Comparison of VSP and sonic–log data in non–vertical wells in a heterogeneous structure

Petr Bulant * & Luděk Klimeš, Department of Geophysics, Faculty of Mathematics and Physics, Charles University, Prague, Czech Republic, http://sw3d.mff.cuni.cz/staff/bulant.htm, http://sw3d.mff.cuni.cz/staff/klimes.htm

Summary

In order to compare the results of sonic–log measurements and of vertical seismic profiling (VSP), the sonic–log velocities are used to estimate the corresponding travel time in the geological structure, which is then compared with the VSP travel time.

In this paper we show how to calculate the sonic–log travel time in the geological structure from the sonic–log velocities while taking into account the effects of the non–vertical propagation of seismic waves due to the source offset and due to the heterogeneous velocity in the structure, together with the effects of well trajectory differing from strictly vertical. We also estimate the errors due to the commonly used assumption of vertical propagation.

Keywords

VSP, sonic–log data, heterogeneous media, travel time, drift curve.

1. Introduction

In borehole geophysics, sonic–log measurements and vertical seismic profiling (VSP) are two different techniques providing the information about the seismic velocity in the structure surrounding a particular well. The two techniques use significantly different frequencies of seismic signals. The results of sonic–log measurements may thus differ from the results of VSP by the velocity dispersion. In order to compare the results of the two measurements, the sonic–log velocities are used to estimate the corresponding *sonic–log travel time* in the geological structure, which is then compared with the VSP travel time.

VSP provides travel times from the surface source to the geophones located in the well. These travel times are measured at frequencies from few tens to few hundreds Hz.

Sonic logging provides very detailed information about the slowness in the structure along the well. The slowness is averaged over the interval covered by receivers, which is typically of the order of 1 m. The slowness is typically measured at frequencies from 2 kHz to 20 kHz.

Comparison of VSP and sonic–log measurements have broadly been addressed in the literature since Gretener (1961). Many references can be found, e.g., in the paper by Stewart, Huddleston & Kan (1984). If both measurements are done for a particular well, so called drift curve may be constructed. The *drift curve* displays the difference of the measured VSP travel times from the sonic–log travel times calculated using the sonic–log slowness.

Gretener (1961) assumes a 1-D layered structure and a straight vertical well. Starting at the shallowest data point common to both sonic–log and VSP measurements, the sonic slowness is integrated over the depth, yielding the travel–time increment for strictly vertical propagation. For the VSP source situated at the well mouth, the drift curve represents the difference of the VSP travel–time increment from the integrated sonic slowness. For an offset between the VSP source and the well mouth, Gretener (1961) assumes a homogeneous background with straight rays and applies a simple cosine correction to the VSP travel times before constructing the drift curve. Since the velocity usually increases with depth, the cosine correction is usually inaccurate.

The difference of the measured VSP travel times from the sonic–log travel times calculated using the sonic–log velocities is typically few “miliseconds per 1000 ft” (Gretener, 1961). This would mean, for P–wave velocity equal to, e.g., 4267 m/s (i.e. 14000 ft/s), the relative travel–time difference of few times 1.4%. For example, if the well in a horizontally layered structure were deviated by 7° from the vertical, the VSP slowness vector at the well were also deviated by 7° from the vertical, and we neglected the deviations, the error of the above mentioned relative travel–time difference might reach 2.2%. We thus should not neglect the error caused by not taking into account the deviation of the well and of the VSP slowness vector from the vertical.

The aim of this paper is to show how to calculate the sonic–log travel time while taking into account the effects of the non–vertical propagation of seismic waves due to the source offset and due to the heterogeneous velocity in the structure, together with the effects of well trajectory differing from strictly vertical. Note that ray tracing in a smooth reference model is used for calculating reference travel times and reference slowness vectors.

