Comparison of VSP and sonic–log data in non–vertical wells in a heterogeneous structure

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Summary

In order to compare the results of sonic–log measurements and of vertical seismic profiling (VSP), sonic–log velocities are used to estimate the corresponding travel time in the geological structure, which is then compared with the VSP travel time.

In this paper we show how to calculate the sonic–log travel time in the geological structure from the sonic–log velocities while taking into account the effects of the non–vertical propagation of seismic waves due to the source offset and due to the heterogeneous velocity in the structure, together with the effects of the well trajectory differing from strictly vertical. We also estimate the errors due to the commonly used assumption of vertical propagation.

Keywords

VSP, sonic–log data, heterogeneous media, travel time, drift curve.

1. Introduction

In borehole geophysics, sonic–log measurements and vertical seismic profiling (VSP) are two different techniques providing information about the seismic velocity in the structure surrounding a particular well. The two techniques use significantly different frequencies of seismic signals. The results of sonic–log measurements may thus differ from the results of VSP by the velocity dispersion. In order to compare the results of the two measurements, the sonic–log velocities are used to estimate the corresponding *sonic–log travel time* in the geological structure, which is then compared with the VSP travel time.

VSP provides travel times from the surface source to the geophones located in the well. These travel times are measured at frequencies from a few tens to a few hundreds Hz.

Sonic logging provides very detailed information about the slowness in the structure along the well. The slowness is averaged over the interval covered by receivers, which is typically of the order of 1 m. The slowness is typically measured at frequencies from 2 kHz to 20 kHz.

Comparisons of VSP and sonic–log measurements have broadly been addressed in the literature since Gretener (1961). Many references can be found, e.g., in the paper by Stewart, Huddleston & Kan (1984). If both measurements are done for a particular well, the so–called drift curve may be constructed. The *drift curve* displays the difference of the measured VSP travel times from the sonic–log travel times calculated using the sonic–log slowness.

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Gretener (1961) assumed a 1-D layered structure and a straight vertical well. Starting at the shallowest data point common to both sonic–log and VSP measurements, the sonic slowness is integrated over the depth, yielding the travel–time increment for strictly vertical propagation. If the VSP source is situated at the well mouth, the drift curve represents the difference of the VSP travel–time increment from the integrated sonic slowness. For the offset between the VSP source and the well mouth, Gretener (1961) assumed a homogeneous background with straight rays and applied a simple cosine correction to the VSP travel times before constructing the drift curve. Since the velocity usually increases with depth, the cosine correction is usually inaccurate.

The difference of the measured VSP travel times from the sonic–log travel times calculated using the sonic–log velocities is typically a few "milliseconds per 1000 ft" (Gretener, 1961). If the P–wave velocity is equal to, e.g., 4267 m/s (i.e. 14000 ft/s), the relative travel–time difference would be a few times 1.4%. For example, if the well in a horizontally layered structure was deviated by 7° from the vertical, the VSP slowness vector at the well would also be deviated by 7° from the vertical, and if we neglected the deviations, the error of the above–mentioned relative travel–time difference might reach 2.2%. Thus we should not neglect the error caused by not taking into account the deviation of the well and of the VSP slowness vector from the vertical.

The aim of this paper is to show how to calculate the sonic–log travel time while taking into account the effects of the non–vertical propagation of seismic waves due to the source offset and due to the heterogeneous velocity in the structure, together with the effects of the well trajectory differing from strictly vertical. Note that ray tracing in a smooth reference model is used to calculate the reference travel times and reference slowness vectors.

2. Sonic–log travel time for a non–vertical well in a heterogeneous structure

We calculate the reference travel time and the reference slowness vector in a reference smooth velocity model. Using the reference slowness vector, we estimate the slowness vector in the locally 1–D strongly heterogeneous structure inferred from sonic logs, and integrate it along the well to obtain the sonic–log travel time from the VSP source.

