TWO-POINT RAY TRACING IN 3-D

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Abstract

Algorithm for determination of all two-point rays of a given elementary wave by means of the shooting method is presented. The algorithm is designed for general 3-D models composed of inhomogeneous geological blocks separated by curved interfaces. It is independent of the initial conditions for rays and of the initial-value ray tracer. The algorithm described have been coded in Fortran 77, using subroutine packages MODEL and CRT for model specification and for initial-value ray tracing.

Key words: two-point ray tracing, shooting method, ray history function

1 Introduction

In modern seismology the ray method is one of the best developed theories. It is based on the high-frequency approximation of elastodynamic equations, it introduces the concepts of the travel time and of the seismic ray, and enables important physical quantities along the ray to be computed. Modern numerical methods enable rays of any 3D seismic waves to be computed in 3D laterally inhomogeneous models with a block structure. They can be used to compute the trajectory, the times of propagation, and the slowness vector along the ray. They also allow the ray-theory amplitudes to be computed and synthetic seismograms to be constructed.

To solve the two-point ray tracing problem means to find all the rays which propagate from the source and pass through the receiver, a point of given coordinates, most frequently located on the surface of the model. These rays, the solutions of the two-point ray tracing problem, are commonly called boundary-value rays or two-point rays. In this study these rays will be referred to as the two-point rays. In solving the two-point problem by shooting, we seek the parameters of the two-point ray, most frequently the shooting angles under which each ray leaves the point source. The solution of two-point ray tracing is of principal significance in all seismological problems, particularly in refraction seismic tomography and in locating the seismic hypocentre.

2 Basics of the Two-point Ray Tracing Method

2.1 Ray Domain, Normalized Ray Domain and Reference Surface

Seismic rays, propagating from a source, are parametrized by ray parameters Γ_1 and Γ_2 . The 2-D set of all ray parameters $(\Gamma_1, \Gamma_2)^T$ is the ray domain. The algorithm is independent of initial conditions for rays: the boundary-value rays, corresponding to an incident wave specified along the bottom and sides of the model, or to an exploding reflector (any 3-D surface or an edge) are sought in the same way as two-point rays leaving the point source. If the source is a point source, parameters Γ_1 and Γ_2 are most frequently the shooting angles of the ray and the ray domain represents the surface of a unit sphere. The ray domain is assumed to be the image of the rectangular normalized ray domain $\langle\gamma_{1min}, \gamma_{1max}\rangle \times \langle\gamma_{2min}, \gamma_{2max}\rangle$ in mapping $(\gamma_1, \gamma_2)^T \to (\Gamma_1, \Gamma_2)^T$, projecting normalized ray parameters $(\gamma_1, \gamma_2)^T$ onto ray parameters $(\Gamma_1, \Gamma_2)^T$. For example, in the case of a point source, this mapping is the projection of a rectangular planar normalized ray domain onto the unit sphere.

The surface, on which the receiver is located, is defined in the model volume. This surface is parametrized by two parameters, x_1, x_2, so that these coordinates can be determined at every point of the surface. The receivers, located on the surface, are defined as points with coordinates $(x_1^k, x_2^k)^T, k = 1...n$, where n is the number of receivers. This surface is most frequently the Earth's surface on the top of the model. Hereinafter this surface will be referred to as the reference surface.

2.2 Ray Code versus Ray History

The incidence of a seismic wave at an interface (the surface between two model blocks) produces four ordinary seismic body waves, reflected P and S, and transmitted P and S waves. In ray tracing, one must know a priori which of the four generated waves to follow at every interface. This a priori information (quantity given by input data) specifying the behaviour of the ray from its initial point to its endpoint, is the ray code. The elementary wave is that part of the wavefield that is described by one specific ray code.

In each computation only the rays of one elementary wave are taken into account, and the rays are thus specified uniquely by the normalized ray parameters $(\gamma_1, \gamma_2)^T$.

Initial-value ray tracing, i.e. the computation of a ray of given normalized ray parameters, assigns to every ray a value of the so-called history function. The history function is an integer-valued function, which assigns the rays to various groups according to their history, i.e. according to the structural blocks and interfaces through which the ray has propagated, as well as to the position of its endpoint (at the bottom of the model, at the sides of the model, at the surface, overcritical incidence at an interface, ...), and the caustics encountered. Rays, which have propagated through the same model blocks, have crossed the same boundaries, have the same phase shift due to caustics, and are incident, e.g., on the surface of the model, are assigned the same value of the history function. The ray history is a function of normalized ray parameters $(\gamma_1, \gamma_2)^T$, because only the rays of one elementary wave are taken into account in each computation.

Initial-value ray tracing also assigns unique coordinates of incidence, $(x_1, x_2)^T$, to all rays incident at the reference surface. These rays are referred to as successful rays. For successful rays, with the same value of the history function, mapping

$(\gamma_1, \gamma_2)^T \rightarrow (x_1, x_2)^T$ is continuous and univalued in models composed of reasonably smooth blocks. This makes it relatively easy to interpolate two-point rays, because derivatives $\frac{\partial \gamma_I}{\partial x_J}; I, J = 1, 2$ are available for these rays.

2.3 Principle of the Two-point Ray Tracing Method

However, mapping $(\gamma_1, \gamma_2)^T \rightarrow (x_1, x_2)^T$ is continuous only within the region with the same value of the history function. Consequently, the fundamental idea of the solution of two-point ray tracing in heterogeneous block models consists in dividing the normalized ray domain into regions with the same value of the history function. The demarcation belts between the different ray histories, corresponding to the numerical uncertainty of the boundaries between the ray histories, have to be kept reasonably narrow, because all two-point rays, situated inside the demarcation belts, will be lost. As soon as the normalized ray domain is decomposed, with the given accuracy, into homogeneous subdomains corresponding to rays of equal ray histories, the two-point rays can be found (using, e.g., the paraxial ray approximation, see Section 3.4.) within the individual homogeneous subdomains corresponding to successful rays. The importance of a sufficiently accurate determination of the boundaries between homogeneous subdomains is illustrated in Figure 1.

The algorithm of solving the two-point problem described is coded in Fortran 77 in the form of a subroutine. To be able to operate it, a main program and subroutines are required, which provide for the specification of the model, determine the metric, and compute the rays (initial-value ray tracing). By specification of the model one understands determining the dimensions of the model, determining the blocks and boundaries between them. Required are also procedures to determine the position of the source, the domain of shooting parameters, the reference surface on which the receivers are located, its parametrization, and the coordinates of the receivers. From the point of view of the shooting algorithm, computing the ray means mapping normalized ray parameters γ to ray parameters Γ (e.g., shooting angles), determining the history function, the value of metric tensor $\mathbf{G}$ (see next section) and, if the ray is incident at the reference surface, computing coordinates x_1, x_2 and the appropriate derivatives $\frac{\partial \gamma_I}{\partial x_J}$.

3 Detailed Description of the 3-D Shooting Algorithm

It is now necessary to introduce a few terms. A triangle in the normalized ray domain is created by three rays of normalized ray parameters $(\gamma_1^i, \gamma_2^i)^T, i = 1, 2, 3$. The triangles are assumed to be right-handed. A basic triangle is an approximately equilateral triangle in the given metric with the length of the side given. The apexes of the basic triangle are formed by basic rays. A homogeneous triangle is a triangle whose apexes are formed by rays with the same value of the history function.

The metric is introduced in the normalized ray domain for several reasons. First of all, it reduces the problem of uniform coverage of the ray domain with basic rays to the problem of coverage of the normalized ray domain with basic triangles. If this special metric were not introduced and if the normalized ray domain were covered by equilateral triangles, the ray domain (e.g., the surface of the unit sphere around the point source) would be covered non-uniformly (e.g., with small density of rays at the equator and high concentration of rays in the neighbourhood of the poles).