2. Sonic–log travel time for a non–vertical well in a heterogeneous structure

We calculate the reference travel time and the reference slowness vector in a reference smooth velocity model. Using the reference slowness vector, we estimate the slowness vector in the locally 1–D strongly heterogeneous structure inferred from sonic logs, and integrate it along the well to obtain the sonic–log travel time from the VSP source.

2.1. Assumptions about the geological structure

To be able to deal with true well trajectories differing from strictly vertical direction, and to be able to consider the VSP source in some distance from the well mouth, we need to make some assumptions about the velocity distribution in the geological structure. We thus assume that a reference smooth velocity model is available, and that the strong variations of the slowness in the geological structure are locally one–dimensional in the direction given by unit vector n_i normal to the local layering. The strong 1-D slowness variations are thus locally functions of "level" $\xi = n_i x_i$. Note that in many cases the reference smooth model is one–dimensional with slowness varying only vertically, and the vector n_i is constant and vertical, but we consider here general smoothly varying $n_i = n_i(\mathbf{x})$ for the sake of generality.

2.2. Reference travel time and slowness vector in a smooth velocity model

We denote by $\tau^0(\mathbf{x})$ the reference travel time calculated from the VSP source in the reference smooth velocity model, and by $p_i^0(\mathbf{x})$ the corresponding slowness vector. They may be calculated, e.g., by ray tracing (Červený, Klimeš & Pšenčík, 1988) or by wavefront tracing (Vinje, Iversen & Gjøystdal, 1993).

2.2.1. Interpolation of the reference travel time and slowness vector

For the calculation of the sonic–log travel time, we need to know the reference slowness vector at points $\mathbf{x}$ along the part of the well where the slowness $u(\mathbf{x})$ has been determined by a sonic log. Moreover, if we know also the reference travel time along the part of the well, we will be able not only to construct the drift curve (21), but also to compare the sonic–log travel time corresponding to sonic–log slowness with the reference travel time corresponding to the smooth reference model.

It is a standard procedure in ray–theory based methods that during the ray tracing all quantities are calculated along the rays, and these quantities are then interpolated between the rays. Assume that we have calculated reference travel time $\tau^0(\mathbf{x})$ and the corresponding reference slowness vector $p_i^0(\mathbf{x})$ at the VSP receivers $\mathbf{x}^{\mathrm{R}}$ situated in the well or in its vicinity (to allow for position measurement uncertainties),

$$\tau^{\mathrm{R}} = \tau^0(\mathbf{x}^{\mathrm{R}}) \quad , \qquad p_i^{\mathrm{R}} = p_i^0(\mathbf{x}^{\mathrm{R}}) \quad . \tag{1}$$

We first interpolate the reference travel time and the corresponding slowness vector along straight lines between receivers to "level" $\xi = n_i x_i$. Along the straight lines, we shall apply the linear interpolation of coordinates x_i^{A} and of slowness–vector components p_i^{A}, and cubic interpolation of reference travel time τ^{A}. We then extrapolate reference travel time from points $\mathbf{x}^{\mathrm{A}}$ of a straight line to points $\mathbf{x}$ of the curved well by the linear Taylor expansion. The slowness vector is extrapolated as a constant vector.

Let x_i^{B}, τ^{B}, p_i^{B} and x_i^{C}, τ^{C}, p_i^{C} be two consecutive multiplets of values x_i^{R}, τ^{R}, p_i^{R}. We denote

$$w^{\mathrm{B}} = \frac{n_i(\mathbf{x})\,[x_i - x_i^{\mathrm{C}}]}{n_j(\mathbf{x})\,[x_j^{\mathrm{B}} - x_j^{\mathrm{C}}]} \tag{2}$$

and

$$w^{\mathrm{C}} = \frac{n_i(\mathbf{x})\,[x_i - x_i^{\mathrm{B}}]}{n_j(\mathbf{x})\,[x_j^{\mathrm{C}} - x_j^{\mathrm{B}}]} = 1 - w^{\mathrm{B}} \quad . \tag{3}$$

