Gaussian packet pre–stack depth migration

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Summary

We present a complete algorithm and the very first numerical examples of a brand new imaging method — the Gaussian packet pre–stack depth migration.

The advantages over the methods based on Gaussian beams should consist in increased accuracy, computational efficiency and a direct relation between the regions in the common shot gather and corresponding regions in the migrated section. The Gaussian packet pre–stack depth migration is especially suitable for a target–oriented imaging.

Key words

Gaussian packet, Gabor function, pre–stack depth migration, target–oriented imaging, multi–valued travel time.

1 Introduction

Gaussian packets are waves whose envelopes at any given time are nearly Gaussian in space. They represent high-frequency asymptotic solutions of the elastodynamic equation, which are concentrated close to the central point of the packet (e.g., Babich & Ulin, 1981; Ralston, 1983; Klimeš, 1989a) .

As well as Gaussian beams, Gaussian packets become inaccurate solutions of the elastodynamic equation if they spread excessively as they propagate. This spreading depends on the complexity of the velocity model and on the initial shape of Gaussian packets. Thus, we should use sufficiently smooth velocity model and *optimize* the initial shape of the packets.

The Gussian packet migration algorithm consists of four basic steps:
a) Preparation of a suitable smooth velocity model (e.g., Žáček, 2001a; Žáček, 2002).
b) Optimization of the shape of Gaussian packets (Klimeš, 1989b; Žáček, 2001b; Žáček, 2001c).
c) Decomposition of the wave field into optimized Gaussian packets (Žáček, 2003a; Žáček, 2003b).
d) Back–propagation of the wave field using Gaussian packets and application of the imaging functional.

The word *optimized* means that the shape of Gaussian packets depends not only on the frequency, but also on the coordinate of the intersection of the central ray of a Gaussian packet with the profile, on its arrival time, and on the component of the slowness vector along the profile.

The migration algorithm is designed here especially for numerical testing in the 2–D acoustic Marmousi model (Versteeg & Grau, 1991). That is why only scalar waves are dealt with in this paper, although the generalization of the Gaussian packet migration to the elastic waves is straightforward.

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2 Basic theory

In the case of the component notation, the lower–case indices take the values $i, j, ... = 1, 2, 3$; the Greek indices take the values $\alpha, \beta, ... = 1, 2, 3, 4$. The Einstein summation over the pairs of identical indices is used. The asterisk (e.g., u^*) denotes the complex conjugate.

We assume the imaging functional for scalar waves in the form of

$$M(u,U) = \int \mathrm{d}\omega \; \frac{\widehat{F}(\omega)\widehat{u}^*(\omega)\widehat{U}(\omega)}{\widehat{u}^*(\omega)\widehat{u}(\omega)} \left[\int \mathrm{d}\omega \; \widehat{F}(\omega)\right]^{-1} \quad , \tag{1}$$

where $\widehat{u}$ is the temporal Fourier transform of the incident wavefield u and $\widehat{U}$ is the temporal Fourier transform of the back–propagated scattered wavefield U. Let us take the spectral filter $\widehat{F}$ in the form of the Fourier transform $\widehat{s}$ of the source time function s, possibly shifted by time $-t_{\mathrm{S}}$ or phase $-\varphi_{\mathrm{S}}$,

$$\widehat{F}(\omega) = \widehat{s}(\omega) \exp(\mathrm{i}\varphi_{\mathrm{S}} - \mathrm{i}\omega t_{\mathrm{S}}) \quad . \tag{2}$$

Assume the incident wavefield $\widehat{u}(\omega)$ approximated by several ray–theory elementary seismograms $\widehat{u}_{\Omega}(\omega)$ corresponding to individual branches Ω of possibly multivalued travel time,

$$\widehat{u}(\omega) = \sum_{\Omega} \widehat{u}_{\Omega}(\omega) \quad , \tag{3}$$

with

$$\widehat{u}_{\Omega}(\omega) = \widehat{s}(\omega)\widehat{a}_{\Omega} \exp(\mathrm{i}\omega\tau_{\Omega}) \quad , \tag{4}$$

where a_{Ω} is the amplitude corresponding to travel time τ_{Ω}. To avoid singularities and to simplify the calculation, we apply approximation

$$\widehat{u}^*(\omega)\widehat{u}(\omega) \approx \widehat{s}^*(\omega)\widehat{s}(\omega) \sum_{\Omega} \widehat{a}^*_{\Omega}\widehat{a}_{\Omega} \quad . \tag{5}$$