The subroutine for two-point ray tracing assumes a fixed normalized ray domain, $\langle 0, 1\rangle \times \langle 0, 1\rangle$, and a fixed length of the side of the basic triangles. The second purpose

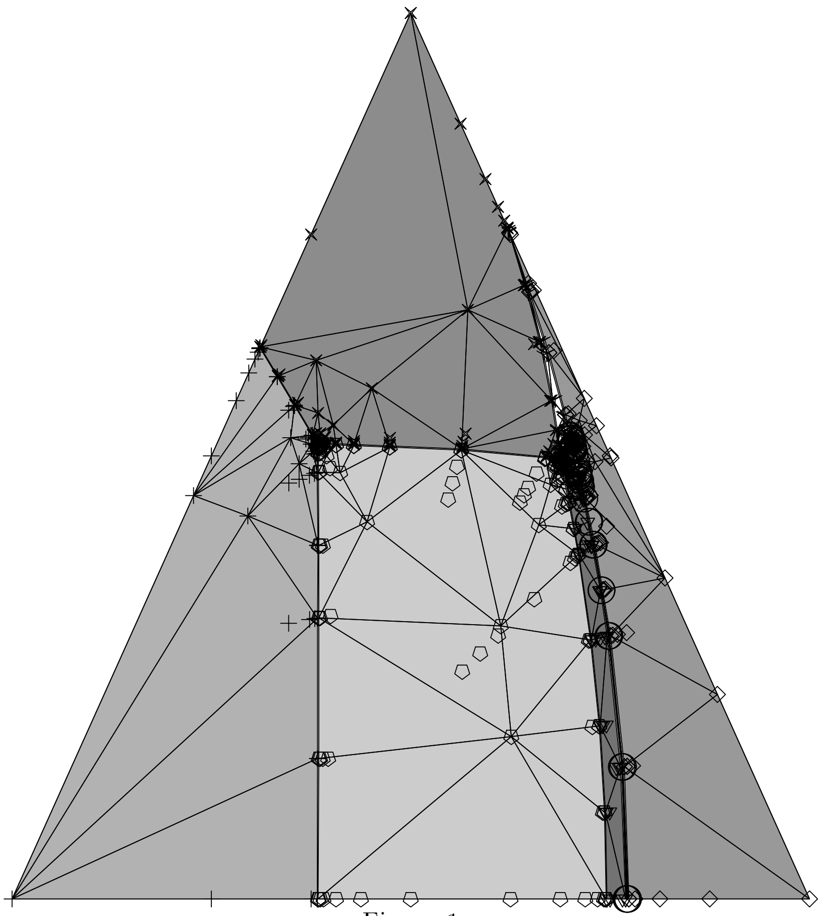

Figure 1
Basic triangle, completely divided into homogeneous triangles. The figure is based on real computation in the model with a lens, see Section 4.1. There are 7 different ray histories in this basic triangle, distinguished by different marks for rays and by different shading for homogeneous triangles. The history corresponding to successful rays refracted in the bottom layer (circle marks) includes two-point ray. The history form a very thin strip. The thickness of this homogeneous subdomain in shooting angles is about 0,00015 π. Very accurate tracing of boundaries between rays with different histories is needed to demarcate this subdomain.

of the metric is to reduce these values to the required dimensions, given by the input data.

The metric also enables the density of the basic rays to be controlled with regard to geometrical spreading.

The metric in the normalized ray domain is introduced as a symmetric positive-definitive 2×2 matrix $\mathbf{G}$, so that the distance, d, between every two close rays on the normalized ray domain with parameters $\gamma^A = (\gamma_1^A, \gamma_2^A)^T, \gamma^B = (\gamma_1^B, \gamma_2^B)^T$ is given as

$$d^2 = |\gamma^B - \gamma^A|^2 = (\gamma^B - \gamma^A)^T G(\gamma^B - \gamma^A). \tag{1}$$

The value of matrix G is approximated by the arithmetic average of its values at points γ^A and γ^B. If the values of the matrix are not known at both points (e. g., in proposing a new ray - the third apex of a new basic triangle) its value is approximated by the value at a known point.

The principle of the solution of the two-point problem is a gradual covering of the normalized ray domain with basic triangles which serve for the initial estimate of the distribution of homogeneous subdomains in the normalized ray domain. The next step is to divide these basic triangles into homogeneous triangles. This is followed by searching for two-point rays inside the homogeneous triangles formed by successful rays. The procedure described above is continued until the whole normalized ray domain has been covered by homogeneous triangles and until the two-point rays inside the homogeneous triangles have been determined.

3.1 The Main Solution Loop

Two-point ray tracing begins with initialisation. The four basic rays are first shot into the corners of the normalized ray domain. The bottom of the normalized ray domain is then covered by basic rays, the distance between which is approximately equal to the length of the side of the basic triangles in the given metric. The solution loop starts after initialisation.

The main solution loop of two-point ray tracing consists of several steps: adding a new basic triangle, dividing it into homogeneous triangles, and finding the two-point rays in these triangles.

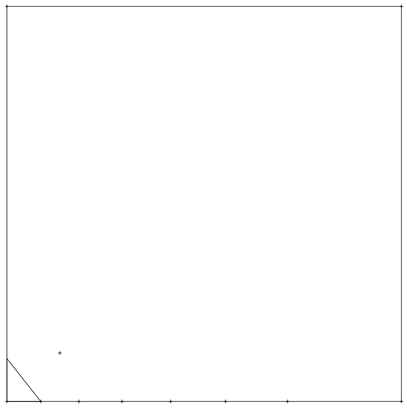

Figure 2
Beginning of covering the normalized ray domain with basic triangles. The basic rays, which form the polyline, are at the bottom of the normalized ray domain. The first basic triangle is in the left hand lower corner of the normalized ray domain. A new basic ray, which will become a apex of the second basic triangle, is to its right.

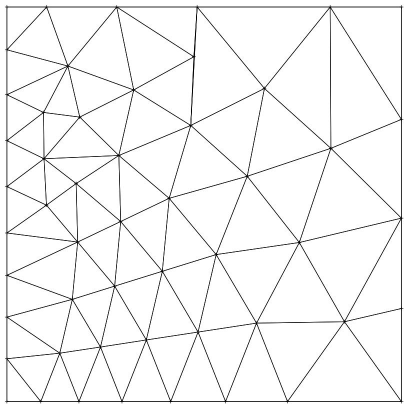

Figure 3
The normalized ray domain covered completely with basic triangles.

The normalized ray domain is covered with basic triangles gradually, beginning from the bottom of the normalized ray domain and proceeding to its top (upper side). After the initialisation, the boundary between the part of the normalized ray domain, covered by basic triangles, and the part not covered, is always defined. This boundary is a polyline joining the apexes of the basic triangles. Its origin is always located on the left hand side of the normalized ray domain, its end always on the right hand side. After the initialisation, the polyline is formed by the rays located at the bottom side of the normalized ray domain. If the polyline as a whole lies on its upper side, the whole normalized ray domain has already been searched, and the calculation of rays of the given wave is terminated. The opening and final coverages of the normalized ray domain with basic rays and basic triangles are illustrated in Figures 2 and 3.

Thus, a new basic triangle is added in each solution loop of the two-point ray tracing. The basic triangle is determined as follows: The lowest ray (with the smallest γ_2) and the lower of its two neighbouring rays are identified on the polyline. A new, approximately equilateral (in the sense of metric G) basic triangle is created on this basic side of the triangle. One has to bear in mind that the third apex of the new basic triangle must neither lie outside the normalized ray domain, nor too close to another ray on the polyline, and that the sides of the new basic triangle must not intersect the polyline. After adding the new basic triangle, it is necessary to update the polyline and, if required, to calculate a new ray - the apex of the new basic triangle.

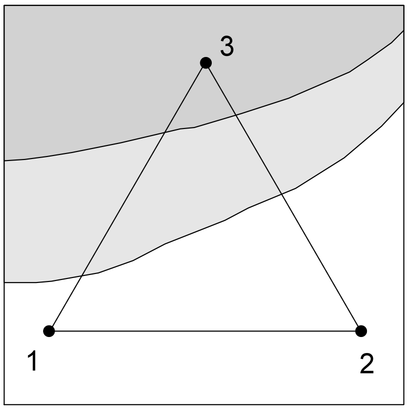

Figure 4
An inhomogeneous basic triangle is formed by basic rays 1,2,3. The regions with different values of the history function are shaded differently. Basic rays are plotted by black circles.

The next step is to determine whether the new basic triangle is homogeneous. If it is not, it is necessary to divide this new inhomogeneous basic triangle into homogeneous triangles. First, the boundary rays are to be found on the sides of the triangle. (The boundary rays are two very close rays with different values of the history function. The pairs of boundary rays serve to separate the homogeneous subdomains.) The boundary rays are sought using the method of halving intervals. The apexes of the triangle together with the boundary rays on its sides are understood to be an inhomogeneous polygon from which areas with the same value of the history function, homogeneous polygons, are gradually separated. These are then divided into homogeneous triangles. First, any apex of the inhomogeneous polygon is taken to initiate the homogeneous polygon, together with all its other neighbouring apexes with the same value of the history function. Then the boundary between the corresponding ray history and other histories

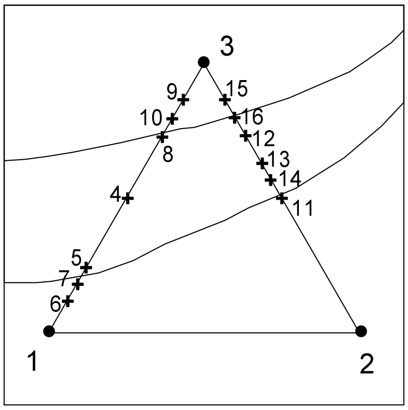

Figure 5
Searching for boundary rays on the sides of the inhomogeneous triangle by halving intervals. Auxiliary rays are plotted as crosses.

