Decomposition of the wavefield into optimized Gaussian packets – work in progress

Karel Žáček

Department of Geophysics, Charles University, Ke Karlovu 3, 121 16 Praha 2, Czech Republic, E-mail: zacek@karel.troja.mff.cuni.cz

Summary

The decomposition of the wavefield into optimized Gaussian packets is of key importance in the Gaussian packet migration. We present the theoretical basics and the first simple numerical examples of our method.

Key words

Gaussian packets, Gabor transform.

1 Introduction

Our long–term project is to explore the properties of a depth migration method based on Gaussian packets. Gaussian packets (GPs) are waves whose envelopes at any given time are *nearly* Gaussian in space and concentrated close to the central point of the packet (e.g, Babich & Ulin, 1981, Ralston, 1983, Klimeš, 1989a). The advantages over the methods based on Gaussian beams should be increased accuracy, computational efficiency and a straight relation between the regions in the common–shot gather and corresponding regions in the migrated section.

The GP migration algorithm consists of five basic steps:
a) preparation of a suitable smooth velocity model (e.g., Žáček, 2002),
b) optimization of the shape of GPs (Klimeš, 1989b, Žáček, 2001),
c) decomposition of the wavefield (common–shot gather) into optimized GPs,
d) extrapolation of the wavefield using GPs (e.g., Klimeš, 1989a) and
e) application of the imaging function.

Currently, we are dealing with the decomposition of the wavefield into *optimized* GPs. The word *optimized* means that the shape of GPs depends on the space–time coordinates of the intersection of the central ray of a GP with the profile and on the component of the slowness vector along the profile.

2 Basic theory

We would like to approximate the "time section" (e.g., common–shot gather) $f(x,t)$ in the form of (Klimeš, personal communication)

$$\widetilde{f}(x',t') = \int \mathrm{d}t_R \int \mathrm{d}\omega \, \exp\left[-\mathrm{i}\omega(t'-t_R)\right] \int \mathrm{d}x_R \int \mathrm{d}p \, \exp\left[\mathrm{i}\omega p(x'-x_R)\right]$$

$$\times \, W(x'-x_R, t'-t_R, x_R, p, t_R, \omega) \; F(x_R, p, t_R, \omega) \quad , \tag{1}$$

where x_R is the coordinate and t_R is the time of the intersection of the central ray of a GP with the profile, p is the component of the slowness vector along the profile, ω is

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the positive circular frequency of a GP, W determines the shape of the envelope of a GP, and the complex–valued function $F(x_R, p, t_R, \omega)$ specifies the amplitude of a GP.

Let us express the amplitude of a GP in the form of integral transform similar to the forward 2–D integral Gabor transform (Gabor, 1946),

$$F(x_R, p, t_R, \omega) = \frac{\omega}{(2\pi)^2} \int \mathrm{d}x \, \exp\left[-\mathrm{i}\omega p(x - x_R)\right] \int \mathrm{d}t \, \exp\left[\mathrm{i}\omega(t - t_R)\right]$$

$$\times \, w(x_R - x, t_R - t, x_R, p, t_R, \omega) \; f(x, t) \quad , \tag{2}$$

The envelope of the trace of a GP along the profile may be expressed as

$$W(x' - x_R, t' - t_R, x_R, p, t_R, \omega) = \exp\left[\mathrm{i}\omega \frac{1}{2} (\mathbf{y}' - \mathbf{y_R})^{\mathrm{T}} \mathbf{K} (\mathbf{y}' - \mathbf{y_R})\right] \quad , \tag{3}$$

where

$$\mathbf{y}' = \begin{pmatrix} x' \\ t' \end{pmatrix} \quad , \tag{4}$$

$$\mathbf{y}_R = \begin{pmatrix} x_R \\ t_R \end{pmatrix} \tag{5}$$

and

$$\mathbf{K} = N \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + N_{44} \begin{pmatrix} p^2 & -p \\ -p & 1 \end{pmatrix} \quad . \tag{6}$$

Let us write the analyzing functions in an analogous form,

$$w(x' - x_R, t' - t_R, x_R, p, t_R, \omega) = \widetilde{a}(N, N_{44}, p, \omega) \, \exp\left[\mathrm{i}\omega \frac{1}{2} (\mathbf{y} - \mathbf{y_R})^{\mathrm{T}} \widetilde{\mathbf{K}} (\mathbf{y} - \mathbf{y_R})\right] \quad , \tag{7}$$

where

$$\mathbf{y} = \begin{pmatrix} x \\ t \end{pmatrix} \quad , \tag{8}$$

and

$$\widetilde{\mathbf{K}} = N \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} + N_{44} \begin{pmatrix} p^2 & p \\ p & 1 \end{pmatrix} \quad . \tag{9}$$