The back–propagated scattered wavefield may be composed of individual Gaussian packets U_{R},

$$\widehat{U}(\omega) = \sum_{\mathrm{R}} \widehat{U}_{\mathrm{R}}(\omega) \quad . \tag{6}$$

Then, the imaging functional takes the form of

$$M(u,U) = \sum_{\mathrm{R}} M(u,U_{\mathrm{R}}) \quad , \tag{7}$$

where

$$M(u,U_{\mathrm{R}}) \approx \int \mathrm{d}\omega \; \frac{\exp(\mathrm{i}\varphi_{\mathrm{S}} - \mathrm{i}\omega t_{\mathrm{S}}) \sum_{\Omega} \widehat{a}^*_{\Omega} \exp(-\mathrm{i}\omega\tau_{\Omega}) \, \widehat{U}_{\mathrm{R}}(\omega)}{\sum_{\Omega} \widehat{a}^*_{\Omega}\widehat{a}_{\Omega}} \left[\int \mathrm{d}\omega \; \widehat{F}(\omega)\right]^{-1} . \tag{8}$$

In the time domain, equation (8) takes the form of

$$M(u,U_{\mathrm{R}}) \approx \frac{\sum_{\Omega} \exp(\mathrm{i}\varphi_{\mathrm{S}}) \, a^*_{\Omega} \, U_{\mathrm{R}}(t_{\mathrm{S}} + \tau_{\Omega})}{F(0) \; \sum_{\Omega} a^*_{\Omega} a_{\Omega}} \quad . \tag{9}$$

Although the back–propagated Gaussian packets are moving from the target zone towards the corresponding receiver, we expect that the central rays of the packets are calculated from the receiver towards the target zone.

Travel time T from the receiver is then used as the parameter along the central ray, whereas $y_i(T)$ and $y_4(T) = t_{\mathrm{R}} - T$ are coordinates and the travel time corresponding to the centre of the Gaussian packet moving towards the receiver.

Because of the back–propagation, the slowness vector of the packet N_i is minus the slowness vector P_i of the ray coming from the receiver,

$$N_i = -P_i \quad . \tag{10}$$

We choose

$$N_4 = -1 \quad . \tag{11}$$

Paraxial approximation of a Gaussian packet propagating towards the corresponding receiver to reach it at time $x_4 = t_{\mathrm{R}}$, centred at point $y_\alpha(T)$, is

$$U_{\mathrm{RT}}(x_4) = A_{\mathrm{RT}} \exp\{\mathrm{i}\omega[N_\alpha r_\alpha + \tfrac{1}{2} N_{\alpha\beta} r_\alpha r_\beta]\} \quad , \tag{12}$$

where

$$N_{\alpha\beta} = \tau_{,\alpha\beta} \quad , \tag{13}$$

$$r_i = x_i - y_i(T) \tag{14}$$

and

$$r_4 = x_4 - y_4(T) = x_4 - (t_{\mathrm{R}} - T) \quad . \tag{15}$$

Matrix $N_{\alpha\beta}$ corresponds to a Gaussian packet moving towards a receiver, although it is, similarly as the travel time y_4 and slowness vector N_α, expressed in terms of the quantities calculated along the ray emanated from the receiver.

Paraxial approximation (12) is accurate in the vicinity of point $y_\alpha(T)$, but becomes inaccurate at large space–time distances from $y_\alpha(T)$. Assuming discrete points $y_i(T)$ with step ΔT along the central ray, we thus limit each paraxial approximation by weighting function

$$W_{\mathrm{T}} = \sqrt{\pi}\, \kappa^{-1} \exp\{-\kappa^2 (N_i r_i)^2 \Delta T^{-2}\} \quad . \tag{16}$$

Sum of these weighting functions is approximately 1 in the vicinity of the central ray and thus

$$U_{\mathrm{R}}(t_{\mathrm{S}} + \tau_\Omega) = \sum_{\mathrm{T}} W_{\mathrm{T}}\, U_{\mathrm{RT}}(t_{\mathrm{S}} + \tau_\Omega) \quad . \tag{17}$$