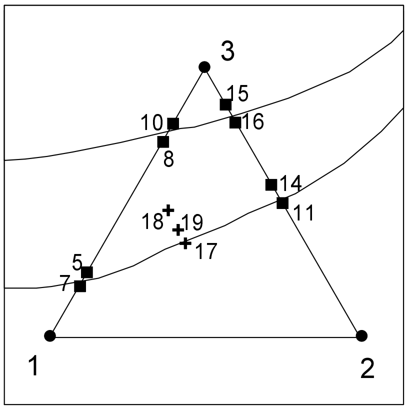

Figure 6
The identified boundary rays are plotted as squares; together with the basic rays they form the inhomogeneous polygon. The initial homogeneous polygon is formed by rays 7,1,2,11. Auxiliary rays 17,18,19 are shot to demarcate the part of the boundary limited by the pairs of boundary rays 5,7 and 11,14.

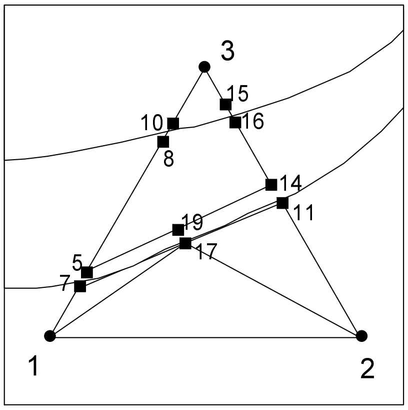

Figure 7
Assume that that the boundary between histories, demarcated by boundary rays 11,14 , 17,19 and 7,5, is approximated with a given accuracy. Then the separated homogeneous polygon formed by rays 7,1,2,11,17 is divided into homogeneous triangles 1,17,7 , 1,2,17 and 2,11,17. Rays 14,16,15,3,10,8,5,19 form the new inhomogeneous polygon.

is traced in order to complete the homogeneous polygon. For the description of the tracing this boundary refer to Section 3.3. Finally, the homogeneous polygon is covered by homogeneous triangles. The method of dividing basic triangles into homogeneous triangles is illustrated in Figures 4 to 7, see also Figure 1.

The last step of the solution loop of two-point ray tracing is the search for the two-point rays inside the homogeneous surface triangles, formed by the rays incident on the reference surface. The ray which is closest to the receiver on the reference surface is selected, and the parameters of a new ray are proposed using paraxial ray approximation. If this new ray is closer to the receiver on the reference surface, it becomes a point of departure for proposing a new ray. This procedure is continued until a ray incident sufficiently close to the receiver is found. This ray is then designated as the two-point ray.

Once all the homogeneous triangles have been searched, the memory is "empted". Only the homogeneous triangles located at the boundary of the covered part of the normalized ray domain, the rays forming the polyline, the two-point rays, and the rays located in the corners of the normalized ray domain are retained in the memory. This is followed by adding a new basic triangle until the whole normalized ray domain is covered.

3.2 Adding a New Basic Triangle

In adding a new basic triangle, the lowest ray on the polyline and the lower of its neighbouring rays are identified first. These rays are marked A and B, from left to right along the polyline. A third ray, the apex of the new triangle, is then sought for this side of the future new basic triangle. The parameters of this ray $\gamma^C = (\gamma_1^C, \gamma_2^C)^T$ are determined using the formula

$$\gamma^C = \frac{\gamma^A + \gamma^B}{2} + \frac{\sqrt{3}}{2}\frac{1}{\sqrt{detG}}\varepsilon G(\gamma^A - \gamma^B)\frac{a}{|\gamma^A - \gamma^B|}, \tag{2}$$

where $\varepsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$, and a is the length of the side of the basic triangle.

Once the parameters of the new ray have been proposed, the ray is checked whether it is outside the normalized ray domain. If so, the ray is moved to the boundary of the normalized ray domain along the line perpendicular to the boundary in the given metric. The new normalized ray parameters γ^D are computed by solving equation

$$(\gamma^D - \gamma^C)G(a,b)^T = 0 \tag{3}$$

which indicates that the scalar product is zero and, therefore, that the vector $(\gamma^D - \gamma^C)$ from the proposed ray C to the new ray position D is perpendicular to vector $(a,b)^T$ parallel with the boundary of the normalized ray domain, e. g., to vector $(a,b)^T = (1,0)^T$ or $(0,1)^T$.

If the new ray is too close to the boundary of the normalized ray domain (i.e. if its distance from the boundary in the given metric is not larger than a selected constant), the ray is moved to the boundary of the normalized ray domain, perpendicularly in the given metric, to avoid creating too narrow triangles at this boundary. The new coordinates of the ray are again computed by solving equation (3).

Finally, it is necessary to check whether the new ray is not too close to another ray on the polyline, because this would lead to creating too narrow triangles. The distance

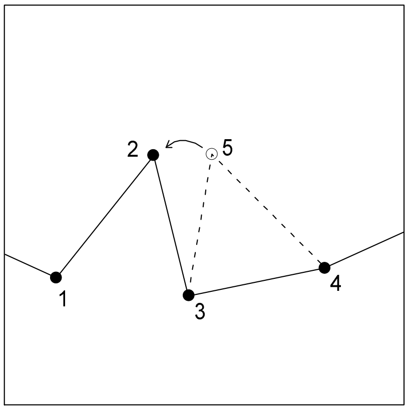

Figure 8
New ray 5 has been proposed for polyline side 3,4. However, creating triangle 3,4,5 would require creating a very narrow triangle 3,5,2. Therefore, ray 5 will be identified with ray 2, and a new triangle 3,4,2 will be formed instead of calculating ray 5. Polyline - bold line; proposed triangle - dashed line; rays on the polyline - black circles; proposed ray - circle.

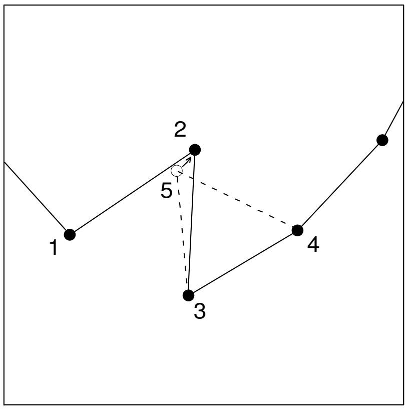

Figure 9
Side 4,5 of the new triangle intersects polyline side 2,3. The new ray will be identified with ray 2 and the new triangle will have apexes 3,4,2. Polyline - bold line; proposed triangle - dashed line; rays on the polyline - black circles; proposed ray - circle.

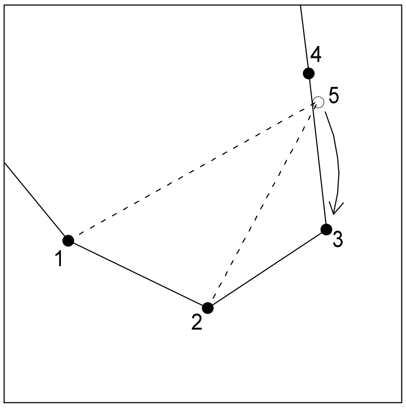

Figure 10
Sides 1,5 and 2,5 of the new triangle intersect polyline side 3,4. The new ray will be identified with ray 3 and the new triangle will have apexes 1,2,3. Polyline - bold line; proposed triangle - dashed line; rays on the polyline - black circles; proposed ray - circle.

is calculated using Eq. (1) in which G is taken to be the same as when proposing the new rays. See Figure 8.

Once this has been done, it is still necessary to check whether the sides of the new triangle intersect the polyline. The points of intersection of both sides of the new triangle with all sides of the polyline are sought. If a point of intersection with a polyline side adjacent to the new triangle is found, the new ray is identified with the more distant of the rays forming this side of the polyline. If a point of intersection with another side of the polyline is found, the new ray is identified with the closer of the rays forming this side of the polyline. See Figures 9 and 10.

After making all the above checks, it is necessary to update the polyline forming the boundary of the area covered by the basic triangles, and to compute a new ray, if required.