2.1. Assumptions about the geological structure

To be able to deal with true well trajectories differing from the strictly vertical direction, and to be able to consider the VSP source at some distance from the well mouth, we need to make some assumptions about the velocity distribution in the geological structure. We thus assume that a reference smooth velocity model is available, and that the strong variations of the slowness in the geological structure are locally one–dimensional in the direction given by unit vector n_i normal to the local layering. The strong 1-D slowness variations are thus locally functions of "level" $\xi = n_i x_i$. Note that in many cases the reference smooth model is one–dimensional with the slowness varying only vertically, and vector n_i is constant and vertical, but here we consider general smoothly varying $n_i = n_i(\mathbf{x})$ for the sake of generality.

2.2. Reference travel time and slowness vector in a smooth velocity model

We denote the reference travel time calculated from the VSP source in the reference smooth velocity model $\tau^0(\mathbf{x})$, and the corresponding slowness vector $p_i^0(\mathbf{x})$. They may be calculated, e.g., by ray tracing (Červený, Klimeš & Pšenčík, 1988) or by wavefront tracing (Vinje, Iversen & Gjøystdal, 1993).

2.2.1. Interpolation of the reference travel time and slowness vector

To be able to calculate of the sonic–log travel time, we need to know the reference slowness vector at points $\mathbf{x}$ along the part of the well where slowness $u(\mathbf{x})$ has been determined by a sonic log. Moreover, if we also know the reference travel time along that part of the well, we will be able not only to construct the drift curve (21), but also to compare the sonic–log travel time corresponding to the sonic–log slowness with the reference travel time corresponding to the smooth reference model.

It is a standard procedure in ray–theory based methods that during ray tracing all quantities are calculated along the rays, and these quantities are then interpolated between the rays. Assume that we have calculated reference travel time $\tau^0(\mathbf{x})$ and the corresponding reference slowness vector $p_i^0(\mathbf{x})$ at the VSP receivers $\mathbf{x}^{\mathrm{R}}$ situated in the well or in its vicinity (to allow for position measurement uncertainties),

$$\tau^{\mathrm{R}} = \tau^0(\mathbf{x}^{\mathrm{R}}) \quad , \qquad p_i^{\mathrm{R}} = p_i^0(\mathbf{x}^{\mathrm{R}}) \quad . \tag{1}$$

We first interpolate the reference travel time and the corresponding slowness vector along straight lines between receivers to "level" $\xi = n_i x_i$. Along the straight lines, we shall apply the linear interpolation of coordinates x_i^{A} and of slowness–vector components p_i^{A}, and the cubic interpolation of reference travel time τ^{A}. We then extrapolate the reference travel time from points $\mathbf{x}^{\mathrm{A}}$ of the straight line to points $\mathbf{x}$ of the curved well by the linear Taylor expansion. The slowness vector is extrapolated as a constant vector.

Let x_i^{B}, τ^{B}, p_i^{B} and x_i^{C}, τ^{C}, p_i^{C} be two consecutive multiplets of values x_i^{R}, τ^{R}, p_i^{R}. We denote

$$w^{\mathrm{B}} = \frac{n_i(\mathbf{x})\,[x_i - x_i^{\mathrm{C}}]}{n_j(\mathbf{x})\,[x_j^{\mathrm{B}} - x_j^{\mathrm{C}}]} \tag{2}$$

and

$$w^{\mathrm{C}} = \frac{n_i(\mathbf{x})\,[x_i - x_i^{\mathrm{B}}]}{n_j(\mathbf{x})\,[x_j^{\mathrm{C}} - x_j^{\mathrm{B}}]} = 1 - w^{\mathrm{B}} \quad . \tag{3}$$

The linear interpolation of coordinates then reads (Bulant & Klimeš, 1999, eq. 2)

$$x_i^{\mathrm{A}} = w^{\mathrm{B}} x_i^{\mathrm{B}} + w^{\mathrm{C}} x_i^{\mathrm{C}} \quad , \tag{4}$$

and the linear interpolation of the slowness vector reads

$$p_k^{\mathrm{A}} = w^{\mathrm{B}} p_k^{\mathrm{B}} + w^{\mathrm{C}} p_k^{\mathrm{C}} \quad . \tag{5}$$

