Smoothing the Marmousi model for Gaussian-packet migrations

Luděk Klimeš

Department of Geophysics, Charles University, Ke Karlovu 3, 121 16 Praha 2, Czech Republic, Fax: +420-2-2191 1214, E-mail: klimes@seis.karlov.mff.cuni.cz

Summary

Construction of a smooth velocity model by smoothing gridded velocities of the "Marmousi model and dataset", using the Sobolev scalar products, is discussed. The smooth velocity model is prepared so as to be suitable for ray tracing and for Gaussian–packet migrations.

Keywords

Model specification, smoothing, objective function, Sobolev scalar products, ray methods.

1 Introduction

This is an example how to fit the gridded velocities by a smooth model, without interfaces, suitable for ray tracing. Models with interfaces will require further study.

Construction of a smooth velocity model, approximating the gridded velocities of the "Marmousi model and dataset" and suitable for ray tracing, is described in this paper. To design the algorithm proposed is this paper, the author took advantage of the experience from extensive numerical calculations performed by Žáček (2000).

The data relevant to this example are located in subdirectory "data/mar" of compact disk SW3D-CD-4. This paper is rather technical and frequently refers to the data files. It is based on the previously written HTML description "data/mar/mar-inv.htm" of the data files.

2 Notes on history file "mar-vel.h"

Since file "velocity.h@" of the "Marmousi model and dataset" is large, history file "mar-vel.h" has been used to create file "mar-vel.dat" located in subdirectory "data/mar" and used by history files "mar-inv.h" and "mar-test.h". History file "mar-vel.h" is thus not designed to be executed with data of subdirectory "mar/mar". It requires file "velocity.h@" of the "Marmousi model and dataset", containing the gridded velocities.

2.1 Data and model parametrization

Velocities in the Marmousi model are discretized on the grid of 751×2301 points with spacing $4\,\mathrm{m} \times 4\,\mathrm{m}$. This is too large number of data for inversion. Since the bicubic B–splines in the model are to be spaced much more sparsely, the set of slownesses inside a cell small with respect to the B–spline cell act nearly like a single average slowness.

We choose the data grid of $76 \times 116 = 8816$ points with spacing $40\,\mathrm{m} \times 80\,\mathrm{m}$ and the B–spline grid of $16 \times 24 = 384$ points with spacing $200\,\mathrm{m} \times 400\,\mathrm{m}$. Program "gmdmgmt.for" then requires $384 \times 8816 + 8816 + 384 \times 384 = 3541616$ storage locations in array RAM and we may perform the calculations with MRAM=4000000 corresponding to the distributed version of "ram.inc".

In: Seismic Waves in Complex 3-D Structures, Report 10 (Department of Geophysics, Charles University, Prague, 2000), pp. 63–74

3 Notes on smoothing (history file "mar-inv.h")

Section 3 is devoted to the description of the algorithm used for smoothing and to the discussion of the respective numerical parameters. Individual iterations of the smoothing algorithm have been numerically realized using history file "mar-inv.h". The form of the history file is described in Section 5.

3.1 Choosing the maximum Lyapunov exponent

The length of synthetic seismograms is $T_{\max} = 2.9\,\mathrm{s}$. This means that the maximum sum of travel times from the source and receiver is limited by $T_{\max}$,

$$T_S + T_R \leq T_{\max} \quad . \tag{1}$$

For average Lyapunov exponent λ, the average number of arrivals from a source times the average number of arrivals from a receiver may be roughly approximated by

$$\tfrac{1}{4}\exp(\lambda\, T_S)\,\exp(\lambda\, T_R) \quad , \tag{2}$$

see equation (82) of Klimeš (2000), and the maximum number of arrivals from a source times the maximum number of arrivals from a receiver may be roughly approximated by

$$\exp(\lambda\, T_S)\,\exp(\lambda\, T_R) \quad . \tag{3}$$

If we choose

$$\lambda \leq \frac{2}{T_{\max}} = 0.690\,\mathrm{s}^{-1} \quad , \tag{4}$$

the product of the average numbers of arrivals from a source and a receiver probably does not exceed $\frac{1}{4}\exp(2) \approx 1.8$ and the product of the maximum numbers of arrivals from a source and a receiver probably does not exceed $\exp(2) \approx 7$.