3.3 Division of an Inhomogeneous Triangle

The division of inhomogeneous triangles is stepwise. First, the boundary rays have to be found on all the sides of the triangle using the method of halving intervals. The apexes of the triangle together with the boundary rays on its sides are understood to be an initial inhomogeneous polygon. This is followed by successive separation of the homogeneous polygons from the inhomogeneous polygon, and by division of the separated homogeneous polygons into homogeneous triangles. This loop is continued until the inhomogeneous polygon is divided up completely.

Any ray of the inhomogeneous polygon is taken as the initial apex of the homogeneous polygon. One then proceeds along the circumference of the inhomogeneous polygon to both sides of the selected ray, and other rays, which display the same value of the history function as the first selected ray, are included in the homogeneous polygon. Once one arrives at the first rays with a different value of the history function, the procedure is terminated. The homogeneous polygon is now formed by at least one ray and at the most by all the rays of the inhomogeneous polygon.

If all the rays of the inhomogeneous polygon form the homogeneous polygon, the homogeneous polygon is complete. The inhomogeneous polygon is erased, and the algorithm continues by dividing the homogeneous polygon into homogeneous triangles.

If the homogeneous polygon is not formed by all the rays of the inhomogeneous polygon, it is necessary to trace the boundary that separates the new homogeneous polygon from the remaining part of the inhomogeneous polygon. This boundary is limited by the two pairs of boundary rays, each pair being formed by the last (or first) ray included in the homogeneous polygon, and by the first (or last) ray not included in the homogeneous polygon. The boundary is demarcated by the pairs of boundary rays. Each new pair of boundary rays is to be found approximately in the middle between two consecutive pairs of the already determined boundary rays. Tracing the boundary by means of adding new pairs of boundary rays between the already determined pairs is continued, until the deviation of the boundary from the straight line is smaller than the given constant (input data) and the distance of the neighbouring pairs of boundary rays is sufficiently small (input data). This step along the traced boundary is longer for pairs displaying the same history on each side of the boundary. For pairs with different history on one side of the traced boundary the step is equal to distance of the boundary rays.

Once all the pairs of boundary rays on the traced boundary have been identified, the rays which display the same value of the history function as the first selected ray (and as

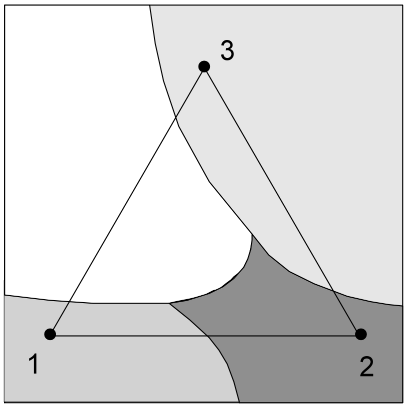

Figure 11
An inhomogeneous basic triangle formed by basic rays 1,2,3 (black circles). The four regions with different values of the history function are shaded differently. This is a fully artificial plot, as well as Figures 12 to 14. Let us note, that in the Figure 19 one can see homogeneous subdomains arranged very similarly as in this plot, which demonstrates suitability of Figures 11 to 14.

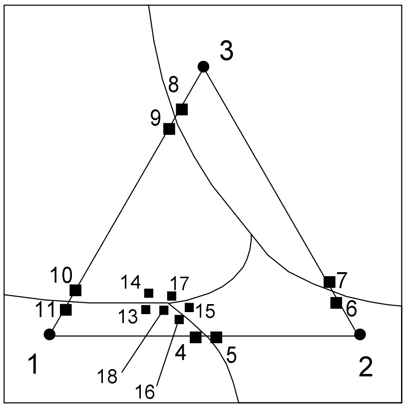

Figure 12
Boundary rays on the circumference of the triangle are identified (bigger squares). The basic homogeneous polygon is formed by rays 11,1,4 and a part of the boundary limited by pairs 10,11 and 4,5 is to be traced. First new pair 13,14 of boundary rays is identified (smaller squares). Since the rays 14 and 5 display different values of the ray history function, this part of the boundary is to be traced more exactly. Pairs 15, 16 and 17,18 are then identified. Tracing of the boundary is then finished, because the distance of rays 15 and 17 is less than the maximum distance between the boundary rays. Homogeneous polygon 11,1,4,16,18,13 may be separated and divided into homogeneous triangles. (To identify all the pairs of boundary rays one must shoot some auxiliary rays. These are not plotted in the figure.)

all the rays of the homogeneous polygon) are added to the homogeneous polygon. The other boundary rays, together with the rays of the original inhomogeneous polygon, not included in the homogeneous polygon, create a new inhomogeneous polygon, ready for the next division.

The division of an inhomogeneous triangle and tracing the boundary between histories is illustrated in Figures 11 to 14. The advantage of tracing the boundary is twofold. The first is in keeping the demarcation belts between the different ray histories reasonably narrow (all two-point rays, situated inside the demarcation belts, will be lost). The second consists in considerably increased probability to identify small homogeneous subdomains. For instance, all the homogeneous subdomains with circumference larger than the distance between the neighbouring pairs of boundary rays will be found. Note that the distance between the neighbouring pairs of boundary rays is given by the

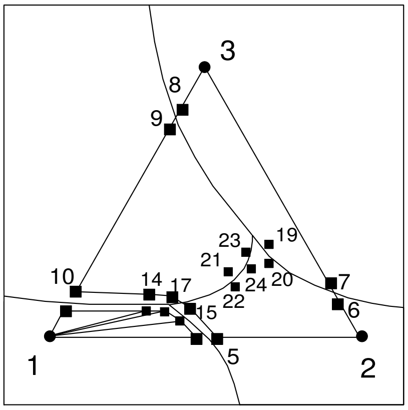

Figure 13
The inhomogeneous polygon is now formed by rays 5,2,6,7,3,8,9,10,14,17,15 (bigger squares). The initial homogeneous polygon chosen for actual separation is formed by rays 15,5,2,6. The boundary is traced by pairs 19,20 , 21,22, and 23,24 (smaller squares). Since the distance between rays 19 and 23 is less than the maximum distance between the boundary rays, homogeneous polygon 15,5,2,6,20,24,22 may be separated and the new inhomogeneous polygon is formed by rays 7,3,8,9,10,14,17,21,23,19.

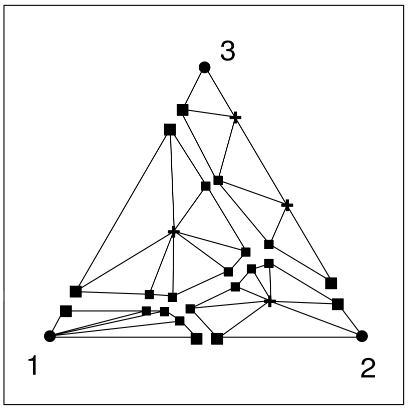

Figure 14
Final division of the inhomogeneous basic triangle into homogeneous triangles. Basic rays - black circles; boundary rays - squares; auxiliary rays traced when dividing homogeneous polygons into homogeneous triangles - crosses. (The demarcation belts are in practice kept much thinner and more pairs of boundary rays than in Figures 11 to 14 are usually needed).

input data, and that metric tensor **G** may contain the information about geometrical spreading along the ray.

If the homogeneous polygon is complete, it should be divided into homogeneous triangles. It is formed by at least one ray and at the most by all the rays of the original inhomogeneous polygon, and by none to several recently traced boundary rays. The subsequent procedure depends on the number of rays which form it. It differs for one, two, three and for four and more rays forming the homogeneous polygon.

If the homogeneous polygon is formed by only one or two rays, a new homogeneous triangle will not be formed, the homogeneous "polygon" is erased, and the algorithm continues by dividing a new inhomogeneous polygon.

If the homogeneous polygon is formed by three rays, these three rays form a new homogeneous triangle, and the algorithm continues by dividing a new inhomogeneous polygon.

If the homogeneous polygon is formed by four or more rays, it is to be covered by new homogeneous triangles. Using eq. (1) the distance between its neighbouring rays is computed for every ray of the homogeneous polygon. The ray whose neighbouring rays are the smallest distance apart is chosen, and parameters of a new ray are proposed using eq. (2), such that the triangle formed by the neighbouring rays and the proposed ray is approximately equilateral. If the proposed ray is not inside divided homogeneous polygon, it is moved to the boundary of the polygon. If the proposed ray is too near the boundary of polygon, it is moved to the boundary. If the proposed ray is too near to some of rays of the homogeneous polygon, it is identified with this ray. All these three checks, which are very similar to the checks done just after proposing the parameters of a new basic ray (see Section 3.2.), must be done to avoid creating too narrow homogeneous triangles. Two new homogeneous triangles are then formed from the proposed ray, the first chosen ray, and his neighbouring rays using the shortest diagonal method. The number of rays of the homogeneous polygon is thus the same or reduced by one or two and the algorithm continues by dividing the homogeneous polygon with regard to the new number of its rays.