The cubic interpolation of reference travel time reads (Bulant & Klimeš, 1999, eq. 25)

$$\tau^{\mathrm{A}} = a(w^{\mathrm{B}})\tau^{\mathrm{B}} + a(w^{\mathrm{C}})\tau^{\mathrm{C}} + (w^{\mathrm{B}})^2 w^{\mathrm{C}} p_i^{\mathrm{B}}[x_i^{\mathrm{C}} - x_i^{\mathrm{B}}] - w^{\mathrm{B}}(w^{\mathrm{C}})^2 p_i^{\mathrm{C}}[x_i^{\mathrm{C}} - x_i^{\mathrm{B}}] \quad , \tag{6}$$

where

$$a(w) = w^2(3 - 2w) \tag{7}$$

(Bulant & Klimeš, 1999, eq. 21).

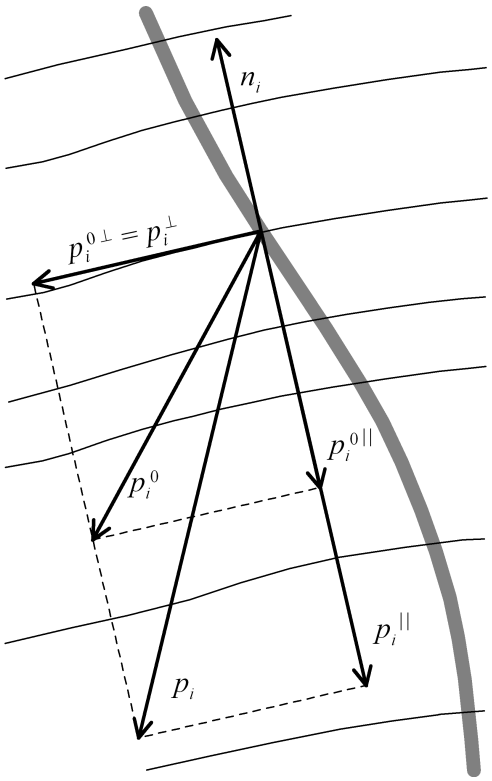

Figure 1. A simple sketch of a well (bold grey line) in a layered geological structure (thin lines). The arrows show the reference slowness vector $p_i^0(\mathbf{x})$, the slowness vector in the geological structure $p_i(\mathbf{x})$, and their projections onto the plane perpendicular to n_i, and onto the direction of n_i.

Then

$$p_i^0(\mathbf{x}) = p_i^{\mathrm{A}} \tag{8}$$

and

$$\tau^0(\mathbf{x}) = \tau^{\mathrm{A}} + p_i^{\mathrm{A}}[x_i - x_i^{\mathrm{A}}] \quad . \tag{9}$$

2.3. Approximate slowness vector in the geological structure

In accordance with our assumption of strong slowness variations in the geological structure appearing only in the direction given by unit vector n_i, we assume the projection of the slowness–vector p_k in the geological structure onto the plane perpendicular to n_i to be approximately equal to the same projection of the reference slowness vector p_k^0,

$$[\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k(\mathbf{x}) = [\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k^0(\mathbf{x}) \quad . \tag{10}$$

The slowness vector in the geological structure may thus be expressed as the sum $p_i^{\perp}(\mathbf{x}) + p_i^{\parallel}(\mathbf{x})$ of its projection $p_i^{\perp}(\mathbf{x}) = [\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k(\mathbf{x})$ onto the plane perpendicular to n_i, and of its projection $p_i^{\parallel}(\mathbf{x}) = n_i(\mathbf{x})n_k(\mathbf{x})p_k(\mathbf{x})$ onto the direction of n_i,

$$p_i(\mathbf{x}) = [\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k(\mathbf{x}) + n_i(\mathbf{x})n_k(\mathbf{x})p_k(\mathbf{x}) \quad , \tag{11}$$

see Figure 1. The first projection is determined by equation (10), and we have to determine component

$$p^{\parallel} = n_k(\mathbf{x})p_k(\mathbf{x}) \tag{12}$$

of the slowness vector $p_k(\mathbf{x})$ in the direction $n_i(\mathbf{x})$,

$$p_i(\mathbf{x}) = p_i^0(\mathbf{x}) - n_i(\mathbf{x})n_k(\mathbf{x})p_k^0(\mathbf{x}) + n_i(\mathbf{x})p^{\parallel} \quad . \tag{13}$$