3.2 Choosing the maximum Sobolev norm of the model

We assume in equation (51) of Klimeš (1999) at least one shift of $-\ln 2$ for a source and one for a receiver, i.e., we assume $\overline{\tau}_{\mathrm{osc}} = 1.45\,\mathrm{s}$ and $K_{\mathrm{osc}} = 2$ in equation (86) of Klimeš (2000). We thus approximate the Lyapunov exponent by

$$\lambda = \langle\sqrt{\mathrm{neg}(V\,v)}\rangle - \frac{2\ln 2}{T_{\max}} \tag{5}$$

(Klimeš 2000, eqs. 84, 85 and 87), i.e.,

$$\langle\sqrt{\mathrm{neg}(V\,v)}\rangle \leq \frac{2\,(1+\ln 2)}{T_{\max}} \quad , \tag{6}$$

where $\mathrm{neg}(f) = \frac{1}{2}(f - |f|)$ is the negative part of f, V is the second velocity derivative perpendicular to a ray and v is the velocity. We make several other rough approximations, analogous to equations (91)–(96) of Klimeš (2000),

$$\langle\sqrt{|V\,v|}\rangle \leq \frac{4\,(1+\ln 2)}{T_{\max}} \quad , \tag{7}$$

$$\langle(V\,v)^2\rangle \leq \left[\frac{4\,(1+\ln 2)}{T_{\max}}\right]^4 \quad , \tag{8}$$

$$\langle (U\,v^3)^2 \rangle \leq \left[\frac{4\,(1+\ln 2)}{T_{\max}} \right]^4 \quad , \tag{9}$$

where U is the second slowness derivative perpendicular to a ray,

$$\langle U^2 \rangle \leq \left[\frac{4\,(1+\ln 2)}{T_{\max}} \right]^4 \frac{1}{\langle v \rangle^6} \quad , \tag{10}$$

$$\|u\|^2 \frac{3}{8} \leq \left[\frac{4\,(1+\ln 2)}{T_{\max}} \right]^4 \frac{1}{\langle v \rangle^6}, \quad , \tag{11}$$

$$\|u\| \leq \sqrt{\frac{8}{3}} \left[\frac{4\,(1+\ln 2)}{T_{\max}} \right]^2 \frac{1}{\langle v \rangle^3} \quad , \tag{12}$$

where $\|u\|$ is the 2-D Sobolev norm given by file "sob22.dat",

$$\|u\| = \sqrt{(u_{,11})^2 + (u_{,22})^2 + \tfrac{2}{3} u_{,11} u_{,22} + \tfrac{4}{3} (u_{,12})^2} \quad . \tag{13}$$

For $\langle v \rangle = 3000\,\mathrm{m\,s^{-1}}$, we wish the maximum Sobolev norm of the model

$$\|u\|_{\max} = 0.330\ 10^{-9}\,\mathrm{s\,m^{-3}} \quad . \tag{14}$$

3.3 Inversion and smoothing

Objective function is taken in the form of

$$y = \left[\frac{|u-u_0|}{\sigma} \right]^2 + [\mathtt{SOBMUL}\|u\|]^2 \quad , \tag{15}$$

where $|u-u_0|$ is the standard slowness deviation of the model from gridded slownesses and $\|u\|$ is the 2-D Sobolev norm given by file "sob22.dat". We specify unit given slowness deviation σ,

$$\sigma = 1\,\mathrm{s\,m^{-1}} \quad . \tag{16}$$

3.3.1 First iteration:

We first perform inversion with `SOBMUL` = 0 (i.e., without smoothing) and obtain the standard slowness deviation of the model

$$|u-u_0| = 0.290\ 10^{-4}\,\mathrm{s\,m^{-1}} \tag{17}$$

stored in file "mar-ud1.out". Note that the standard slowness deviation from the densely sampled grid $4\,\mathrm{m} \times 4\,\mathrm{m}$ is larger, $0.340\ 10^{-4}\,\mathrm{s\,m^{-1}}$. The Sobolev norm of the model smoothed just by the projection onto the B–splines is large in comparison with (12),

$$\|u\| = 3.735\ 10^{-9}\,\mathrm{s\,m^{-3}} \quad , \tag{18}$$

see file "mar-un1.out". The value of the objective function is

$$y = 0.841\ 10^{-9} \quad . \tag{19}$$

3.3.2 Second iteration:

We need to choose SOBMUL for this iteration. If we imagine that the second term in (15) for the last iteration may equal the first term in (15) for the first iteration, we may choose

$$\mathtt{SOBMUL} = \frac{|u - u_0|}{\sigma \|u\|_{\max}} \quad , \tag{20}$$

where $|u - u_0|$ is taken from the first iteration. Then

$$\mathtt{SOBMUL} = \frac{0.290\ 10^{-4}}{0.330\ 10^{-9}\,\mathrm{s\,m^{-3}}} = 87878\,\mathrm{m^3\,s^{-1}} \tag{21}$$

and we enter

$$\mathtt{SOBMUL} = 90000\,\mathrm{m^3\,s^{-1}} \tag{22}$$

for the second iteration. After the inversion, we obtain the standard slowness deviation of

$$|u - u_0| = 0.408\ 10^{-4}\,\mathrm{s\,m^{-1}} \tag{23}$$

stored in file "mar-ud1.out", and the Sobolev norm

$$\|u\| = 0.134\ 10^{-9}\,\mathrm{s\,m^{-3}} \quad , \tag{24}$$

stored in file "mar-un1.out". The value of the objective function is

$$y = 1.66\ 10^{-9} + 0.145\ 10^{-9} \quad . \tag{25}$$

3.3.3 Third iteration:

We choose

$$\mathtt{SOBMUL} = \mathtt{SOBMUL}_{\mathrm{previous}}\,\frac{\|u\|}{\|u\|_{\max}} \quad , \tag{26}$$