Parameter $\widetilde{a}$ depends on N, N_{44}, p and ω as

$$\widetilde{a} = \frac{\omega}{2\pi} \frac{N\sqrt{-\mathrm{i}N}\sqrt{-\mathrm{i}K}}{N - p^2 K} \quad , \tag{10}$$

where

$$K = \frac{N N_{44}}{N + p^2 N_{44}} \quad . \tag{11}$$

N and N_{44} are free parameters determining the shape of GPs. Since we want to use *optimized* GPs,

$$N = N(x_R, t_R, p) \tag{12}$$

and

$$N_{44} = N_{44}(x_R, t_R, p) \quad . \tag{13}$$

N and N_{44} must be *smooth* functions of x_R, t_R and p. Complex-valued parameters N may be obtained as described in Žáček (2001).

3 Algorithm

Due to the dependence of analyzing functions w on x_R, t_R, p and ω, the use of FFT in the computation would not be efficient. We would need a serie of Fourier transforms of the time section for every single value of $F(x_R, p, t_R, \omega)$. Therefore, we discretize integrals (1) and (2) and calculate them by a simple summation (naturally, with some computational tricks).

Maximum steps x_R, t_R, p and ω for replacement of integrals (1) and (2) by their discretized counterparts are (Klimeš, personal communication)

$$\Delta x_R = \kappa \sqrt{\frac{-\mathrm{Im}\,[(N - p^2 K) N^{-2}]}{\omega}} \quad , \tag{14}$$

$$\Delta t_R = \kappa \sqrt{\frac{-\mathrm{Im}\,[(N - p^2 K) N^{-1} K^{-1}]}{\omega}} \quad , \tag{15}$$

$$\Delta p = \kappa \sqrt{\frac{\mathrm{Im} N}{\omega}} \tag{16}$$

and

$$\Delta \omega = \kappa \sqrt{\omega \mathrm{Im} K} \quad . \tag{17}$$

We take $\kappa^2 = \frac{\pi}{2}$.

4 Numerical examples

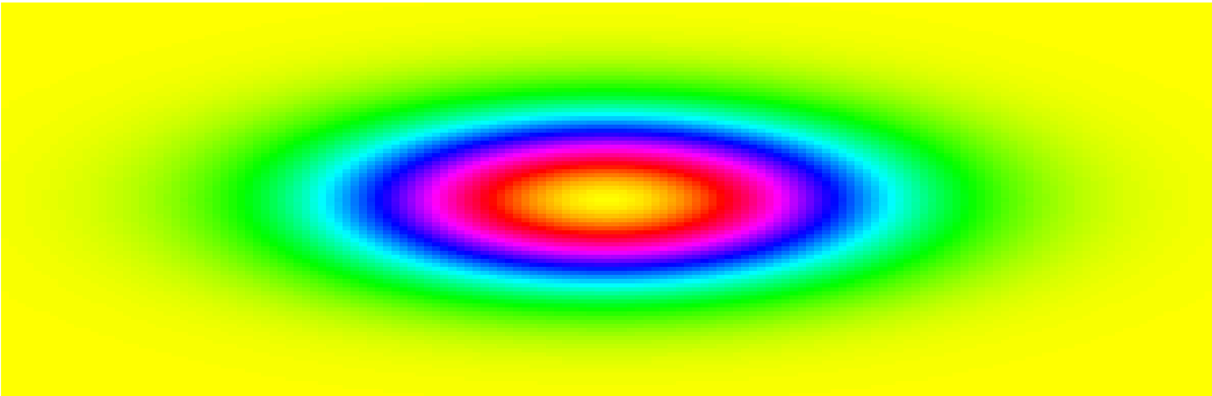

Figure 1. The envelope of a Gaussian packet.

We present two simple examples of the decomposition into *uniform* GPs. The first example is the decomposition of the Gabor signal (Gabor, 1946), the second is the decomposition of a simple monochromatic signal. Compare Figure 2a with Figure 2d, and Figure 3a with Figure 3d. As you can see in Figure 2, the approximation of a signal by GPs is accurate for smooth localized functions. Note the significant difference between Figures 3a and 3d, which is caused by sharp edges. This difference is controllable by the use of higher frequencies in the decomposition. However, the price we have to pay is increasing computation time.