Component $p^{\|}$ of the slowness vector in the direction n_i can be calculated from slowness $u(\mathbf{x})$ determined by a sonic log using eikonal equation

$$p_i(\mathbf{x})p_i(\mathbf{x}) = [u(\mathbf{x})]^2 \quad . \tag{14}$$

Inserting (13) into (14), we obtain

$$p^{\|} = \mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})]\sqrt{[u(\mathbf{x})]^2 - p_k^0(\mathbf{x})p_k^0(\mathbf{x}) + [n_k(\mathbf{x})p_k^0(\mathbf{x})]^2} \quad . \tag{15}$$

Then

$$p_i(\mathbf{x}) = p_i^0(\mathbf{x}) + n_i(\mathbf{x})[p^{\|} - n_k p_k^0(\mathbf{x})] \quad . \tag{16}$$

2.4. Sonic–log travel time

At a selected point $\mathbf{x} = \mathbf{x}^{\mathrm{select}}$, usually at the shallowest point for which the sonic–log slowness is measured, we put

$$\tau(\mathbf{x}^{\mathrm{select}}) = \tau^0(\mathbf{x}^{\mathrm{select}}) \quad . \tag{17}$$

Then we use the trapezoidal quadrature along the well,

$$\tau(\mathbf{x}^{\mathrm{new}}) = \tau(\mathbf{x}) + \frac{1}{2}[p_i(\mathbf{x}^{\mathrm{new}}) + p_i(\mathbf{x})]\,[x_i^{\mathrm{new}} - x_i] \quad , \tag{18}$$

and obtain the sonic–log travel time $\tau(\mathbf{x})$ along the part of the well where slowness $u(\mathbf{x})$ has been determined by a sonic log.

2.5. Travel–time differences and the drift curve

Along the part of the well where the sonic–log slowness $u(\mathbf{x})$ has been determined, we may plot the differences

$$\tau(\mathbf{x}) - \tau^0(\mathbf{x}) \tag{19}$$

of the sonic–log travel time from the reference travel time.

Along the part of the well where the VSP receivers are located, we may plot the differences

$$\tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{R}}) - \tau^0(\mathbf{x}^{\mathrm{R}}) \tag{20}$$

of the VSP travel time from the reference travel time.

Along the part of the well where both the sonic–log slowness $u(\mathbf{x})$ and VSP travel times have been measured, we may plot the differences

$$\tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{R}}) - \tau(\mathbf{x}^{\mathrm{R}}) \tag{21}$$

and refer to the plot as the *drift curve*.

Note that in other formulations the calculation of the sonic–log travel time is usually initiated at the shallowest point $\mathbf{x}^{\mathrm{Rselect}}$ for which both the VSP and sonic–log measurements are available by putting $\tau(\mathbf{x}^{\mathrm{Rselect}}) = \tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{Rselect}})$. We believe that our initiation (17) of calculation of τ is more appropriate, because the resulting drift curve does not reflect the systematic shift caused by a possible error of the VSP travel time at the initial point $\mathbf{x}^{\mathrm{Rselect}}$.

3. Error of sonic–log travel time due to assumption of "vertical propagation"

Under "vertical propagation" we understand the assumption that

$$p_i(\mathbf{x}) = n_i(\mathbf{x})u(\mathbf{x})\ \mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})] \quad , \tag{22}$$

i.e., the assumption that all the rays are always parallel with vector n_i. As already mentioned in Section 2.1, vector n_i will in most cases be vertical, and assumption (22) will then be the assumption of vertical propagation.