i.e.,

$$\mathtt{SOBMUL} = 90000\,\mathrm{m^3\,s^{-1}}\,\frac{0.1336\ 10^{-9}\,\mathrm{s\,m^{-3}}}{0.330\ 10^{-9}\,\mathrm{s\,m^{-3}}} \quad , \tag{27}$$

which is approximately

$$\mathtt{SOBMUL} = 36000\,\mathrm{m^3\,s^{-1}} \quad . \tag{28}$$

After the inversion, we obtain the standard slowness deviation of

$$|u - u_0| = 0.377\ 10^{-4}\,\mathrm{s\,m^{-1}} \tag{29}$$

stored in file "mar-ud1.out", and the Sobolev norm

$$\|u\| = 0.328\ 10^{-9}\,\mathrm{s\,m^{-3}} \quad , \tag{30}$$

stored in file "mar-un1.out". The value of the objective function is

$$y = 1.42\ 10^{-9} + 0.139\ 10^{-9} \quad . \tag{31}$$

The model is shown in Figure 1.

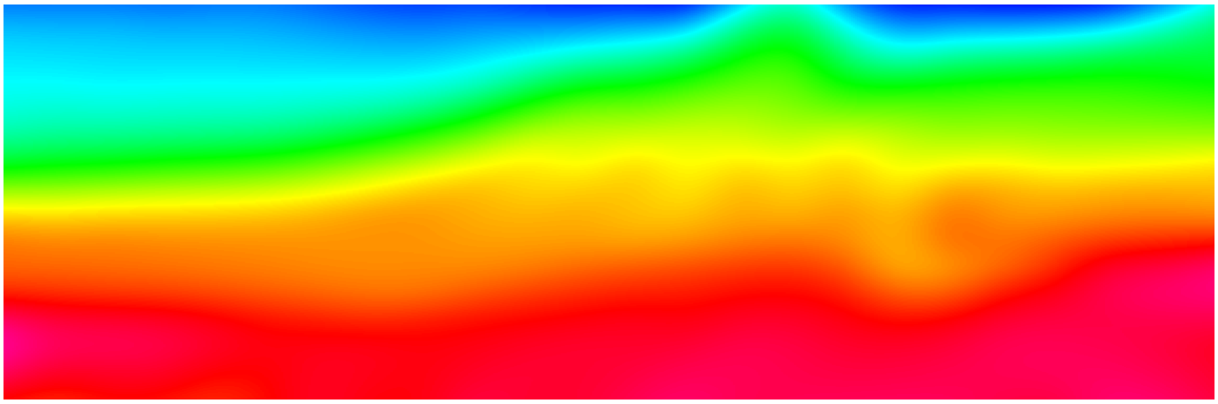


Figure 1. Selected smooth marmousi model, resulting from the third iteration, discretized on the grid of spacing $8\,\mathrm{m} \times 16\,\mathrm{m}$.

3.3.4 Subsequent iterations:

The iterations may be repeated until we arrive at the desired value of $\|u\|$. Then, we may repeat the inversion with sparser or denser B–spline grid. If the B–spline grid is sufficiently dense, a denser B–spline grid does not influence the resulting model. If the resulting model does not change with a sparser grid, we may switch to the sparser B–spline grid.

Since Sobolev norm (30) from the third iteration already meets our idea (14), we keep the value of `SOBMUL` from the third iteration and proceed to the tests of the B–spline grid density. We performed the following tests of the B–spline grid density:

B–spline grid	$200\,\mathrm{m} \times 400\,\mathrm{m}$	$250\,\mathrm{m} \times 400\,\mathrm{m}$	$300\,\mathrm{m} \times 400\,\mathrm{m}$	$375\,\mathrm{m} \times 400\,\mathrm{m}$
$\lvert u - u_0 \rvert / (\,\mathrm{s\,m}{-}1)$	$0.376725\ 10^{-4}$	$0.377243\ 10^{-4}$	$0.379596\ 10^{-4}$	$0.384356\ 10^{-4}$
$\|u\| / (\,\mathrm{s}/m^{-3})$	$0.328061\ 10^{-9}$	$0.325954\ 10^{-9}$	$0.316512\ 10^{-9}$	$0.296923\ 10^{-9}$