Finally,

$$M(u, U) = \sum_{\mathrm{R}} \sum_{\mathrm{T}} M(u, W_{\mathrm{T}} U_{\mathrm{RT}}) \quad , \tag{18}$$

where the image of a single paraxial approximation of a Gaussian packet is, according to equations (9) and (17),

$$M(u, W_{\mathrm{T}} U_{\mathrm{RT}}) = \frac{\sqrt{\pi} \sum_\Omega a_\Omega^* \, A_{\mathrm{RT}} \, \exp\{\mathrm{i}\varphi_{\mathrm{S}} + \mathrm{i}\omega(N_\alpha r_\alpha + \frac{1}{2} N_{\alpha\beta} r_\alpha r_\beta) - \kappa^2 (N_i r_i)^2 \Delta T^{-2}\}}{F(0)\ \kappa\ \sum_\Omega a_\Omega^* a_\Omega} \ , \tag{19}$$

with

$$r_4 = t_{\mathrm{S}} + \tau_\Omega(x_i) + T - t_{\mathrm{R}} \quad . \tag{20}$$

3 Algorithm

First of all, we need to prepare a suitable velocity model. Required model should be sufficiently smooth and preserve original travel times. We smooth the second derivatives of slowness by minimizing the relevant Sobolev norm (Žáček, 2001a; Žáček, 2002).

Then, we have to choose the initial shape of Gaussian packets. Since we want to prevent Gaussian packets from excessive spreading, we optimize their initial shape by minimizing their width along the whole ray path (Klimeš, 1989b; Žáček, 2001b; Žáček, 2001c).

The next step is the decomposition of the common shot gather into optimized Gaussian packets (Žáček, 2003a; Žáček, 2003b). Each Gaussian packet arriving at the receivers is represented by the Gabor function, whose shape depends on the shape of the Gaussian packet. We decompose the time section into the Gabor functions. The complex valued amplitude of the back–propagated Gaussian packet then equals the amplitude of the corresponding Gabor function. The amplitudes depend upon the coordinate of the intersection of the central ray of a Gaussian packet with the profile, its arrival time, the component of the slowness vector along the profile, and the frequency.

Once we have an appropriate velocity model and the initial shapes and amplitudes of Gaussian packets, we can back–propagate the wave field and construct the migrated image. To do so, we have to generate a suitable system of rays and calculate expressions (18) and (19).

4 Target–oriented imaging

Each Gabor function from the time section generates its localized image in the depth section. In this way, we obtain a one–to–one relation between the Gabor functions from the time section and their localized images from the depth section.

Let us say that we are specifically interested in a particular area in the subsurface structure. We call this area a target zone. Unlike Gaussian beams, Gaussian packets are, at any given time, concentrated close to the central point of the packet. Thus, we can consider just those packets, which fall into the target zone.

Moreover, we can choose Gaussian packets contributing to the target zone, multiply their amplitudes by corresponding weighting factors, and use them in the re–composition of the common shot gather. In such a case, we obtain the wave field scattered from the target zone only. These data could be useful in the process of interpretation, AVO analysis, etc.

5 Numerical examples

We tested our method on the Marmousi data set (Versteeg & Grau, 1991). The smoothed velocity model is in Figure 1. In Figures 2, 3, 4, 5 and 6, the colour coded amplitudes are normalized and do not represent the true amplitudes. We plot the areas outside the target zone in gray colour.

The wave field was decomposed in dependency upon frequency, upon the coordinate of the intersection of the central ray of a Gaussian packet with the profile, upon its arrival time t_{R}, and upon the component of the slowness vector along the profile. We used uniform Gaussian packets in the decomposition of the wave field.