By inserting assumption (22) into the trapezoidal quadrature (18) along the well, we obtain the approximation of the sonic–log travel time for "vertical propagation" in the form of

$$\tilde{\tau}(\mathbf{x}^{\mathrm{new}}) = \tilde{\tau}(\mathbf{x}) + \frac{1}{2}[u(\mathbf{x}^{\mathrm{new}}) + u(\mathbf{x})]\ n_i(\mathbf{x})[x_i^{\mathrm{new}} - x_i]\ \mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})] \quad . \tag{23}$$

We denote the tangent vector to the well $t_i(\mathbf{x})$,

$$\Delta\tau(\mathbf{x}) = p_i(\mathbf{x})\, t_i(\mathbf{x}) \tag{24}$$

the increment of the sonic–log travel time in the direction along the well, and

$$\Delta\tilde{\tau}(\mathbf{x}) = n_i(\mathbf{x})\, t_i(\mathbf{x})\ u(\mathbf{x})\ \mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})] \tag{25}$$

the increment of the approximate sonic–log travel time in the direction along the well under the assumption of "vertical propagation". The local relative sonic–log travel–time error is then

$$\varrho\tau = \frac{\Delta\tilde{\tau} - \Delta\tau}{\Delta\tau} \quad . \tag{26}$$

We may calculate and inspect this relative error along the part of the well where the slowness $u(\mathbf{x})$ has been determined by a sonic log.

Note that the calculation of the sonic–log travel time described in Sections 2.1 to 2.5 excludes this error due to the assumption of "vertical propagation", which is the reason why we consider this method of calculation of the sonic–log travel time to be superior to Gretener's (1961) method.

Let us now approximate error (26). We decompose the slowness vector into the vector parallel with n_i and into vector $p_i^\perp$ perpendicular to n_i:

$$p_i = n_i p^\parallel + p_i^\perp \quad . \tag{27}$$

Then

$$\Delta\tau = n_i t_i p^\parallel + p_i^\perp t_i \tag{28}$$

and

$$\Delta\tilde{\tau} = n_i t_i p^\parallel \sqrt{1 + p_i^\perp p_i^\perp\, (p^\parallel)^{-2}} \quad . \tag{29}$$

We see that $\varrho\tau = 0$ if $p_i^\perp = 0$. Note that vector $p_i^\perp = p_i^{0\perp}$, unlike $p^\parallel$, varies smoothly. If we insert (28) and (29) into (26) and neglect $p_i^\perp t_i$ in the denominator, we may decompose relative error (26) into two terms,

$$\varrho\tau = \left[\sqrt{1 + \frac{p_i^\perp}{p^\parallel}\frac{p_i^\perp}{p^\parallel}} - 1\right] - \frac{p_i^\perp}{p^\parallel}\frac{t_i}{n_k t_k} \quad . \tag{30}$$

For small $|p_i^\perp|/p^\parallel$, the first term may further be approximated. Finally, introducing the decomposition