Once remaining inhomogeneous polygon is formed by two or less rays, it is declared divided and its division is terminated.

3.4 Identifying Two-point Rays within a Homogeneous Triangle

Any three rays, apexes of a homogeneous triangle, which are incident on the reference surface, create a triangle on this surface. The source of the surface triangle in mapping

$(\gamma_1, \gamma_2)^T \to (x_1, x_2)^T$ is not an exact triangle; it is deformed, but its apexes are identical with those of the corresponding homogeneous triangle. If a receiver is located within the surface triangle, it is very probable that the two-point ray running to the receiver lies within the normalized ray domain inside the corresponding homogeneous triangle, or inside neighbouring homogeneous triangles of the same history function. We shall call these neighbouring triangles adjacent. All homogeneous triangles which have at least one apex in common with the source triangle are in this case considered to be adjacent.

Only the triangles, whose apexes do not lie on the polyline between the part of normalized ray domain, covered by basic triangles, and the part not yet covered, are searched for two-point rays. The search of the triangles lying by the polyline is postponed, because for these triangles future adjacent triangles from the actually uncovered part of the ray domain are not yet at disposal.

The search for two-point rays within a homogeneous surface triangle begins by searching for all adjacent triangles. These triangles are, of course, formed by rays with the same value of the history function as the triangle being treated and, therefore, their images are on the reference surface. This is followed by identifying the receivers located in the surface triangle under consideration.

If a receiver is identified in the surface triangle, one has to check whether a two-point ray, originating in the given triangle or in the adjacent triangles on the normalized ray domain, has already been identified for this receiver. If it has not, the actual search for the two-point ray is to be commenced.

First of all, the ray closest to the receiver on the reference surface is selected among all the rays belonging to the given surface triangle and the adjacent triangles. Given the parameters, coordinates of incidence, and derivatives $\frac{\partial \gamma_I}{\partial x_J}$ for the selected ray, and given the coordinates of the receiver, the parameters of the new ray are proposed using the formula

$$\gamma_I^{new} = \gamma_I^{closest} + d\gamma_I, \tag{4}$$

where $d\gamma_I = (\frac{\partial \gamma_I}{\partial x_J})^{closest} dx_J^{closest} = (\frac{\partial \gamma_I}{\partial x_J})^{closest}(x_J^{receiver} - x_J^{closest})$.

The new ray must lie in the normalized ray domain within one of the triangles being considered. If this is not so, its parameters have to be adjusted using formula

$$\gamma_I^{new} = \gamma_I^{closest} + \frac{d\gamma_I}{2} \tag{5}$$

If the ray is still not within one of the triangles being considered, the two-point ray is probably not located there either, and the algorithm continues by searching for the next receiver located on the reference surface within the triangle being considered.

If the new ray is sufficiently close to the receiver, it is designated two-point. If it is further away from the receiver, but still closer to the receiver than the original closest ray, the new ray becomes the selected initial ray for further interpolation. If the new ray lies further away from the receiver than the original closest ray, the interpolation is commenced from the next closest ray. If there is no such other ray, the algorithm continues by searching for the next receiver located on the reference surface within the triangle being considered.

Once the triangle have been searched, and the two-point rays identified where possible, the triangle is marked as having been searched. (Nevertheless, two-point rays may still be sought (and identified) in it later, because it may become adjacent to some homogeneous triangle which has not yet been searched).

3.5 Splitting a Homogeneous Triangle with a Ray of Different History Function

The value of the history function of the new ray may be different from the value of the homogeneous triangle in which the new ray is located (i.e. from the value of the history function of the rays forming the triangle). In such a case, three new triangles are created from the homogeneous triangle. This division is illustrated in Figures 15 and 16. Since the division leads to the creation of three inhomogeneous triangles, the algorithm reverts to dividing these three triangles into homogeneous triangles immediately.

4 Numerical Examples

Algorithms for solving the two-point problem in a 3D medium have been developed. Using these algorithms, subroutines controlling the shooting have been coded. This chapter is devoted to the description of numerical examples of using the two-point ray tracing program in two different models of seismic media.

The two-point ray tracing procedures determine the initial conditions for computing the rays. The subroutine packages MODEL and CRT are used to calculate the rays in inhomogeneous three-dimensional models with a block structure. For detailed information about the models and about ray tracing itself, refer to Červený at al. (1988).

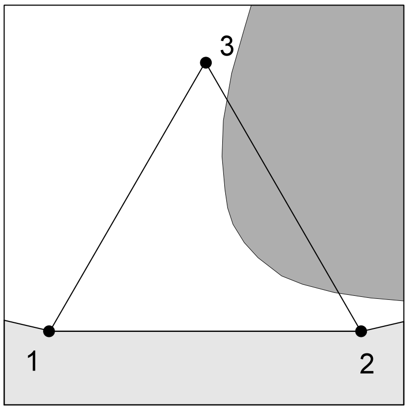

Figure 15
The new basic triangle also covers a part of the homogeneous subdomain (dark grey colour) with different ray history than the apexes of the triangle display. However, all three apexes of the triangle display the same history, and the triangle is marked as homogeneous. (The light grey colour represents the part of the normalized ray domain covered by other basic triangles.)

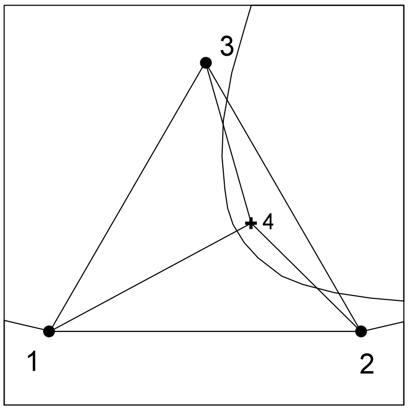

Figure 16
Ray 4 indicates the need to divide the originally homogeneous triangle 1,2,3 into three inhomogeneous triangles 1,2,4 , 2,3,4 , and 3,1,4. The algorithm will continue with the division of these three triangles immediately.

4.1 Model with Lenticular Inclusion

The two-point ray tracing is firstly tested in an inhomogeneous two-layer model with a homogeneous lenticular inclusion in the upper constant velocity-gradient layer. The model is the demo model of the MODEL package. The model is illustrated in Figure 17. The relative velocity of propagation of seismic waves is scaled in grey colour, ranging from light grey for the lowest velocity on the model surface to dark grey at the bottom interface. The lenticular inclusion has a higher velocity than its neighbourhood, and the velocity is highest in the lower layer. Structural interfaces of the first order are covered by smooth surfaces. The dimensions of the model are: length 60 km, width 40 km, and height 30 km, including the free space.

Two-point ray tracing was performed for the refracted elementary wave, i.e. the wave whose rays are transmitted without conversion at any interface. The normalized ray domain of the computation covers the whole ray domain $\langle -\frac{\pi}{2}, \frac{\pi}{2}\rangle \times \langle 0, 2\pi\rangle$. Since not all the rays computed in 3D shooting can be plotted distinctly, Figure 18 shows only the two-point rays found in the computation in this model. Figure 18 also demonstrates the position of the seismic source and receivers.