First of all, we present the migrated images of individual Gabor functions according to equation (19) summed along the central ray, see Figure 2. These images correspond

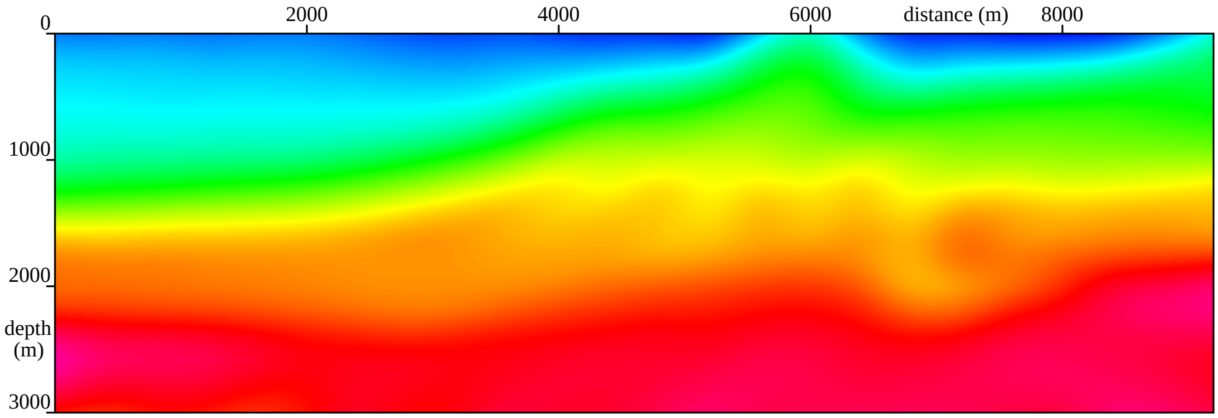

Figure 1. Smoothed Marmousi model.

to various times in the common shot gather t_{R}. Please note that we show the migrated images of Gabor functions from the common shot gather, not back propagating Gaussian packets.

In Figure 3, you can find the images corresponding to the same travel times as in the previous figure, superimposed for all frequencies. We would like to remind you that the amplitudes are normalized.

The sum of all images with the same central ray is in Figure 4, superimposed image for all central rays corresponding to the same receiver is displayed in Figure 5. Finally, we show the migrated image of the first common shot gather of the Marmousi data set in Figure 6. This figure was calculated using expression (18). Although we can recognize some features of the Marmousi model, the result is still troublesome.

Presented examples demonstrate the current state of our research. Since we are intensively testing and refining our method, we cannot judge its accuracy and efficiency yet.

6 Conclusions

The Gaussian packet pre–stack depth migration presents a new and previously untested imaging method. It can handle multi–valued travel times and allows a target–oriented approach.

We need to study and answer these basic questions:
a) How smooth should the velocity model be?
b) What is the influence of the optimization of the shape of Gaussian packets upon the imaging process?
c) What is the true accuracy and efficiency of the Gaussian packet migration?

We would also like to inspire and encourage other researchers and possibly shift their attention to the packet domain. We believe that it could show even greater potential than the well–known beam domain.