B-spline grid	$200\,\mathrm{m} \times 400\,\mathrm{m}$	$200\,\mathrm{m} \times 460\,\mathrm{m}$	$200\,\mathrm{m} \times 511\,\mathrm{m}$	$200\,\mathrm{m} \times 575\,\mathrm{m}$
$\lvert u - u_0 \rvert / (\,\mathrm{s\,m}{-}1)$	$0.376725\ 10^{-4}$	$0.377246\ 10^{-4}$	$0.377508\ 10^{-4}$	$0.378320\ 10^{-4}$
$\|u\| / (\,\mathrm{s\,m}^{-3})$	$0.328061\ 10^{-9}$	$0.326122\ 10^{-9}$	$0.325090\ 10^{-9}$	$0.322291\ 10^{-9}$

We might use a B–spline grid up to 1.25 times sparser in both directions than $200\,\mathrm{m} \times 400\,\mathrm{m}$. Coarser grids than $250\,\mathrm{m} \times 500\,\mathrm{m}$ are smoothed by insufficiently sampled splines. Since the spline smoothing is much worse than smoothing with the Sobolev scalar products, we should avoid sparser B–spline grids than $250\,\mathrm{m} \times 500\,\mathrm{m}$ for the smoothed Marmousi model.

After the above tests, we decided that it is no use to change the B–spline grid. We thus terminate the iterations, rename file "mar-mod.out" with the resulting model to "mar-mod.dat" and proceed to more detailed tests of the smoothed model which is displayed in Figure 1.

4 Notes on history file "mar-test.h"

4.1 Statistical properties of the model

We discretize the velocity and its first and second derivatives in the smoothed model "mar-mod.dat". To test the code, we calculate the standard slowness deviation once more from the gridded velocity and obtain the value of

$$|u - u_0|_{\mathrm{grid}40\times 80} = 0.377\ 10^{-4}\,\mathrm{s\,m^{-1}} \quad . \tag{32}$$

stored in file "mar-ud2.out". Similarly, we calculate the Sobolev norm of the slowness from the gridded velocity and its derivatives and obtain the value of

$$\|u\|_{\mathrm{grid}40\times 80} = 0.330\ 10^{-9}\,\mathrm{s\,m^{-3}} \quad , \tag{33}$$

stored in file "mar-un2.out". Note that for denser grid $8\,\mathrm{m} \times 16\,\mathrm{m}$ we obtain

$$|u - u_0|_{\mathrm{grid}8\times 16} = 0.413\ 10^{-4}\,\mathrm{s\,m^{-1}} \tag{34}$$

and

$$\|u\|_{\mathrm{grid}8\times 16} = 0.328\ 10^{-9}\,\mathrm{s\,m^{-3}} \quad . \tag{35}$$

The difference between $\|u\|_{\mathrm{grid}8\times 16}$ and the value of $\|u\|$ stored in file "mar-un1.out" is $0.0001\ 10^{-9}\,\mathrm{s\,m^{-3}}$. For original dense grid $4\,\mathrm{m} \times 4\,\mathrm{m}$ we have

$$|u - u_0|_{\mathrm{grid}4\times 4} = 0.418\ 10^{-4}\,\mathrm{s\,m^{-1}} \quad . \tag{36}$$

The standard relative slowness deviation, which is equivalent to the standard relative travel–time deviation for very short rays, is

$$|u/u_0 - 1|_{\mathrm{grid}40\times 80} = 0.130 \quad , \tag{37}$$

see file "mar-ur2.out", and

$$|u/u_0 - 1|_{\mathrm{grid}4\times 4} = 0.145 \quad . \tag{38}$$

We may also plot gridded slowness "mar-u0.ps" having been fit, and slowness "mar-u.ps" in the smoothed model, see history file "mar-test.h".

4.2 Average Lyapunov exponent for the model

Average Lyapunov exponent for the 2-D model, calculated by program "modle2d.for", has the value of

$$\lambda = 0.519\,\mathrm{s^{-1}} \tag{39}$$

and is stored in file "mar-lem.out" in the form of line in order to be plotted by program "pictures.for" into the graph of the angular dependence of the directional Lyapunov exponents "mar-led.out", see PostScript figure "mar-le.ps". The frame of the figure is stored in file "mar-lef.out". The Lyapunov exponent is averaged over the angles with a uniform weight, in accordance with the isotropic Sobolev scalar products used for smoothing. Although the Lyapunov exponent is little bit better than we formerly required, it is not as small as to decrease the smoothness of the model without testing ray tracing and the widths of Gaussian beams in the model. The product of the maximum numbers of arrivals from a source and a receiver thus probably does not exceed $\exp(0.519 \times 2.9) = \exp(1.5) \approx 5$.

4.3 Ray tracing and widths of Gaussian beams

The source and receiver points in the Marmousi data set are equally spaced with the interval of 25 m. The leftmost point at $x_2 = 425$ m corresponds to receiver 1 of shot 1. The rightmost point at $x_2 = 8975$ m corresponds to shot 240. There are thus 343 different source and receiver points from 425 m to 8975 m. We index them by integers from 1 to 343. File "mar-srp.dat" contains position of formal point 0 and the interval vector between the points. Program "srp.for" may be used to generate positions of points 1 to 343.