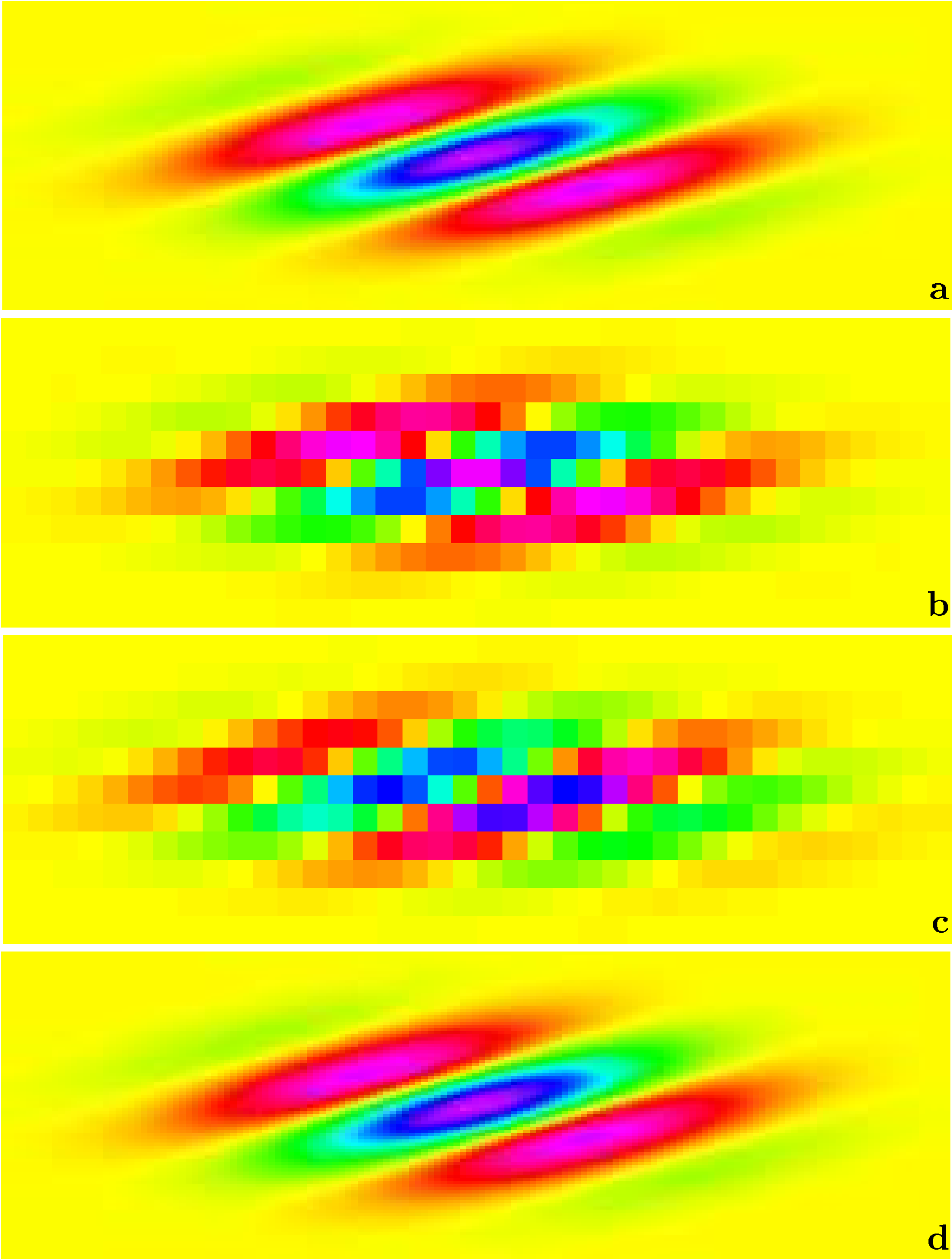

Figure 2. A simple example of a Gaussian packet decomposition.
(a) A synthetic "time section", Gabor signal. The *yellow* colour corresponds to 0.
(b) An example of the amplitudes F of GPs, see equation (2), for a certain value of ω and p, real part.
(c) An example of the amplitudes F of GPs, see equation (2), for a certain value of ω and p, imaginary part.
(d) An approximation of the time section composed from GPs, see equation (1).