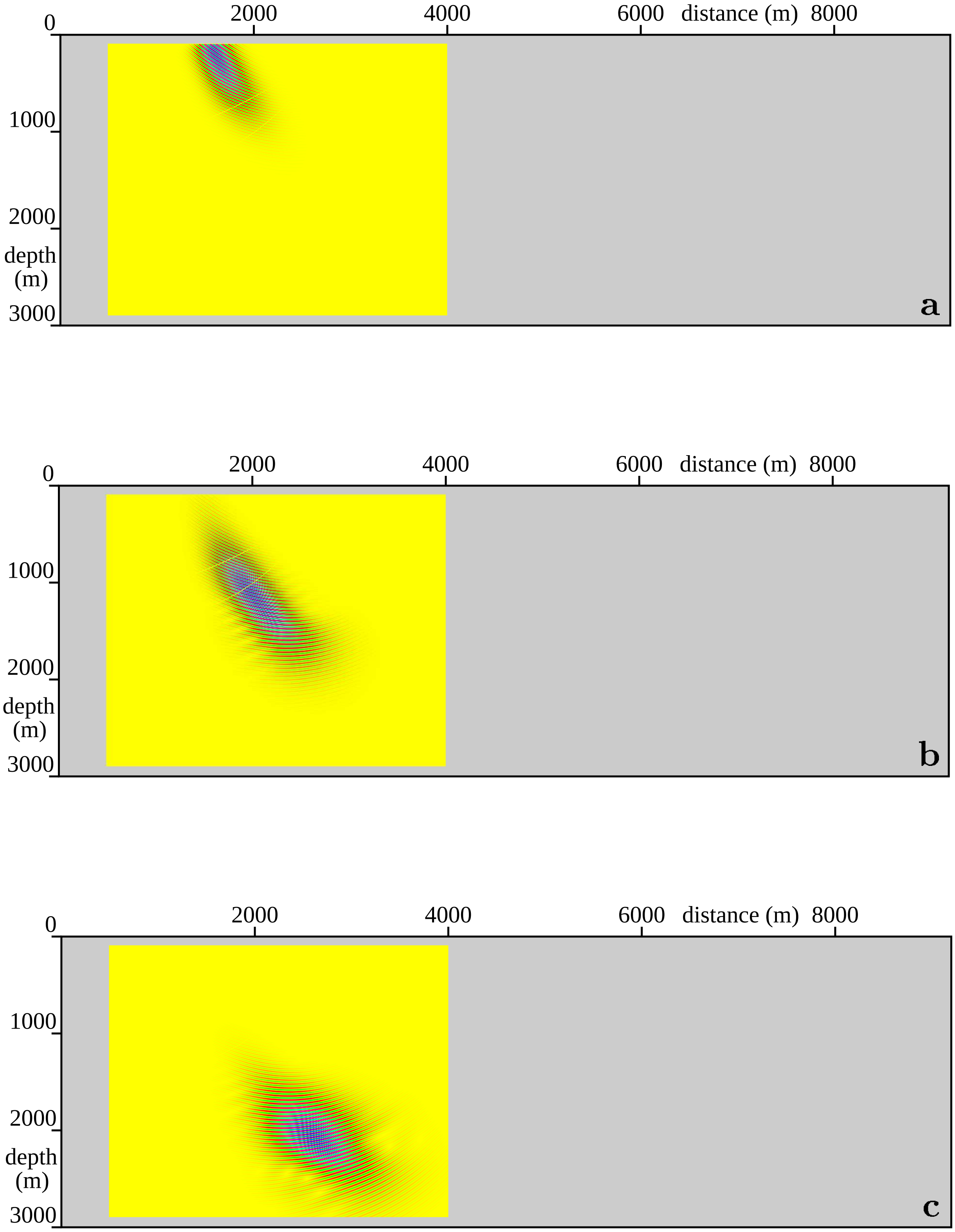

Figure 2. Migrated images of individual Gabor functions for various times t_{R}. Each figure represents the migrated image for a single receiver, a single central ray, a single time t_{R} and a single frequency. Yellow colour corresponds to 0. Gray colour denotes the area outside the target zone.