Ray tracing and the calculation of the gridded numbers of arrivals and the widths of Gaussian beams for a specified source point is performed by history file "mar-crt.h". We selected 18 source positions 1, 20, 40, 60, 80, 100, 120, 140, 160, 180, 200, 220, 240, 260, 280, 300, 320, 335. The results for all specified individual sources are accumulated in common grid files (data cubes) and are displayed in Figure 2. The calculation of rays and travel times is terminated at the travel time of 2.3 s. For greater travel times from a source or a receiver, the sum of both travel times would exceed the maximum travel time of 2.9 s for the measurement configuration under consideration. The numbers of arrivals are small and the travel–time triplications occur only at the ends of rays terminated at travel time 2.3 s. The numbers of arrivals are thus fine for calculations, as we expected from the mean Lyapunov exponent for the model.

We now calculate the standard halfwidths of Gaussian beams to check the suitability of the model for Gaussian–beam and Gaussian–packet calculations. Standard halfwidth a of Gaussian beam of crossection

$$\exp\left(-\frac{q^2}{2a^2}\right) \quad , \tag{40}$$

multiplied by the square root of $\omega = 2\pi f$, is interpolated between rays and displayed in colours,

$$W = a\sqrt{2\pi f} \quad . \tag{41}$$

The yellow colour corresponds to $W_{\text{yellow}} = 0\,\text{m}\,\text{s}^{-\frac{1}{2}}$, the green colour to

$$W_{\text{green}} = 3000\,\text{m}\,\text{s}^{-\frac{1}{2}} \quad . \tag{42}$$

Trapezoidal frequency filter applied to seismograms is determined by frequencies 0Hz, 10Hz, 35Hz and 55Hz. Gaussian beam halfwidth corresponding to the green colour is at lower corner frequency of 10Hz

$$a_{\text{green}} = 378\,\text{m} \quad , \qquad a_{\text{cyan}} = 756\,\text{m} \quad ; \tag{43}$$

at central frequency of 25 Hz

$$a_{\text{green}} = 239\,\text{m} \quad , \qquad a_{\text{cyan}} = 478\,\text{m} \quad ; \tag{44}$$

and at the upper corner frequency of 35 Hz

$$a_{\text{green}} = 202\,\text{m} \quad , \qquad a_{\text{cyan}} = 404\,\text{m} \quad . \tag{45}$$

Since the halfwidth should be smaller than the B–spline intervals, acceptable halfwidths are between yellow and green. The values between the cyan and red are awfully bad (2 to 5 times greater values than for the green).

Three rightmost columns in Figure 2 correspond to three different initial conditions for Gaussian beams, constant along the surface of the model.

Unlike all previous tests, the standard halfwidths of Gaussian beams indicate that the model is at the edge of applicability of Gaussian beams and Gaussian packets, because of too small frequencies for the high–frequency asymptotic methods. We also see that the initial conditions for the Gaussian beams should not be constant along the model surface and should be carefully optimized. Until doing that, we cannot be sure whether the smoothed model is sufficiently smooth or must be smoothed more.

5 Brief description of history file "mar-inv.h"

In this section, we denote the matrix or vector stored in file "file.out" by **file**. Note that initials "dm" stand for a diagonal matrix, initials "sm" stand for a symmetric matrix, and initials "gm" stand for a general matrix including a vector.