Then the linear interpolation of coordinates reads (Bulant & Klimeš, 1999, eq. 2)

$$x_i^{\mathrm{A}} = w^{\mathrm{B}} x_i^{\mathrm{B}} + w^{\mathrm{C}} x_i^{\mathrm{C}} \quad , \tag{4}$$

and the linear interpolation of the slowness vector reads

$$p_k^{\mathrm{A}} = w^{\mathrm{B}} p_k^{\mathrm{B}} + w^{\mathrm{C}} p_k^{\mathrm{C}} \quad . \tag{5}$$

The cubic interpolation of reference travel time reads (Bulant & Klimeš, 1999, eq. 25)

$$\tau^{\mathrm{A}} = a(w^{\mathrm{B}})\tau^{\mathrm{B}} + a(w^{\mathrm{C}})\tau^{\mathrm{C}} + (w^{\mathrm{B}})^2 w^{\mathrm{C}} p_i^{\mathrm{B}} [x_i^{\mathrm{C}} - x_i^{\mathrm{B}}] - w^{\mathrm{B}} (w^{\mathrm{C}})^2 p_i^{\mathrm{C}} [x_i^{\mathrm{C}} - x_i^{\mathrm{B}}] \quad , \tag{6}$$

where

$$a(w) = w^2(3 - 2w) \tag{7}$$

(Bulant & Klimeš, 1999, eq. 21).

Then

$$p_i^0(\mathbf{x}) = p_i^{\mathrm{A}} \tag{8}$$

and

$$\tau^0(\mathbf{x}) = \tau^{\mathrm{A}} + p_i^{\mathrm{A}} [x_i - x_i^{\mathrm{A}}] \quad . \tag{9}$$

2.3. Approximate slowness vector in the geological structure

In accordance with our assumption of strong slowness variations in the geological structure appearing only in the direction given by unit vector n_i, we assume the projection of the slowness–vector p_k in the geological structure onto the plane perpendicular to n_i to be approximately equal to the same projection of the reference slowness vector p_k^0,

$$[\delta_{ik} - n_i(\mathbf{x}) n_k(\mathbf{x})] p_k(\mathbf{x}) = [\delta_{ik} - n_i(\mathbf{x}) n_k(\mathbf{x})] p_k^0(\mathbf{x}) \quad . \tag{10}$$

The slowness vector in the geological structure may thus be expressed as the sum $p_i^{\perp}(\mathbf{x}) + p_i^{\parallel}(\mathbf{x})$ of its projection $p_i^{\perp}(\mathbf{x}) = [\delta_{ik} - n_i(\mathbf{x}) n_k(\mathbf{x})] p_k(\mathbf{x})$ onto the plane perpendicular to n_i, and of its projection $p_i^{\parallel}(\mathbf{x}) = n_i(\mathbf{x}) n_k(\mathbf{x}) p_k(\mathbf{x})$ onto the direction of n_i,

$$p_i(\mathbf{x}) = [\delta_{ik} - n_i(\mathbf{x}) n_k(\mathbf{x})] p_k(\mathbf{x}) + n_i(\mathbf{x}) n_k(\mathbf{x}) p_k(\mathbf{x}) \quad , \tag{11}$$

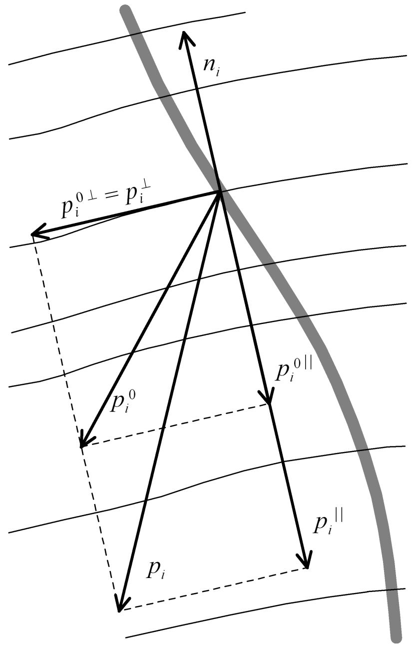

Figure 1. A simple sketch of a well (thick grey line) in a layered geological structure (thin lines). The arrows show the reference slowness vector $p_i^0(\mathbf{x})$, the slowness vector in the geological structure $p_i(\mathbf{x})$, and their projections onto the plane perpendicular to n_i, and onto the direction of n_i.