$$t_i = n_i t^\parallel + t_i^\perp \quad , \tag{31}$$

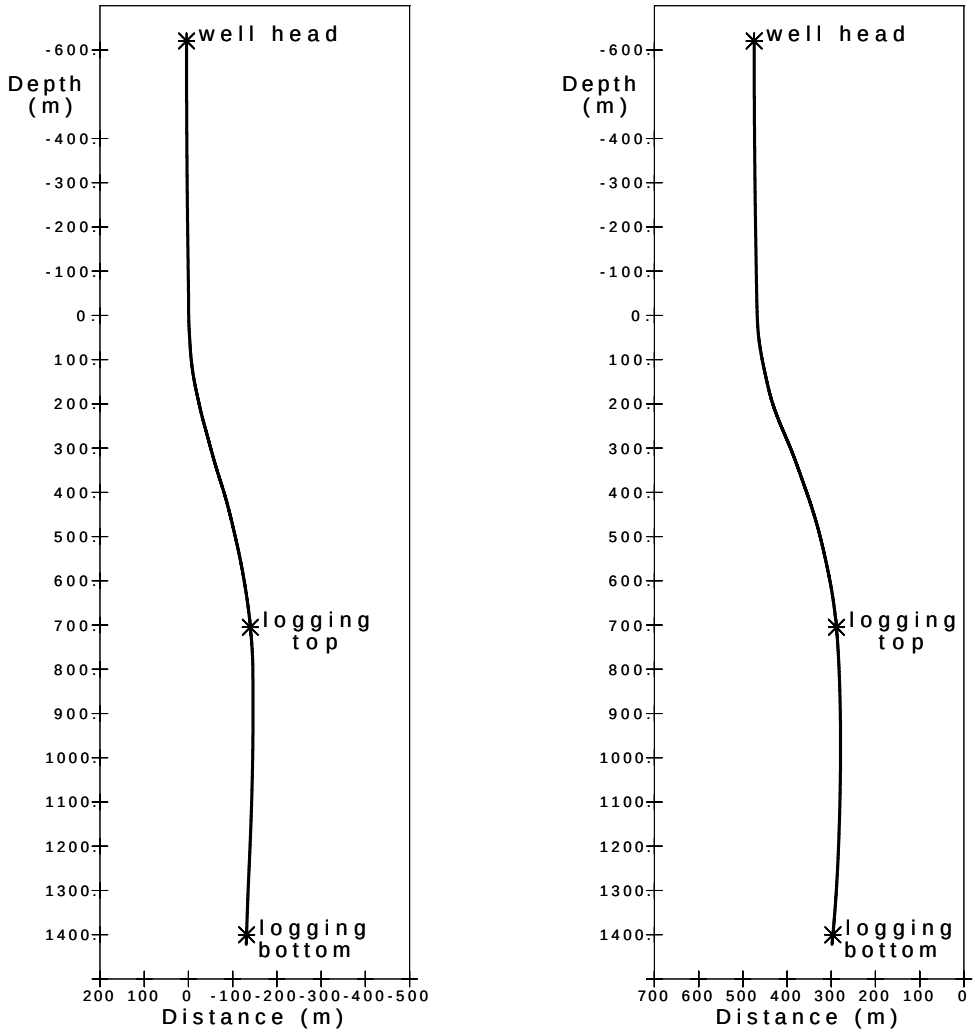

Figure 2. The trajectory of the well in which the measurements were carried out. The left plot shows the projection onto the x-z plane, the right plot the projection onto the y-z plane.

we arrive at

$$\varrho\tau = \frac{1}{2}\frac{p_i^{\perp}}{p^{\parallel}}\frac{p_i^{\perp}}{p^{\parallel}} - \frac{p_i^{\perp}}{p^{\parallel}}\frac{t_i^{\perp}}{t^{\parallel}} \quad . \tag{32}$$

The first error term is independent of the well inclination and depends on the "horizontal" offset of the VSP source from the studied part of the well only. The second error term also depends on the well inclination and may be much larger than the first term for wells deviated from the "vertical". Equation (32) provides a better understanding of the sonic–log travel–time error caused by neglecting the non–verticality of the well and by not taking into account the non–verticality of the VSP slowness vector in a heterogeneous structure. The equation, for example, allowed us to estimate the 2.2% error mentioned in the Introduction for the case of the 7° non–vertical deviation of the well and slowness.

4. Numerical example

The authors have been motivated to carry out this study when they faced the task of comparing the results of sonic logging and of VSP in a well which considerably deviated from the vertical, see Figure 2. The depth scale of Figure 2 is in metres with