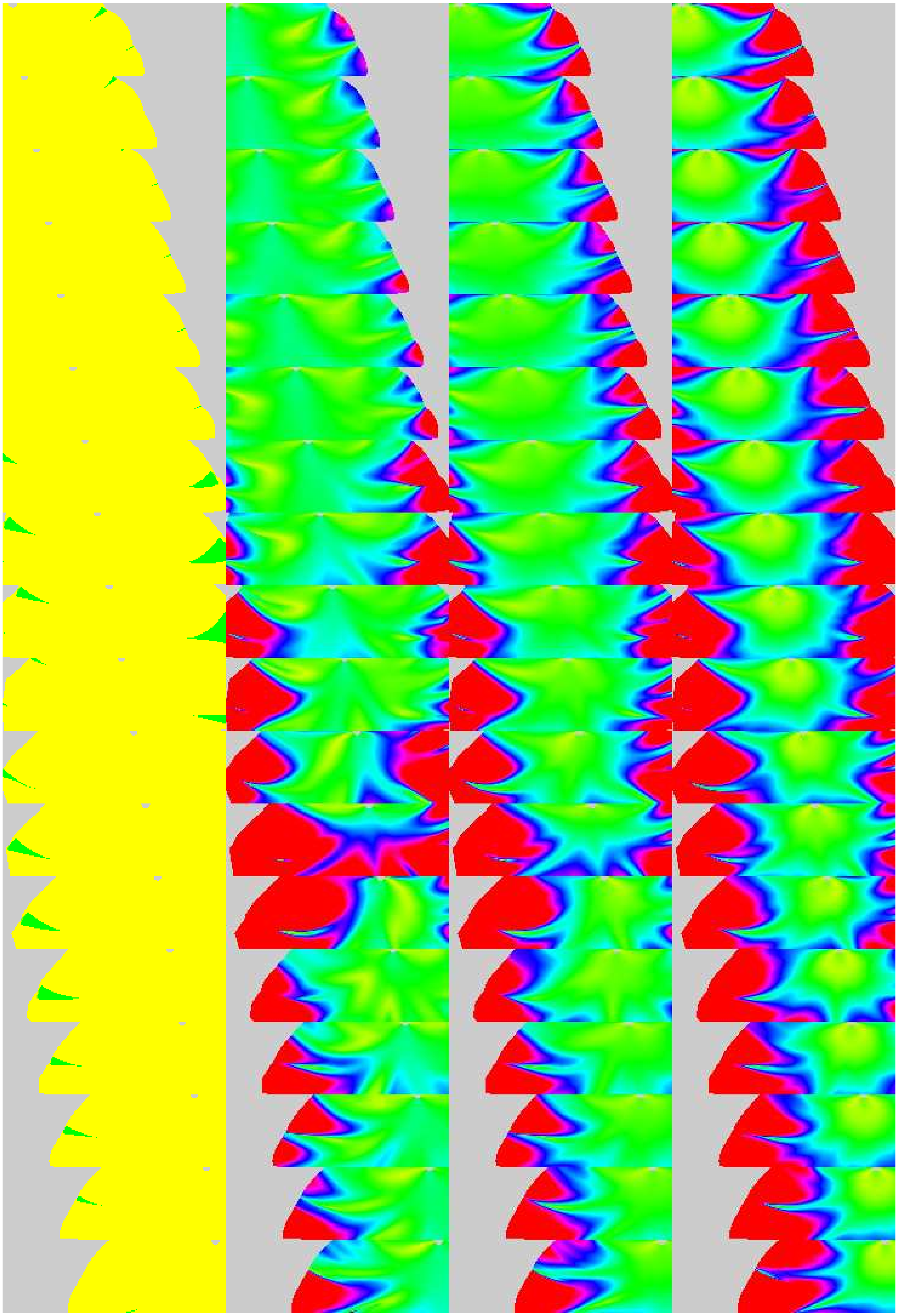
History file "mar-inv.h" performs a single iteration described in this paper. Individual iterations differ just by the value of parameter SOBMUL, which should be updated between iterations. The value of SOBMUL in the distributed version of the history file corresponds to the third iteration, producing the final model. Since the inversion is linear, all iterations can use the same initial model. To test the density of the B–spline grid, file "mar-mod0.dat" with the initial model should be modified accordingly or replaced by another file.

```
# Smoothing Marmousi velocity model
# ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
# Input files required
 #chk.pl: "data/mar/" "mar-vel.dat"
 #chk.pl: "data/mar/" "mar-mod0.dat"
 #chk.pl: "model/"    "sob22.dat"

# Inversion
# ~~~~~~~~~
# Initial and updated models
  MODEL='mar-mod0.dat'  MODOUT='mar-mod.out'  NEGPAR=0

# Gridded slowness to fit (intervals 40m*80m)
  N1=76  N2=116  D1=40  D2=80  O1=0  O2=0
  CAL='inv.cal' GRD1='mar-vel.dat' GRD2='mar-u0.out'
  grdcal: 'mar-inv.h' /
```

Figure 2 (right page). Eighteen rows in the figure correspond to source positions 425 m, 920 m, 1420 m, 1920 m, 2420 m, 2920 m, 3420 m, 3920 m, 4420 m, 4920 m, 5420 m, 5920 m, 6420 m, 6920 m, 7420 m, 7920 m, 8420 m, 8775 m. *First column*: Numbers of travel times. Areas with no travel times are grey, areas with 1 arrival are yellow, areas with 3 arrivals are green. Travel times up to 2.3 s are calculated. *Second, third and fourth columns*: Standard halfwidths of Gaussian beams multiplied by $\sqrt{2\pi f}$, displayed in colours along the central rays of beams. Yellow corresponds to $W = 0\,\mathrm{m\,s^{-1/2}}$, green to $W = 3000\,\mathrm{m\,s^{-1/2}}$, cyan to $W = 6000\,\mathrm{m\,s^{-1/2}}$ and so on. Individual columns differ by the initial conditions for Gaussian beams. *Second column*: The second derivatives of the complex–valued travel times of Gaussian beams along the surface of the model equal $(\mathrm{i}\,0.050)10^{-6}\,\mathrm{m^{-2}\,s}$. *Third column*: The second derivatives along the surface equal $(\mathrm{i}\,0.250)10^{-6}\,\mathrm{m^{-2}\,s}$. *Fourth column*: The second derivatives along the surface equal $(0.400+\mathrm{i}\,0.250)10^{-6}\,\mathrm{m^{-2}\,s}$.



