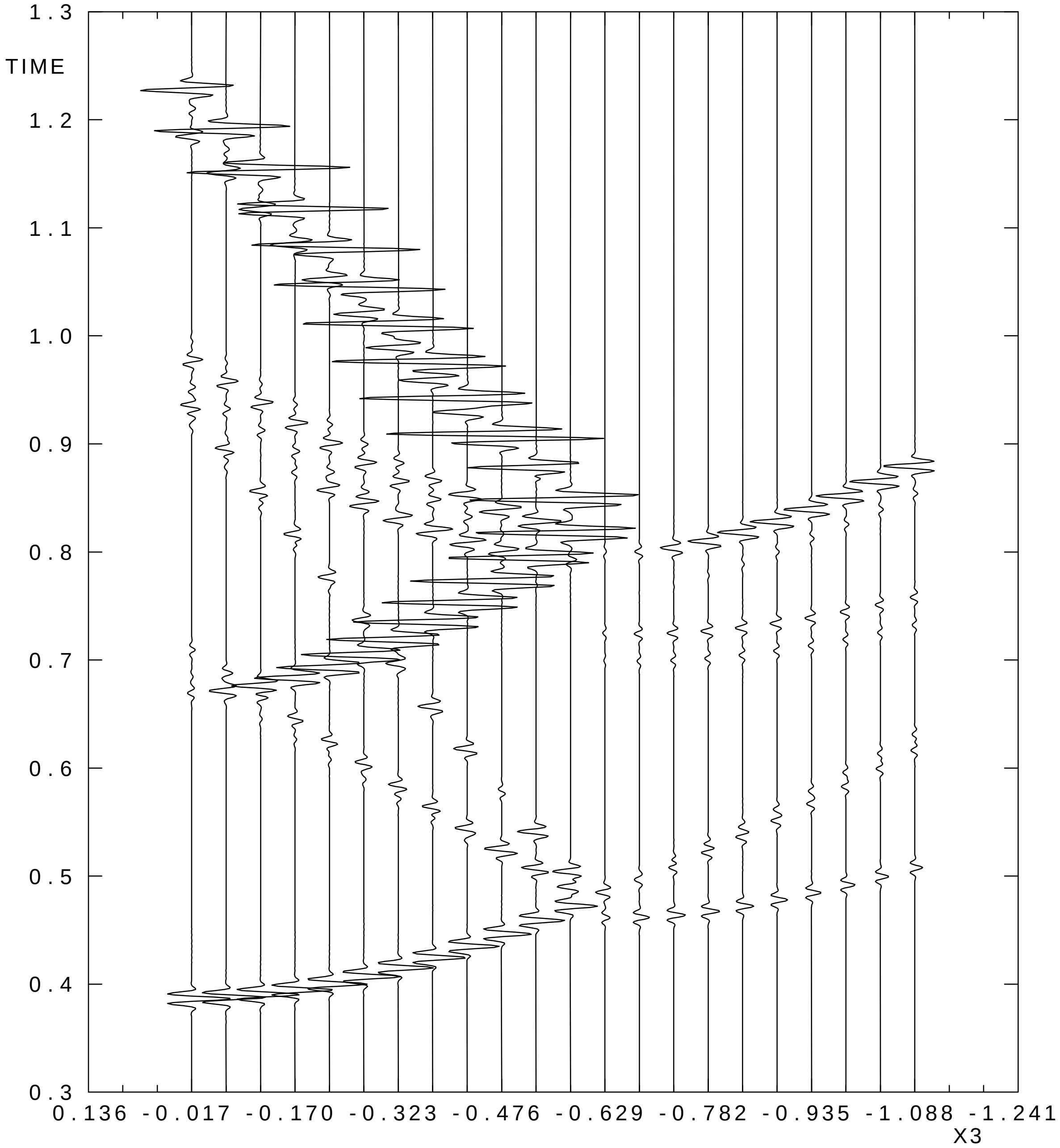

Figure 3. Ray–theory synthetic seismograms without attenuation, calculated in the frequency domain. Horizontal component.