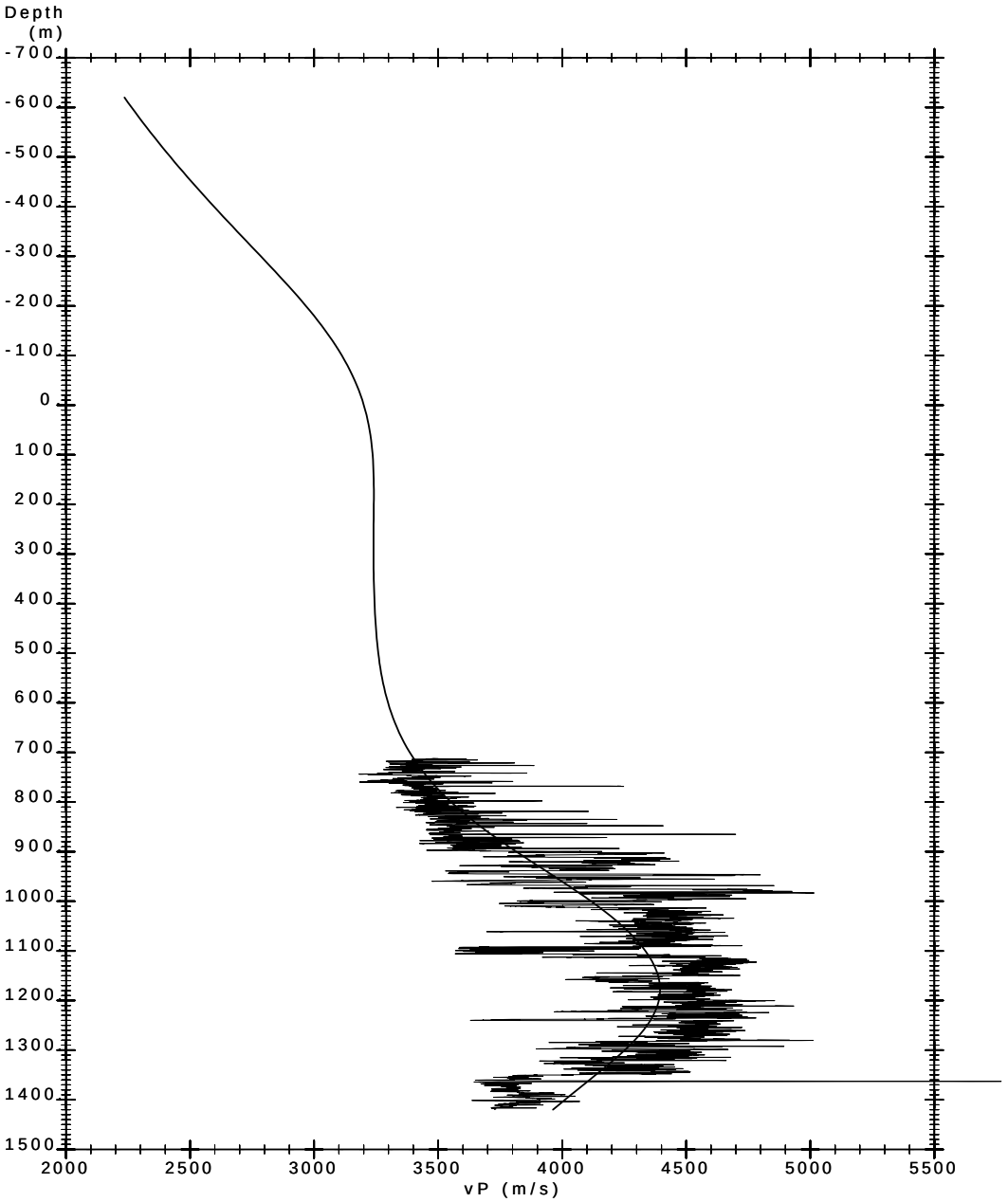

Figure 3. The P-wave velocity measured by sonic logging (thin line) in the well, and the P-wave velocity (bold line) in the smooth reference velocity model.

zero depth at sea level. The well head was at a depth of -620.27 m, the bottom of the well at a depth of 1421.76 m. Sonic logging was performed between the depths of 712.24 m and 1419.39 m at steps of 0.1528 m (0.5 ft) along the well. There were 64 VSP receivers in the depth range from -498.33 m to 1370.37 m at steps of 30.24 m (100 ft) along the well.

The well is located in Texas, North America, where the geological structure is dominated by approximately horizontal sedimentary layers. Thus the reference smooth velocity model chosen was 1-D with vertical velocity variations only, see Figure 3. The model was constructed from sonic–log velocities and from VSP travel times using the method of minimizing the Sobolev norm composed of the second velocity derivatives according to Bulant (2002).