Since the interpolation of slowness by bicubic splines is prescribed in the model, we converted the gridded velocity into slowness, to make the inversion linear. Gridded slowness, also represented here in terms of vector $\mathbf{u}_0$ stored in file "mar-u0.out", will thus be fit.

```
# Calculating matrices for inversion
  M1='m1.out'  M2='m2.out'  MODL2=' '  MODSOB='modsob.out'
  SOBOLEV='sob22.dat'  SOBW01=1  # minimizing second slowness derivatives
  invsoft: 'mar-inv.h' /
```

Symmetric positive–semidefinite matrix **modsob** (stored in file "modsob.out") contains the Sobolev scalar products of m_1 B-splines. Number m_1 of the model parameters is stored in file "m1.out". The Sobolev scalar products correspond to the second partial derivatives of the slowness and are defined by input file "sob22.dat".

```
# Unit given standard deviation of slowness: ERRMUL=SQRT(N1*N2)
  GRD='mar-u0.out'  INDFUN=1  MPAR=1  POWERM=-1  ERRMUL=93.9
  GM1='gm1.out'  GM2='gm2.out'  GM3=' '  DM1='dm1.out'
  invpts: 'mar-inv.h' /
  GRD=' '
```

Each column of general $m_1 \times m_2$ matrix **gm1** corresponds to a given slowness value to be fit. Number m_2 of given slowness values is stored in file "m2.out". Each column of general matrix **gm1** contains the derivative of the slowness with respect to B-spline coefficients. Vector **gm2** contains the differences of the given slowness values from the values in the initial model "mar-mod0.dat". Diagonal covariance matrix **dm1** contains the variances of the input values, all equal to ERRMUL^2. We have specified ERRMUL equal to the square root of the number of given slowness values, so that

$$|\mathbf{u}_{INI} - \mathbf{u}_0| = \sqrt{\mathbf{gm2}^{\mathrm{T}}\,\mathbf{dm1}^{-1}\,\mathbf{gm1}}$$

is the standard deviation between the initial model and the given slowness values.

Minimization of objective function (15) then yields increments **gm5** of the B-spline coefficients in the form of

$$\mathbf{gm5} = (\mathbf{gm1}\ \mathbf{dm1}^{-1}\,\mathbf{gm1}^{\mathrm{T}} + \mathtt{SOBMUL}^2\,\mathbf{modsob})^{-1}\,\mathbf{gm1}\ \mathbf{dm1}^{-1}\,\mathbf{gm2}$$

```
# Matrix operations
  N1=0  N2=1  N3=1  M1='m2.out'
  CAL='inv.cal' GRD1='dm1.out' GRD2='dm2.out'
  grdcal: 'mar-inv.h' /
```

We have calculated diagonal matrix $\mathbf{dm2} = \mathbf{dm1}^{-1}$.

```
  M1='m1.out'  M2='m2.out'  GM1='gm1.out'  DM1='dm2.out'  SM1='sm1.out'
  gmdmgmt: 'mar-inv.h' /
```

We have calculated symmetric matrix $\mathbf{sm1} = \mathbf{gm1}\ \mathbf{dm2}\ \mathbf{gm1}^{\mathrm{T}}$.

```
  N1=0  N2=0  N3=1  M1='m1.out'  M2='m1.out'  SOBMUL=36000
  CAL='addsob.cal' GRD1='sm1.out' GRD2='modsob.out' GRD3='sm2.out'
  grdcal: 'mar-inv.h' /
```

We have calculated symmetric matrix $\mathbf{sm2} = \mathbf{sm1} + \texttt{SOBMUL}^2\,\mathbf{modsob}$. Note that the value of parameter SOBMUL should be updated for individual iterations. The above value of SOBMUL corresponds to the third iteration, producing the final model.

```
  M1='m1.out'                 SM1='sm2.out'  SM2='sm3.out'
  sminv: 'mar-inv.h' /
```

We have calculated symmetric matrix $\mathbf{sm3} = \mathbf{sm2}^{-1}$.

```
  M1='m2.out'  M2=' '         DM1='dm2.out'  GM1='gm2.out'  GM2='gm3.out'
  dmgm: 'mar-inv.h' /
```

We have calculated vector $\mathbf{gm3} = \mathbf{dm2}\,\mathbf{gm2}$.

```
  M1='m1.out'  M2='m2.out'  GM1='gm1.out'  GM2='gm3.out'  GM3='gm4.out'
  gmgm: 'mar-inv.h' /
```

We have calculated vector $\mathbf{gm4} = \mathbf{gm1}\,\mathbf{gm3}$.

```
  M1='m1.out'  M2=' '         SM1='sm3.out'  GM1='gm4.out'  GM2='gm5.out'
  smgm: 'mar-inv.h' /
```

We have calculated resulting vector $\mathbf{gm5} = \mathbf{sm3}\,\mathbf{gm4}$ of the increments of the B-spline coefficients.

```
# Updating the model
  M1='m1.out'  MODNEW='gm5.out'
  modmod: 'mar-inv.h' /
```

We have updated the initial model. The new model is 'mar-mod.out'.