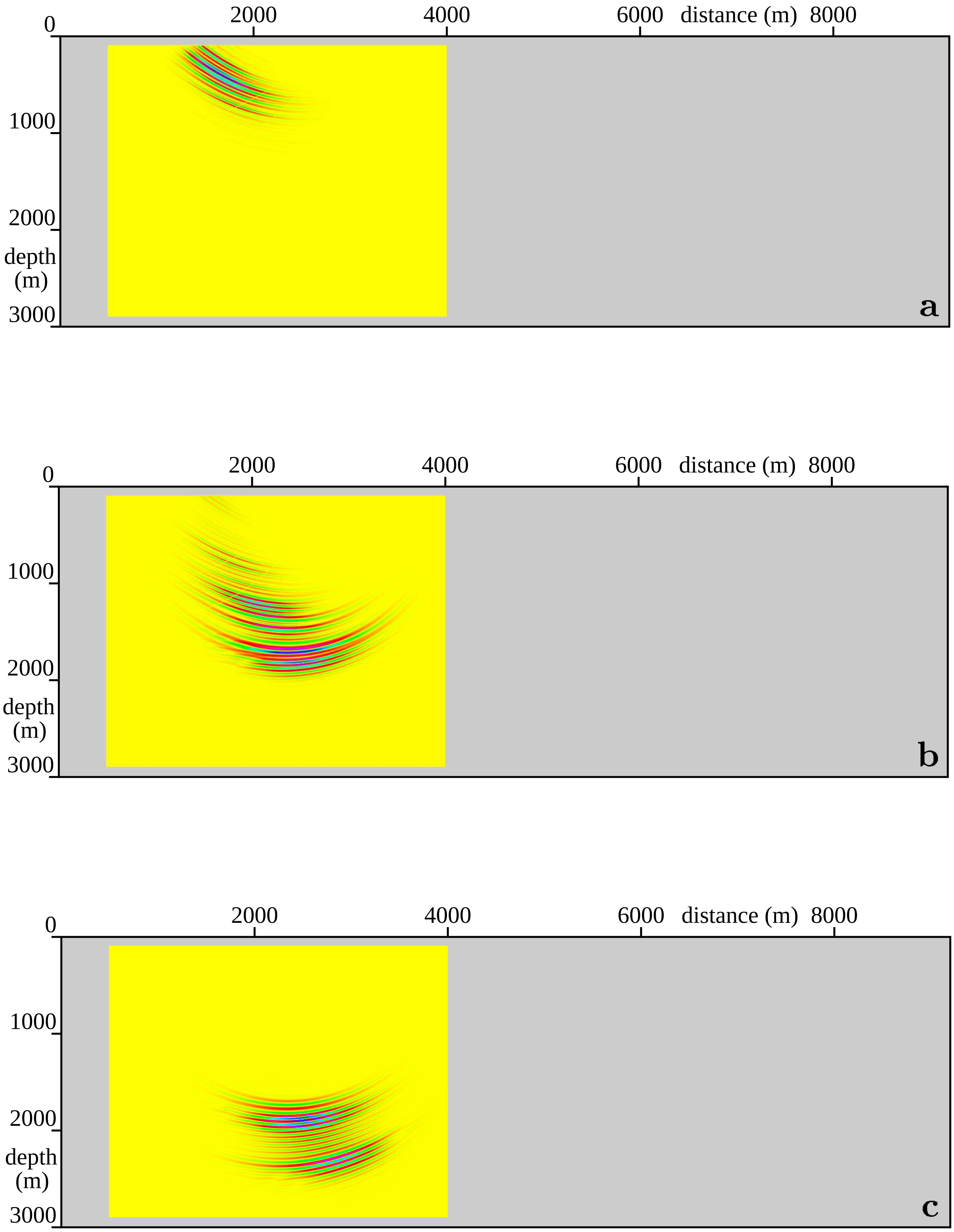

Figure 3. Migrated images of individual Gabor functions for various times t_{R}, superimposed for all frequencies. Each figure represents the migrated image for a single receiver, a single central ray, a single time t_{R} and all frequencies. Yellow colour corresponds to 0. Gray colour denotes the area outside the target zone.

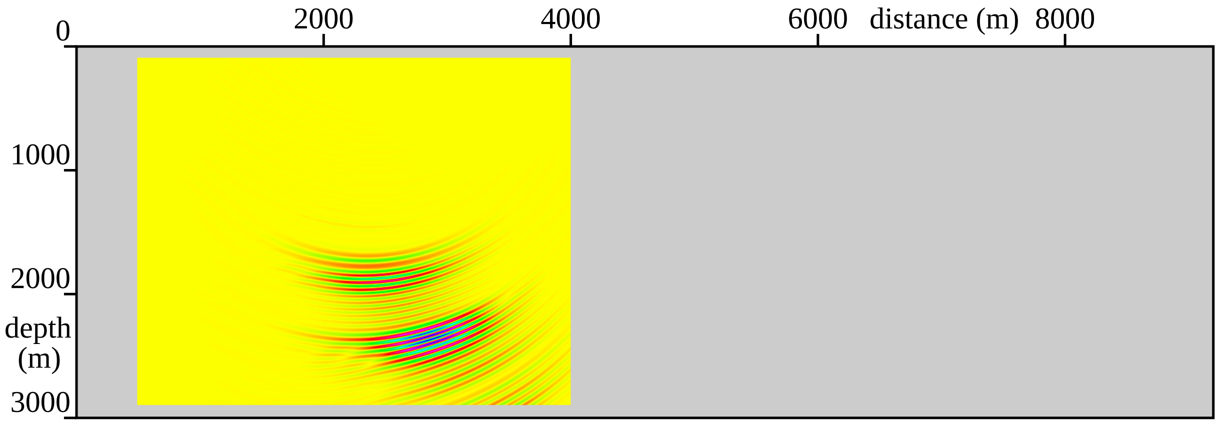

Figure 4. Superimposed migrated image for a single receiver, a single central ray, all times t_{R} and all frequencies. Yellow colour corresponds to 0. Gray colour denotes the area outside the target zone.