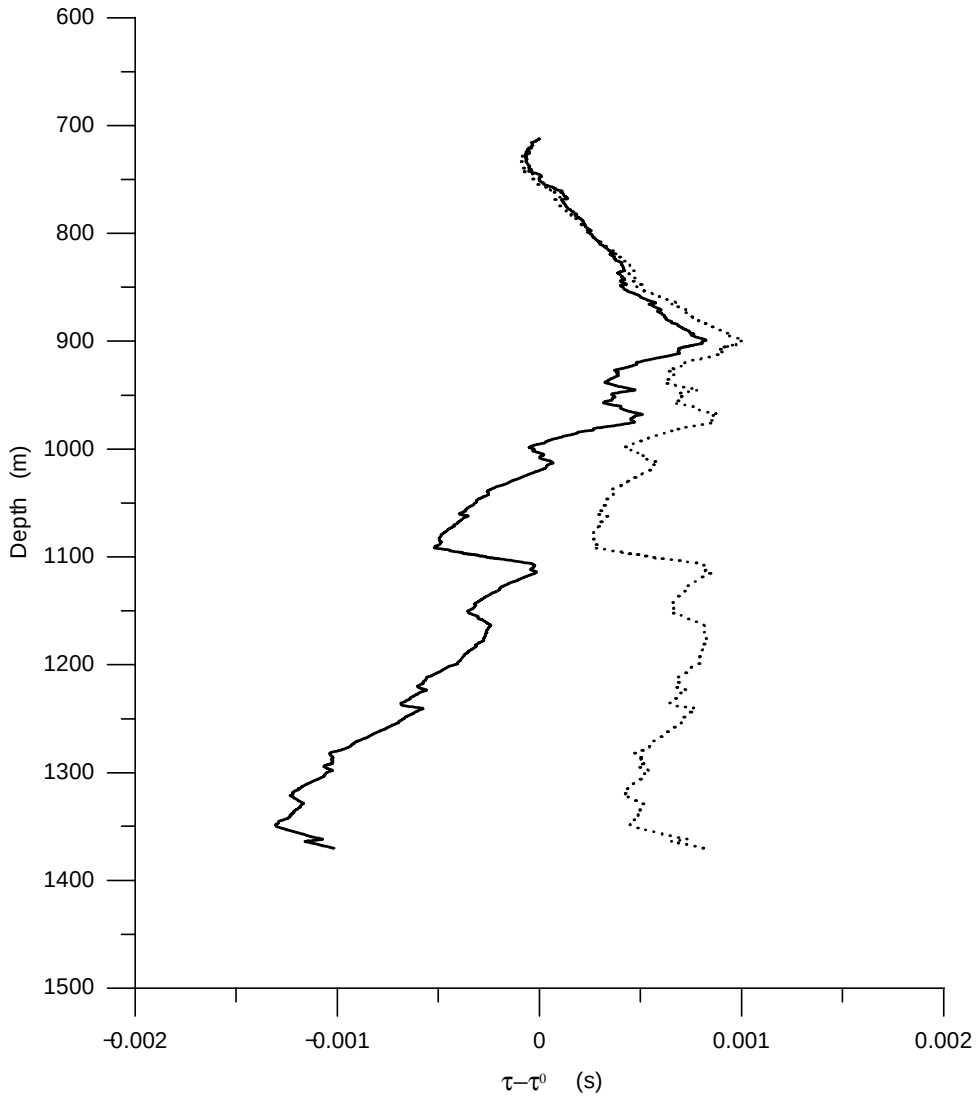

Figure 4. The difference of the sonic–log travel time from the reference travel time (solid line), and the same difference calculated under the assumption of vertical propagation (dotted line).

The difference (19) of the sonic–log travel time from the reference travel time in the smooth velocity model was calculated according to equation (18), and is plotted as the solid line in Figure 4. We have also calculated the approximate sonic–log travel time under the assumption of vertical propagation using equation (23), and plotted its difference from the reference travel time by the dotted line in Figure 4.

5. Conclusions

We have proposed the method for calculating the sonic–log travel time in the geological structure from the values of velocity measured by sonic logging, while taking into account the effects of the non–vertical propagation of seismic waves due to the source offset and due to the heterogeneous velocity in the structure, together with the effects of the well trajectory differing from strictly vertical. We also estimated the errors due to the commonly used assumption of vertical propagation.

In the numerical example, the difference of the sonic–log travel time from the reference travel time is approximately -1 millisecond over the 700 m of the depth

interval of the measurements. The same travel–time difference under the assumption of vertical propagation is almost +1 millisecond. The error caused by the assumption of vertical propagation for this particular well is thus almost twice as large as the value of the difference of the sonic–log travel time from the reference travel time.

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