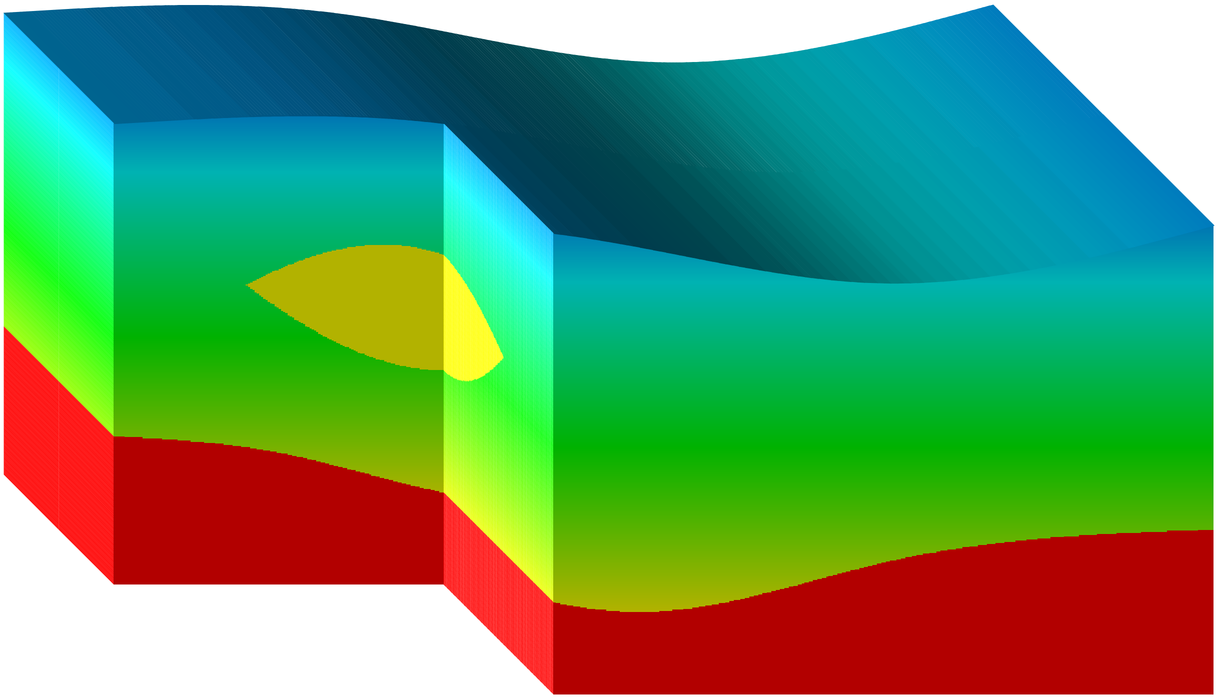

Figure 17
Two-layer model with a homogeneous lenticular inclusion in the upper constant velocity gradient layer. The relative velocity of propagation of seismic waves is scaled in grey colour, ranging from light grey for the lowest velocity to dark grey for the highest one. The structural interfaces of the first order are covered by smooth surfaces. The dimensions of the model are: length 60 km, width 40 km, and height 30 km including the free space.

9057 rays with 26 different values of the history function were computed. The computation took 3 minutes and 40 seconds on a 90 Mhz PC Pentium. Figure 19 shows the coverage of the normalized ray domain $\langle \gamma_{1min}, \gamma_{1max} \rangle \times \langle \gamma_{2min}, \gamma_{2max} \rangle$ with rays. The rays with different values of the history function are symbol-coded. The enlarged symbols indicate two-point rays. Figure 20 shows the same coverage of the normalized ray domain as Figure 19, but the rays are symbol-coded according to their types: basic rays, boundary rays, two-point rays and auxiliary rays.

The Earth's surface was taken to be the reference surface. Figure 21 shows the distribution of the successful rays on the reference surface; the rays are symbol-coded according to the value of their history function as in Figure 19. The large diamonds indicate the nine receivers located on the reference surface. The receivers for which two-point rays were identified, are marked by appropriate enlarged symbols.

Rays of three different histories were incident on the reference surface.
These were:

a) rays which had propagated directly to the surface (marked by triangles in Figures 19 and 21);
b) rays which had propagated to the surface through the lenticular inclusion (marked $\times$);
c) rays with large geometrical spreading refracted at the boundary in the right hand lower part of the model (marked by circles).

As can be seen in Figure 21, the two-point ray tracing program was successful, probably all the two-point rays were found. Note that two-point rays were also found for weakly refracted rays c), although the relative width of the homogeneous subdomain

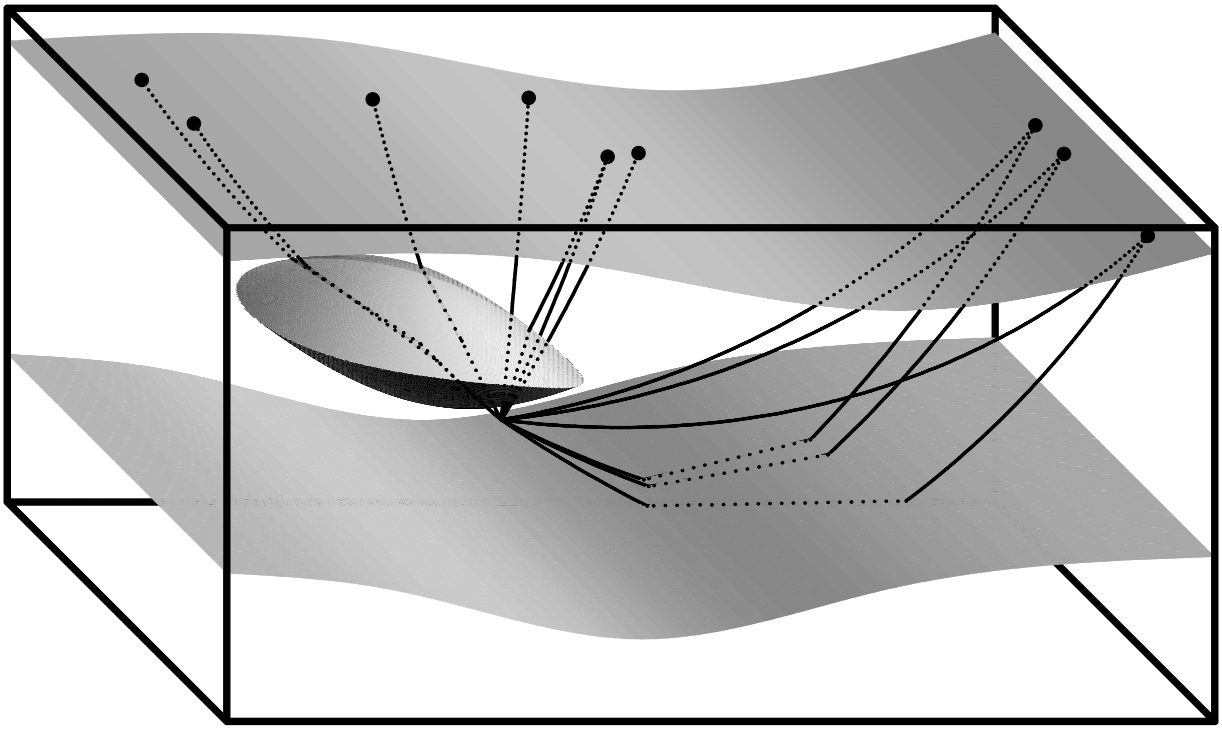

Figure 18
The two-point rays found in the computation in the model with lenticular inclusion. The first five rays from the left are rays that have propagated through the lens, the next five rays are direct rays, and the last three are rays weakly refracted at the lower layer. The figure also shows the position of the seismic source and receivers.

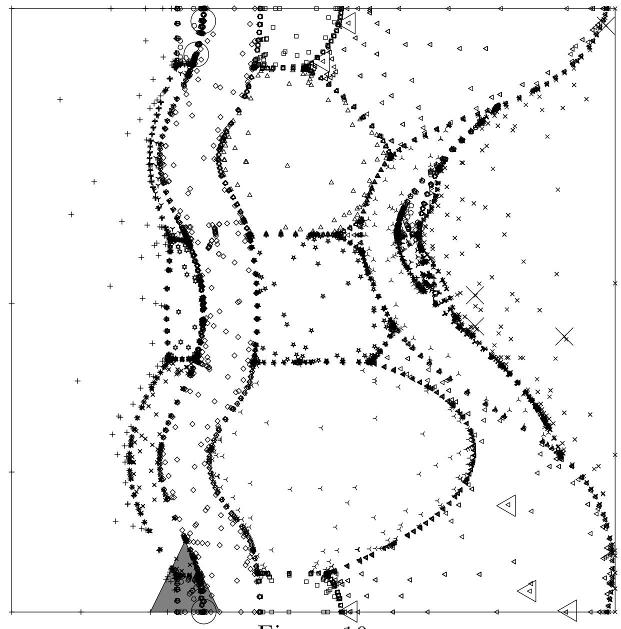

Figure 19
Coverage of the normalized ray domain $\langle \gamma_{1min}, \gamma_{1max} \rangle \times \langle \gamma_{2min}, \gamma_{2max} \rangle$ with seismic rays traced in computation in the model with a lens. The normalized ray domain of the computation covers the whole ray domain $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle \times \langle 0, 2\pi \rangle$. Rays with different values of the history function are plotted using different symbols. The enlarged symbols indicate two-point rays. Enlarged detail of the shaded triangle is shown in Figure 1.

of these rays is about 0.03 per cent of the dimension of the whole ray domain. This is