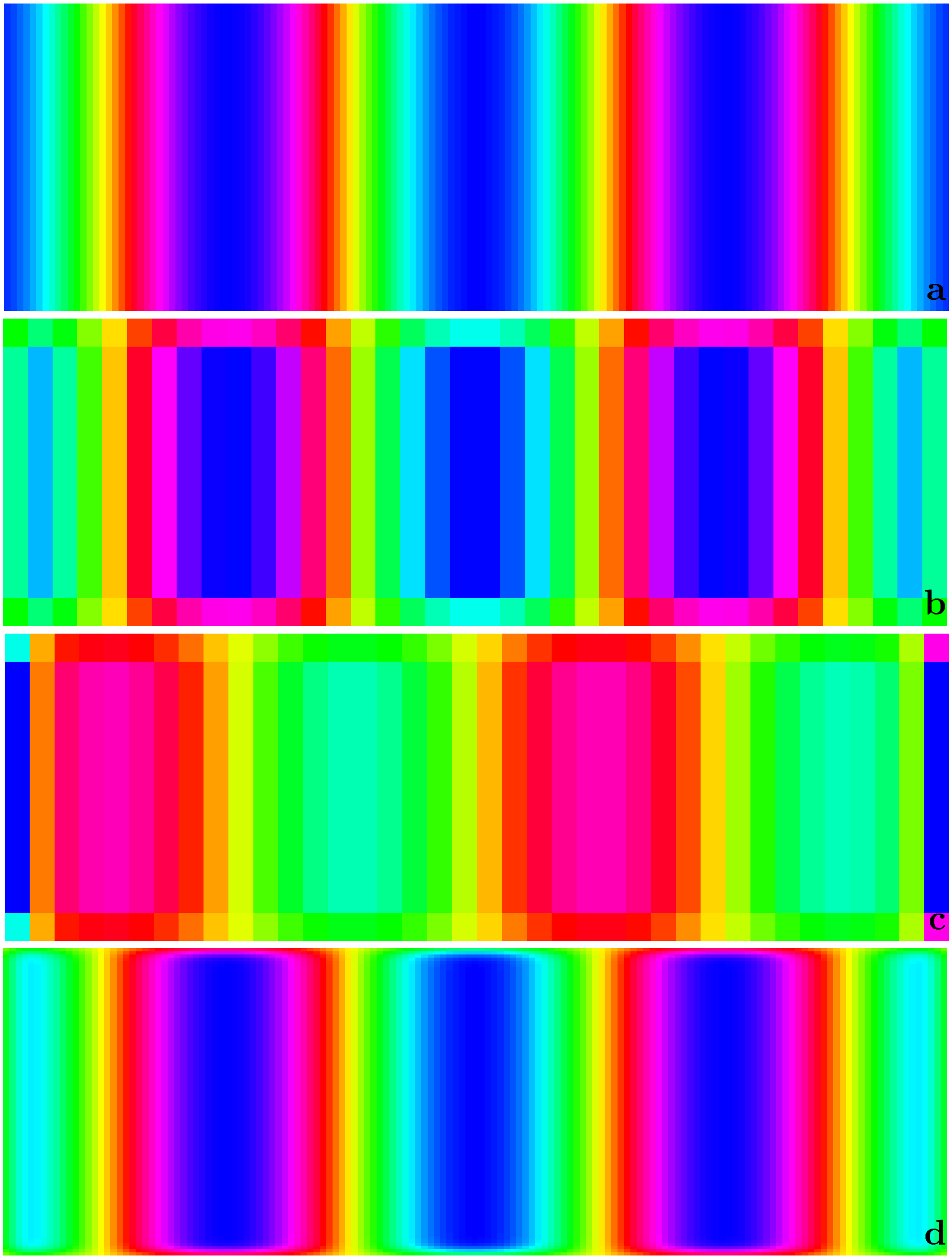

Figure 3. A simple example of a Gaussian packet decomposition.
(a) A synthetic "time section", monochromatic signal. The *yellow* colour corresponds to 0.
(b) An example of the amplitudes F of GPs, see equation (2), for a certain value of ω and p, real part.
(c) An example of the amplitudes F of GPs, see equation (2), for a certain value of ω and p, imaginary part.
(d) An approximation of the time section composed from GPs, see equation (1).

5 Future prospects

We will apply our method on a pseudo–real data (e.g., the Marmousi data set). We will study the influence of the optimization of the shape of GPs on the decomposition of the wavefield.

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