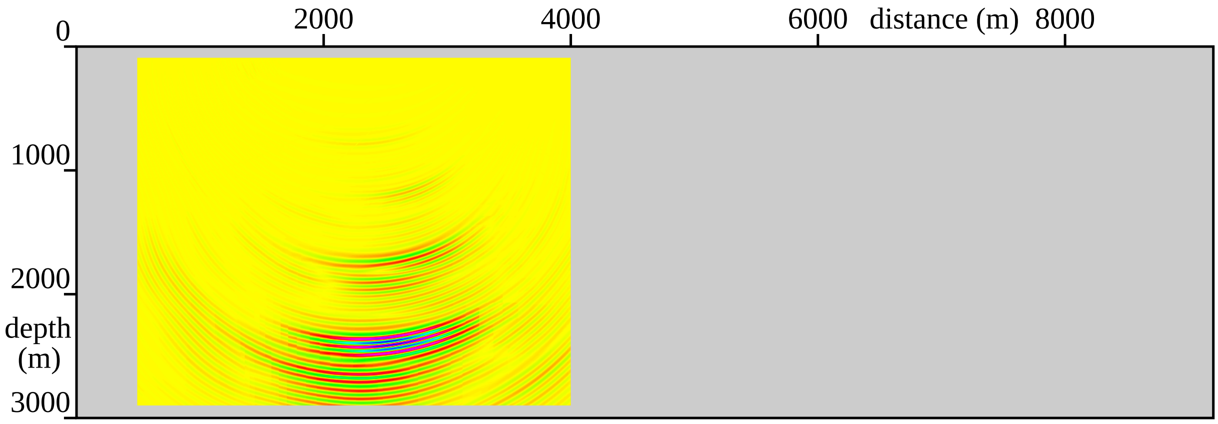

Figure 5. Superimposed migrated image for a single receiver, all central rays, all times t_{R} and all frequencies. Yellow colour corresponds to 0. Gray colour denotes the area outside the target zone.

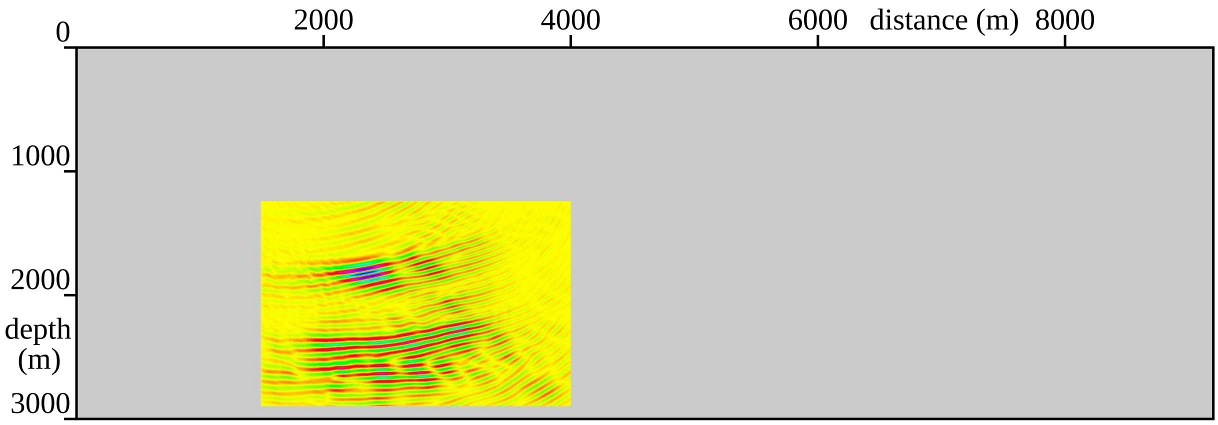

Figure 6. Superimposed migrated image for all receivers, all central rays, all times t_{R} and all frequencies. This is the migrated image of the first common shot gather of the Marmousi data set. Yellow colour corresponds to 0. Gray colour denotes the area outside the target zone.

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