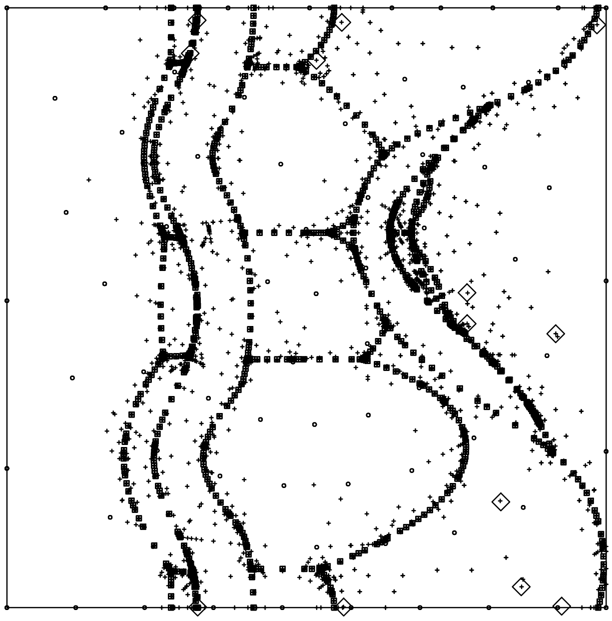

Figure 20
The coverage of the normalized ray domain with seismic rays. The rays are symbol-coded according to their types: basic rays by circles, boundary rays by squares, two-point rays by large diamonds, and auxiliary rays by crosses.

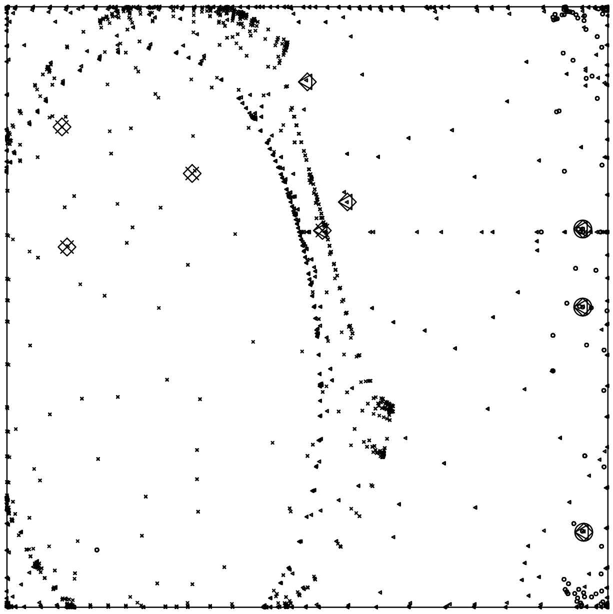

Figure 21
The distribution of the computed successful rays on the reference surface. The large diamonds indicate receivers. The rays are symbol-coded according to the value of their history function as in Figure 19, the two-point rays are marked by appropriate larger symbols.

the advantage of the accurate determination of the boundaries between the rays with different ray histories.

4.2 Model of Western Bohemia

The model of western Bohemia is used as the second example of using the two-point ray tracing program. This model consists of one laterally inhomogeneous block, whose surface corresponds to the actual topology of western Bohemia. It is illustrated in Figure 22. The relative velocity of propagation of seismic waves is scaled in grey colour, ranging from light grey for the lowest velocity to dark grey for the highest velocity. The dimensions of the model are: length 60 km, width 70 km, and height 13.1 km including the free space.

Figure 22 also demonstrates the position of the seismic source and of 559 receivers, which correspond to real measurements carried out in western Bohemia. Figure 23 shows the two-point rays found in this model.

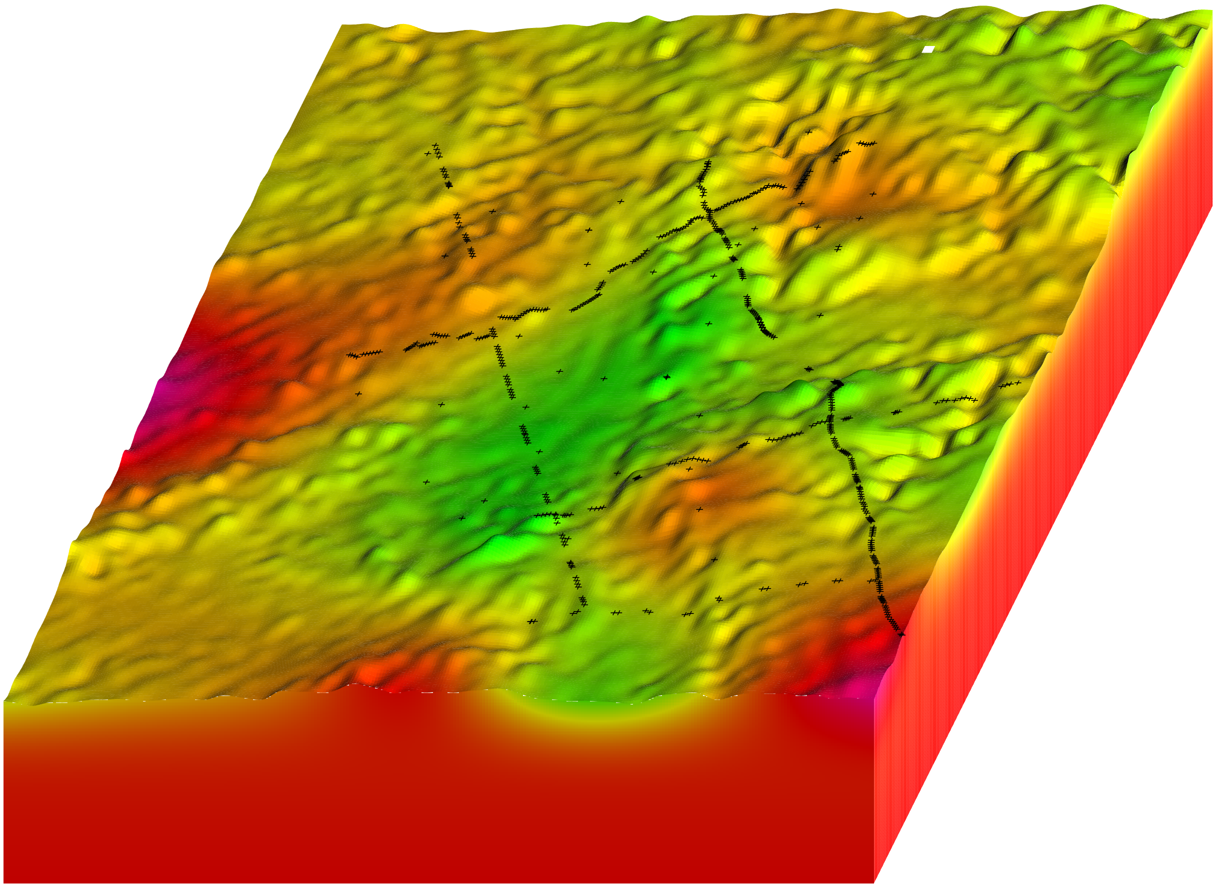

Figure 22
Western Bohemia a priori model with location of the source (white square) and the receivers (white circles). The surface of the model corresponds to the actual topology of western Bohemia. The relative velocity of propagation of seismic waves is scaled in grey colour, ranging from light grey for the lowest velocity to dark grey for the highest velocity. The dimensions of the model are: length 60 km, width 70 km, and height 13.1 km.

Only refracted rays are considered (the model is one-block). The normalized ray domain of the computation covers the whole ray domain $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle \times \langle 0, 2\pi \rangle$, which ensures the search of the whole model for two-point rays.