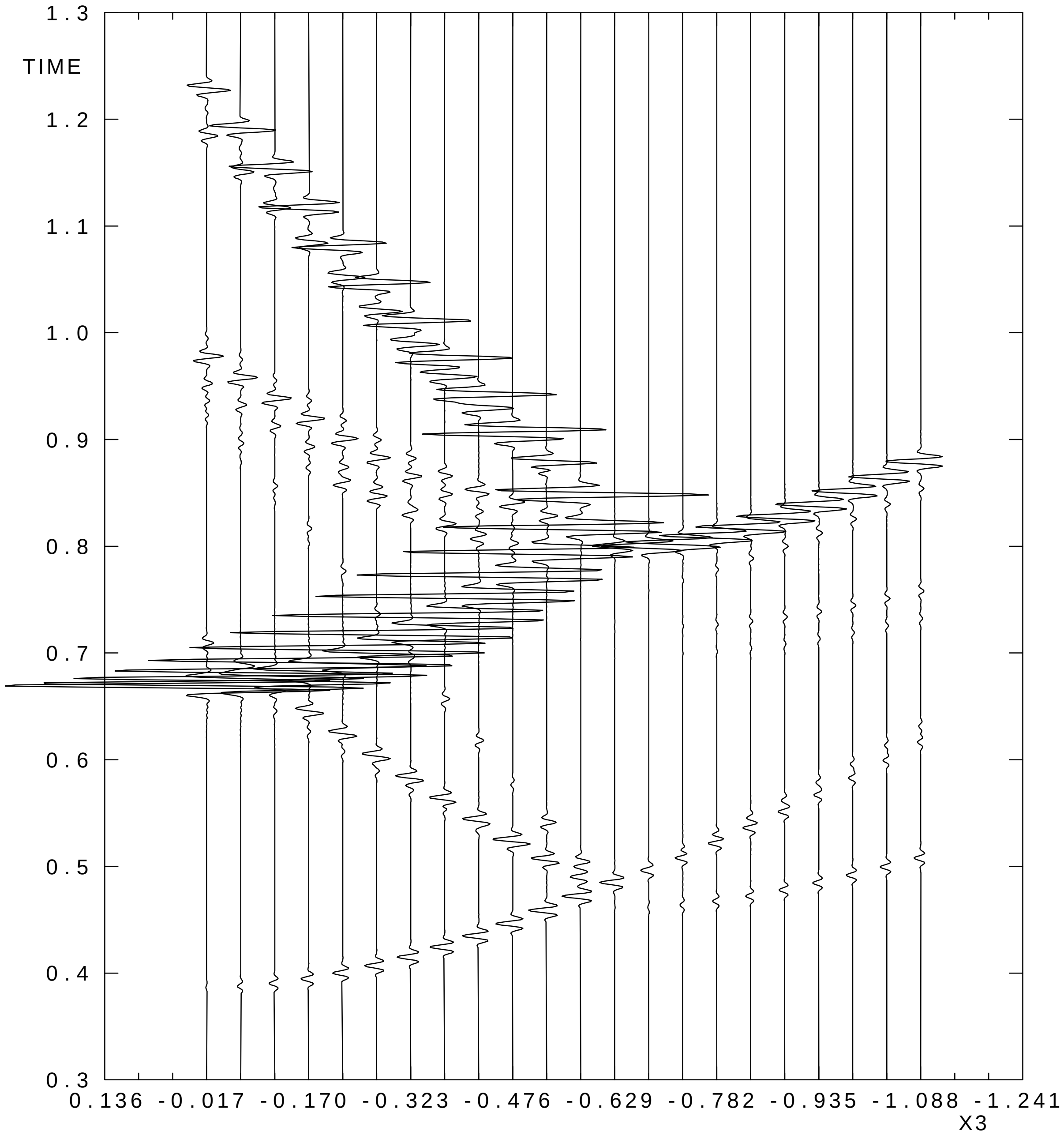

Figure 4. Ray–theory synthetic seismograms without attenuation, calculated in the frequency domain. The same scaling as in Fig. 3. Vertical component.