The differences between the given slowness values and the new model are

$$\mathbf{u}_0 - \mathbf{u} = \mathbf{gm2} - \mathbf{gm1}^{\mathrm{T}}\,\mathbf{gm5}$$

The standard deviation between the given slowness values and the new model is

$$|\mathbf{u}_0 - \mathbf{u}| = \sqrt{(\mathbf{gm2} - \mathbf{gm1}^{\mathrm{T}}\,\mathbf{gm5})^{\mathrm{T}}\,\mathbf{dm1}^{-1}\,(\mathbf{gm2} - \mathbf{gm1}^{\mathrm{T}}\,\mathbf{gm5})}$$

```
# Calculating components of the residual objective function
# ~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~~
# Standard slowness deviation 'mar-ud1.out' of the updated model
  M1=' '  M2='m1.out'  M3='m2.out'
  GM1='gm5.out'  GM2='gm1.out'  GM3='gm8.out'
  gmgm: 'mar-inv.h' /
  M3=' '
```

We have calculated vector $\mathbf{gm8} = \mathbf{gm1}^{\mathrm{T}}\,\mathbf{gm5}$.

```
  N1=0  N2=1  N3=1  M1='m2.out'
  CAL='sub.cal' GRD1='gm2.out' GRD2='gm8.out' GRD3='gm9.out'
  grdcal: 'mar-inv.h' /
```

We have calculated vector $\mathbf{gm9} = \mathbf{gm2} - \mathbf{gm8}$.

```
  M1=' '  M2='m2.out'  GM1='gm9.out'  DM1='dm2.out'  SM1='gm0.out'
  gmdmgmt: 'mar-inv.h' /
```

We have calculated square $\mathbf{gm0} = \mathbf{gm9}^{\mathrm{T}}\,\mathbf{dm2}\,\mathbf{gm9}$ of the standard deviation.

```
  N1=0  N2=1  N3=1  M1=' '
  CAL='sqrt.cal' GRD1='gm0.out' GRD2='mar-ud1.out' GRD3=
  grdcal: 'mar-inv.h' /
```

We have calculated standard deviation $|\mathbf{u}_0 - \mathbf{u}| = \sqrt{\mathbf{gm0}}$, stored in file "mar-ud1.out".
The Sobolev norm of the new model is

$$|\mathbf{u}_0 - \mathbf{u}| = \sqrt{(\mathbf{gm2} - \mathbf{gm1}^{\mathrm{T}}\,\mathbf{gm5})^{\mathrm{T}}\,\mathbf{dm1}^{-1}\,(\mathbf{gm2} - \mathbf{gm1}^{\mathrm{T}}\,\mathbf{gm5})}$$

```
# Sobolev norm 'mar-un1.out' of the slowness in the updated model
  M1='m1.out'  M2=' '  SM1='modsob.out'  GM1='gm5.out'  GM2='gm6.out'
  smgm: 'mar-inv.h' /
```

We have calculated vector $\mathbf{gm6} = \mathbf{sm5}\,\mathbf{gm5}$.

```
  M1=' '  M2='m1.out'  GM1='gm5.out'  GM2='gm6.out'  GM3='gm7.out'
  gmgm: 'mar-inv.h' /
```

We have calculated square $\mathbf{gm7} = \mathbf{gm5}^{\mathrm{T}}\,\mathbf{gm6}$ of the Sobolev norm.

```
  N1=0  N2=1  N3=1  M1=' '
  CAL='sqrt.cal' GRD1='gm7.out' GRD2='mar-un1.out'
  grdcal: 'mar-inv.h' /
```

We have calculated Sobolev norm $\|\mathbf{u}\| = \sqrt{\mathbf{gm7}}$ of the new model, stored in file "mar-un1.out".

```
# Important output files
# ~~~~~~~~~~~~~~~~~~~~~~
# 'mar-u0.out'...  Gridded slowness being fit.
# 'mar-mod0.dat'...Smoothed model.
# 'mar-ud1.out'... Standard slowness deviation of the model.
# 'mar-un1.out'... Sobolev norm of the slowness in the model.
```

References

Klimeš, L.: (1999): Lyapunov exponents for 2-D ray tracing without interfaces. In: Seismic Waves in Complex 3-D Structures, Report 8, pp. 83–96, Dep. Geophys., Charles Univ., Prague.

Klimeš, L. (2000): Sobolev scalar products in the construction of velocity models. In: Seismic Waves in Complex 3-D Structures, Report 10, pp. 15–40, Dep. Geophys., Charles Univ., Prague.

Žáček, K. (2000): Smoothing the Marmousi model. In: Seismic Waves in Complex 3-D Structures, Report 10, pp. 41–62, Dep. Geophys., Charles Univ., Prague.