2720 rays with 6 different values of the history function corresponding to 6 different boundary surfaces of the block were computed. The computation took 2 minutes and 40 seconds on a 90 Mhz PC Pentium. Figure 24 shows the coverage of the normalized

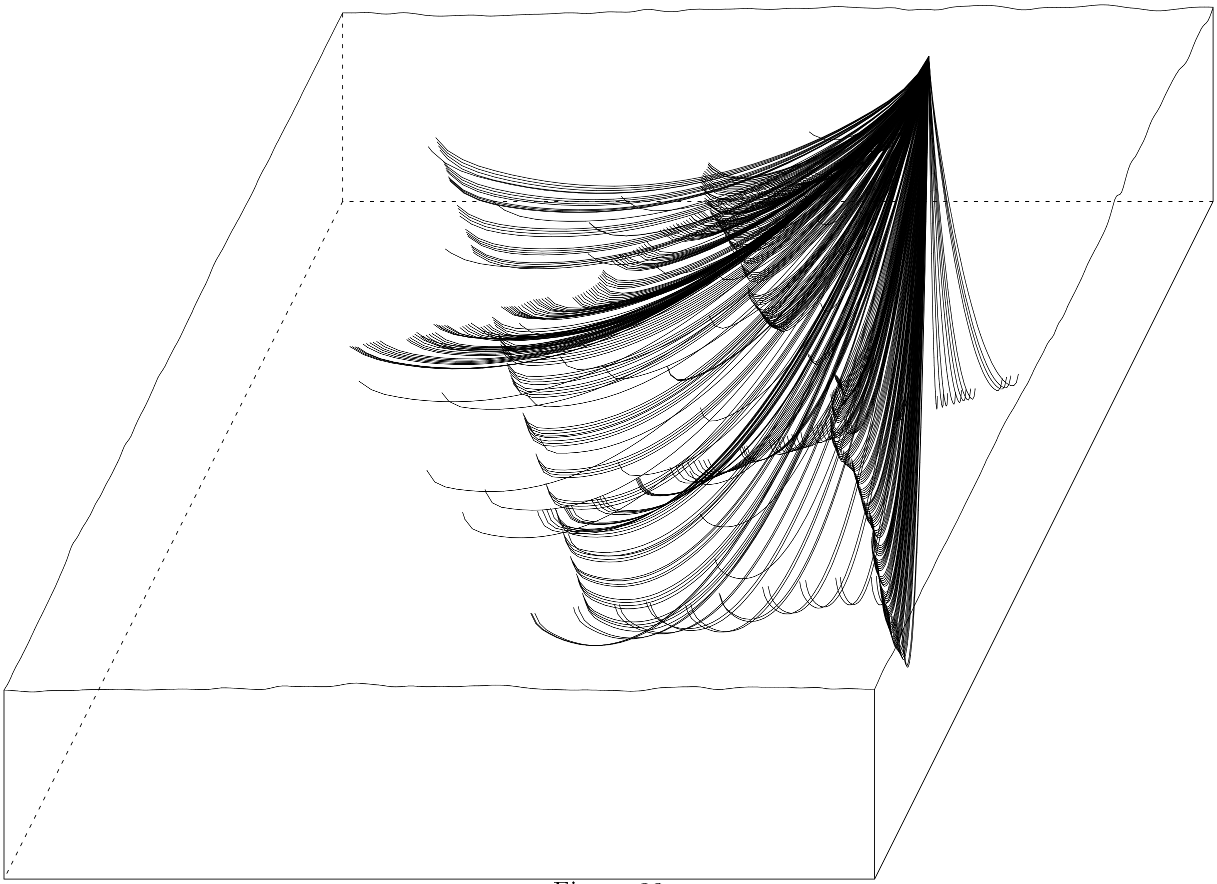

Figure 23
The two-point rays identified during the computation in the model of western Bohemia.

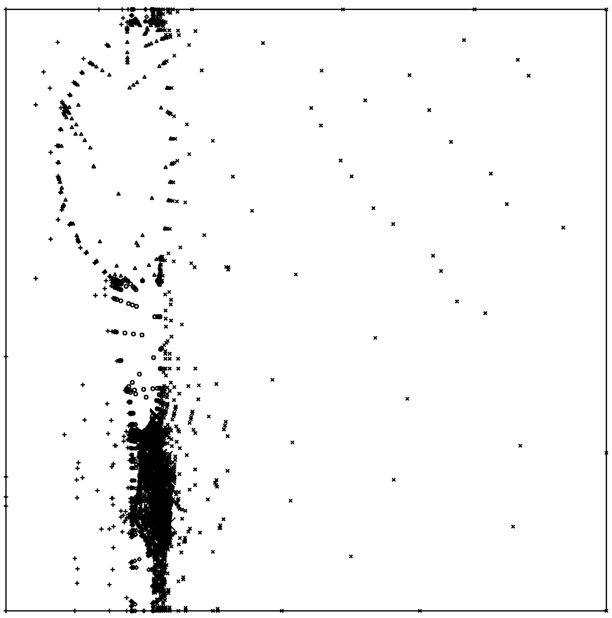

Figure 24
The coverage of the normalized ray domain with seismic rays. The normalized ray domain of computation covers ray domain $\langle -\frac{\pi}{2}, \frac{\pi}{2} \rangle \times \langle 0, 2\pi \rangle$. The rays with different values of the history function are plotted using different symbols. Larger symbols indicate two-point rays.

ray domain $\langle \gamma_{1min}, \gamma_{1max} \rangle \times \langle \gamma_{2min}, \gamma_{2max} \rangle$ with computed rays. The rays with different

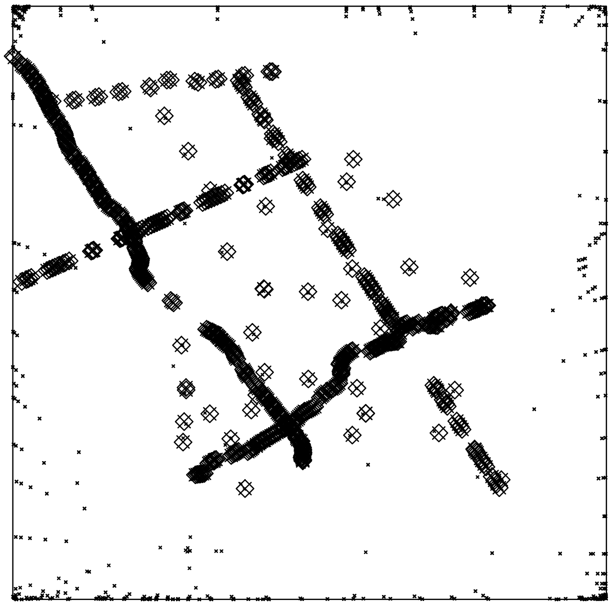

Figure 25
The distribution of successful rays on the reference surface. The larger diamonds indicate receivers, the larger X-crosses indicate two-point rays.

values of the history function are symbol-coded. The enlarged symbols indicate two-point rays.

The Earth's surface is taken to be the reference surface. Figure 25 shows the distribution of the successful rays on the reference surface; the large diamonds again indicate receivers, and the enlarged crosses indicate the two-point rays identified. Almost all the two-point rays to the receivers were found. Only two receivers near the periphery of the model were not found.

5 Conclusion

Algorithms for solving two-point ray tracing problems in a 3D heterogeneous medium have been developed. Special emphasis was put on the sufficiently accurate determination of boundaries between rays with different ray histories, and on finding all the homogeneous subdomains of the ray domain. Consequently, the algorithm is very reliable in computing all two-point rays of the given elementary waves to the given receivers.

The shooting algorithm designed, is thus closely related to controlled initial-value ray tracing, whose aim is not to hit given receivers, but to cover the model with a sufficiently dense system of rays (or to cover the reference surface with a sufficiently dense system of ray endpoints). The demarcation of ray histories and the triangularization of homogeneous subdomains, corresponding to rays of equal ray histories, play the principal role in controlled initial-value ray tracing. In this way the space between two consecutive wavefronts is decomposed into ray cells, as in the wavefront tracing method (Vinje at al. 1996), and the travel times and all other calculated quantities may routinely be interpolated within the individual ray cells. Unlike the weighting of paraxial ray approximations, this method is capable of complying strictly with the boundaries of shadow zones. Controlled initial-value ray tracing, basically equivalent to wavefront tracing, is thus a very promising tool of fast calculation of most energetic elementary travel times at gridpoints of dense rectangular point grids, e.g., for ray-theory based migration techniques, or the nonlinear determination of seismic hypocentres.

The shooting algorithm is, to some extent, independent of the initial-value ray tracer. "To some extent" means that the ray tracer has to provide the shooting algorithm with required information on traced rays. The algorithm is also independent of initial conditions for rays. Similarly, point receivers can be replaced by a different boundary condition.

The corresponding Fortran 77 computer code for 3-D two-point ray tracing and controlled initial-value ray tracing is being developed on the basis of package MODEL to specify smooth block models of geological structures, and package CRT for initial-value ray tracing in such models (Červený at al. 1988). Two examples of seismic models were used to demonstrate and discuss the efficiency and effectiveness of the two-point ray tracing program. One can see that the two-point ray tracing method described is relatively demanding with regard to the number of rays to be traced. The number of rays, which have to be computed, is not significantly affected by the number of receivers, nor by the number of two-point rays identified, because these mostly involve auxiliary rays required to divide the normalized ray domain into regions with the same value of the history function.

As can be seen from the computation in the model with lenticular inclusion, the accurate tracing of the boundary between two regions of the normalized ray domain with different values of the history function ensures the method highly reliable within very small (or narrow) regions of the normalized ray domain.

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