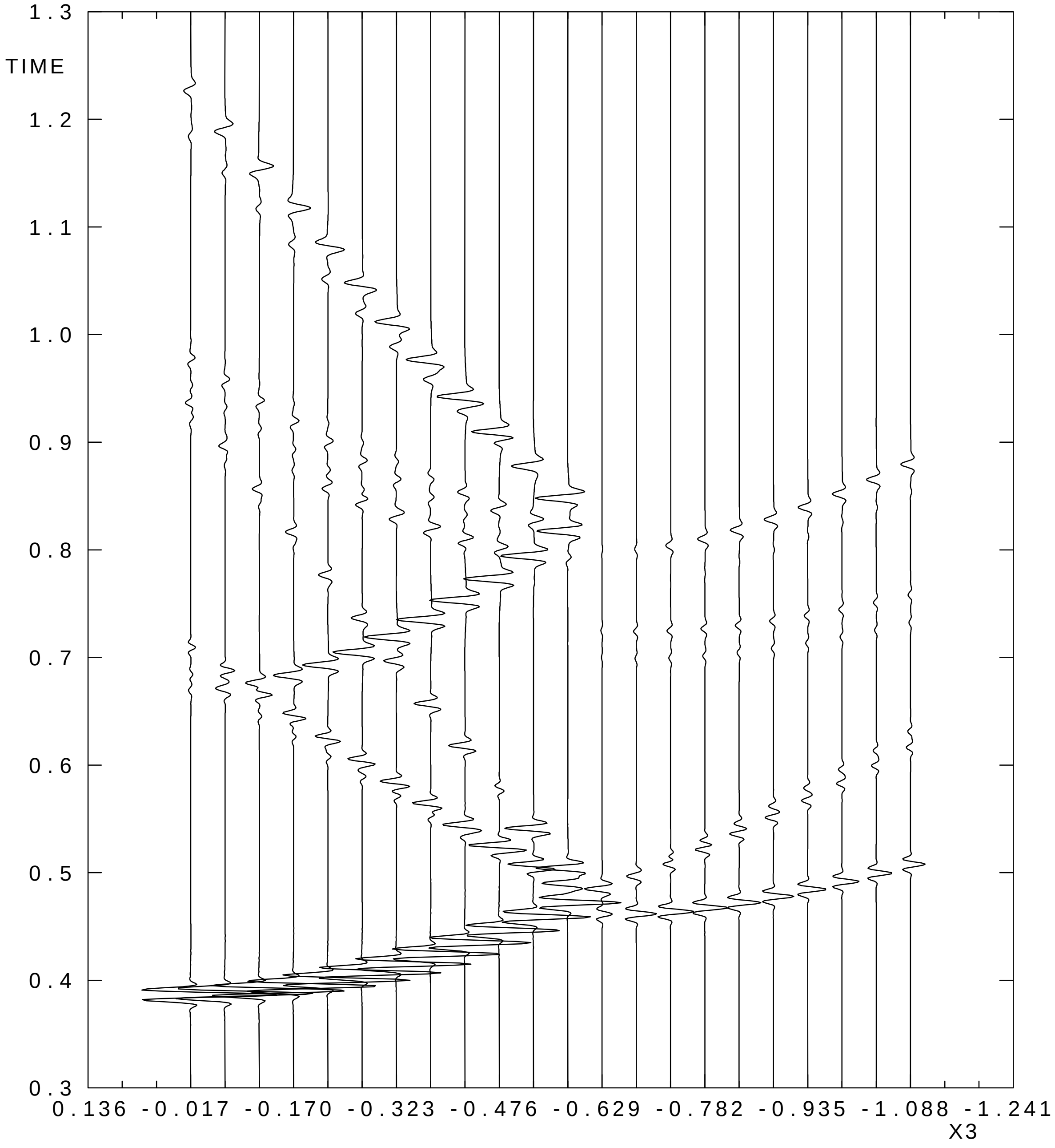

Figure 5. Ray–theory synthetic seismograms with noncausal attenuation, calculated in the frequency domain. Scaling 4 times enlarged with respect to Fig. 3. Horizontal component.

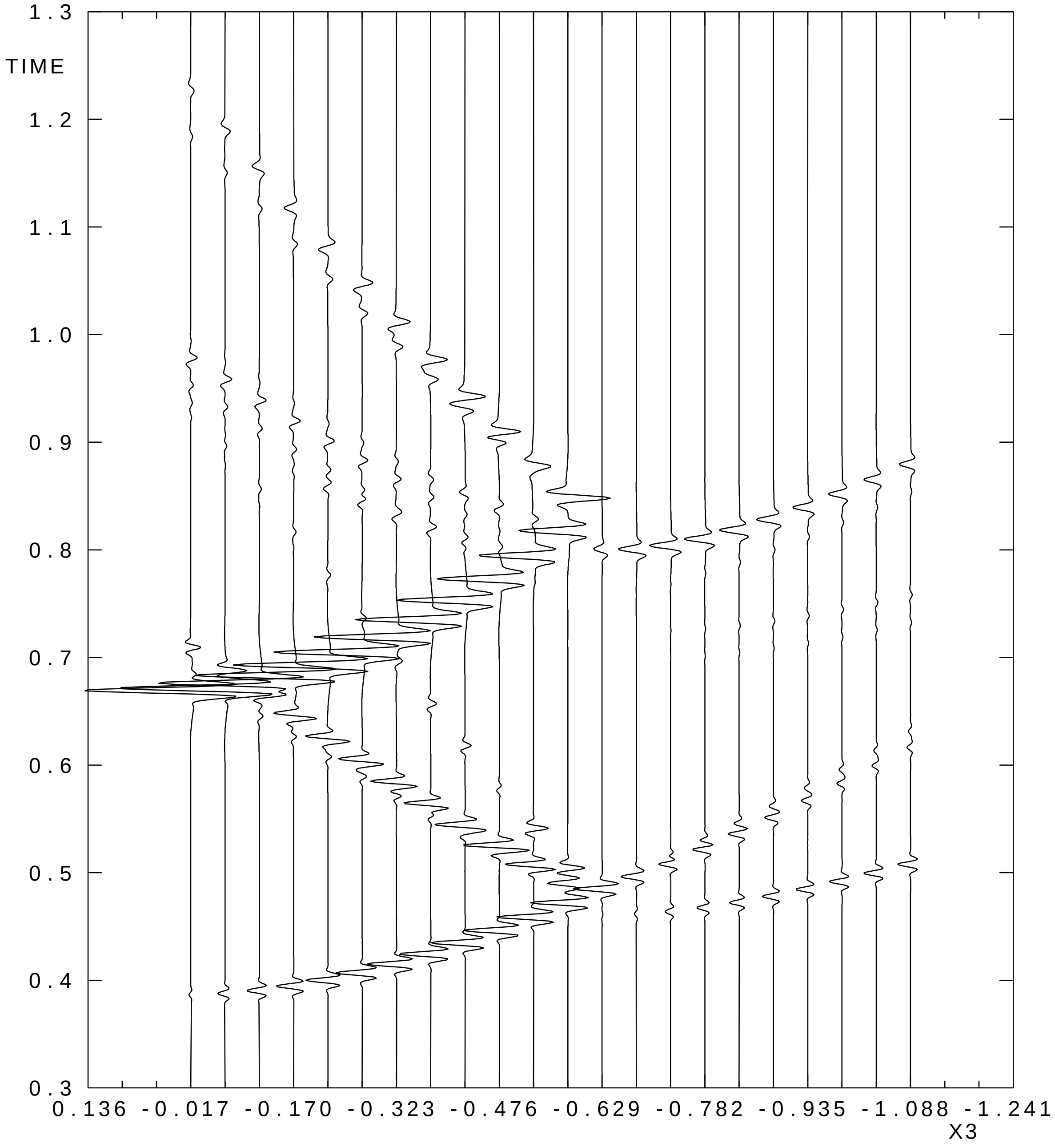

Figure 6. Ray–theory synthetic seismograms with noncausal attenuation, calculated in the frequency domain. Scaling 4 times enlarged with respect to Fig. 3. Vertical component.

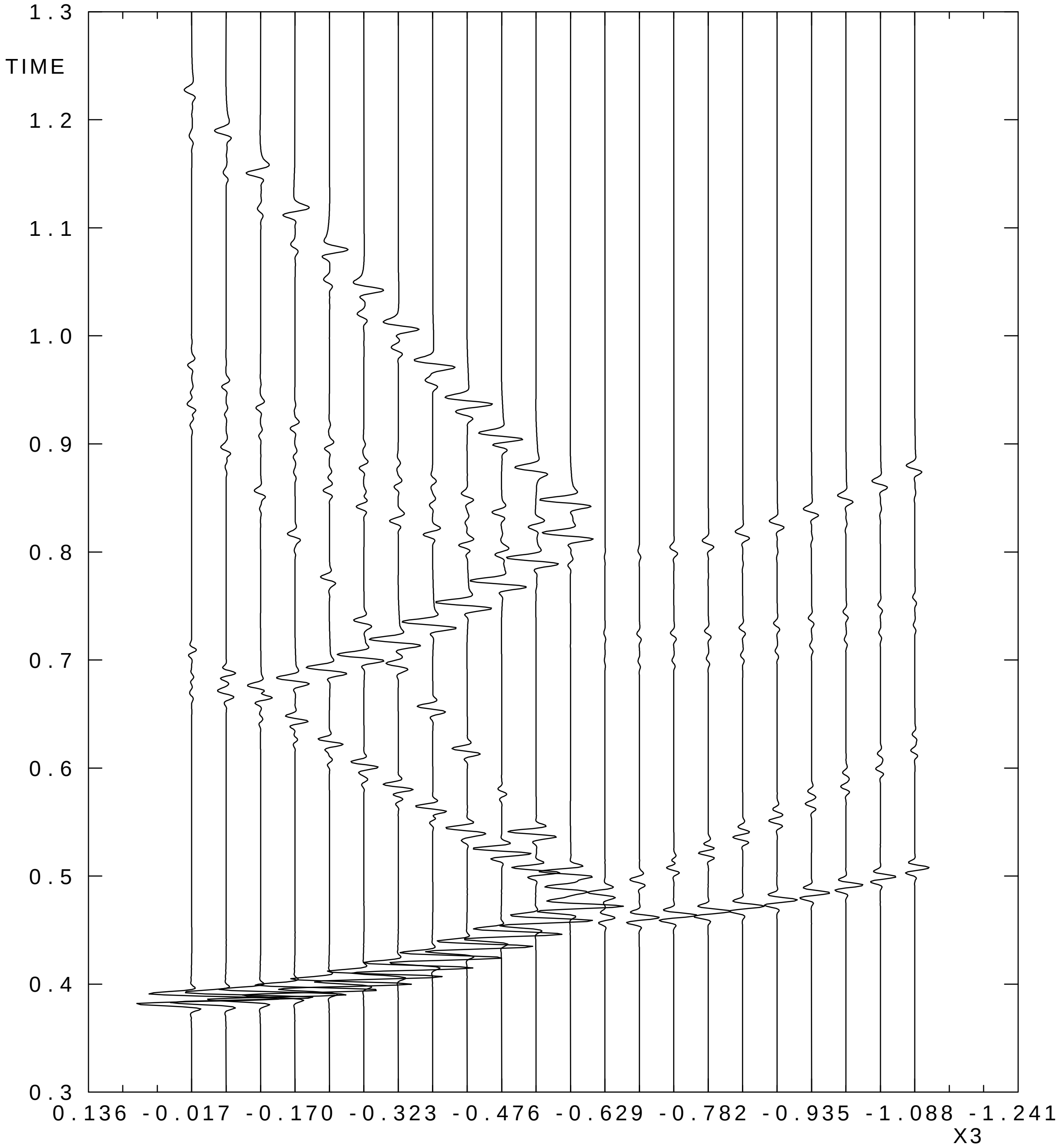

Figure 7. Ray–theory synthetic seismograms with causal attenuation, calculated in the frequency domain. Scaling 4 times enlarged with respect to Fig. 3. Horizontal component.

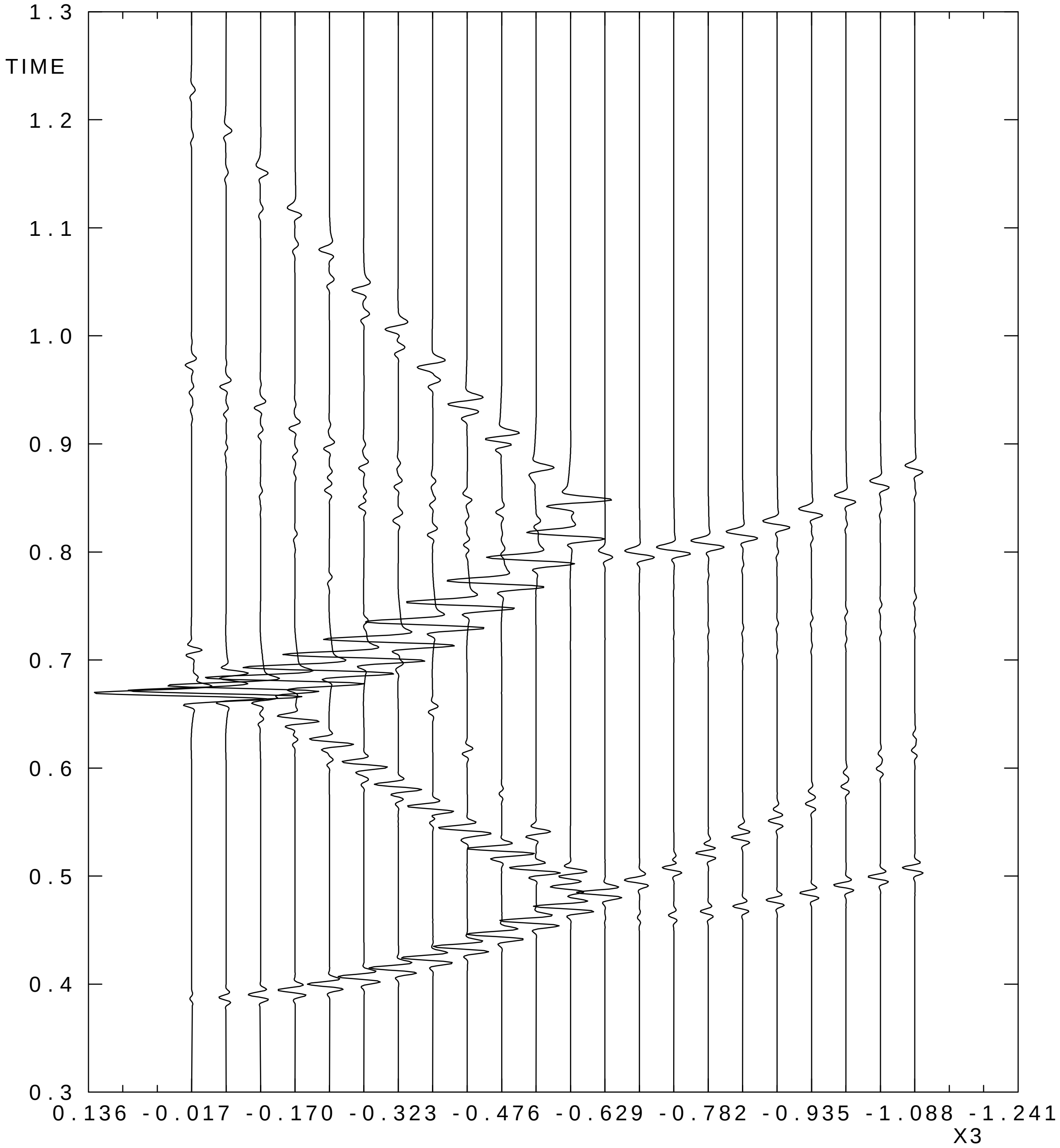

Figure 8. Ray–theory synthetic seismograms with causal attenuation, calculated in the frequency domain. Scaling 4 times enlarged with respect to Fig. 3. Vertical component.