see Figure 1. The first projection is determined by equation (10), and we have to determine component

$$p^{\|} = n_k(\mathbf{x}) p_k(\mathbf{x}) \tag{12}$$

of the slowness vector $p_k(\mathbf{x})$ in the direction $n_i(\mathbf{x})$,

$$p_i(\mathbf{x}) = p_i^0(\mathbf{x}) - n_i(\mathbf{x}) n_k(\mathbf{x}) p_k^0(\mathbf{x}) + n_i(\mathbf{x}) p^{\|} \quad . \tag{13}$$

Component $p^{\|}$ of the slowness vector in the direction n_i can be calculated from slowness $u(\mathbf{x})$ determined by a sonic log using eikonal equation

$$p_i(\mathbf{x}) p_i(\mathbf{x}) = [u(\mathbf{x})]^2 \quad . \tag{14}$$

Inserting (13) into (14), we obtain

$$p^{\|} = \mathrm{sgn}[n_i(\mathbf{x}) p_i^0(\mathbf{x})] \sqrt{[u(\mathbf{x})]^2 - p_k^0(\mathbf{x}) p_k^0(\mathbf{x}) + [n_k(\mathbf{x}) p_k^0(\mathbf{x})]^2} \quad . \tag{15}$$

Then

$$p_i(\mathbf{x}) = p_i^0(\mathbf{x}) + n_i(\mathbf{x}) [p^{\|} - n_k p_k^0(\mathbf{x})] \quad . \tag{16}$$

2.4. Sonic–log travel time

At a selected point $\mathbf{x} = \mathbf{x}^{\mathrm{select}}$, usually at a shallowest point for which the sonic–log slowness is measured, we put

$$\tau(\mathbf{x}^{\mathrm{select}}) = \tau^0(\mathbf{x}^{\mathrm{select}}) \quad . \tag{17}$$

Then we use trapezoidal quadrature along the well,

$$\tau(\mathbf{x}^{\mathrm{new}}) = \tau(\mathbf{x}) + \frac{1}{2} [p_i(\mathbf{x}^{\mathrm{new}}) + p_i(\mathbf{x})] \, [x_i^{\mathrm{new}} - x_i] \quad , \tag{18}$$

and obtain sonic–log travel time $\tau(\mathbf{x})$ along the part of the well where the slowness $u(\mathbf{x})$ has been determined by a sonic log.

2.5. Travel–time differences and the drift curve

Along the part of the well where the sonic–log slowness $u(\mathbf{x})$ has been determined, we may plot the differences

$$\tau(\mathbf{x}) - \tau^0(\mathbf{x}) \tag{19}$$

of the sonic–log travel time from the reference travel time.

Along the part of the well where the VSP receivers are located, we may plot the differences

$$\tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{R}}) - \tau^0(\mathbf{x}^{\mathrm{R}}) \tag{20}$$

of the VSP travel time from the reference travel time.

Along the part of the well where both the sonic–log slowness $u(\mathbf{x})$ and VSP travel times have been measured, we may plot the differences

$$\tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{R}}) - \tau(\mathbf{x}^{\mathrm{R}}) \tag{21}$$

and refer to the plot as the *drift curve*.

Note that in other formulations the calculation of sonic–log travel time is usually initialized at the shallowest point $\mathbf{x}^{\mathrm{Rselect}}$ for which both the VSP and sonic–log measurements are available by putting $\tau(\mathbf{x}^{\mathrm{Rselect}}) = \tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{Rselect}})$. We believe that our initialization (17) of calculation of τ is more appropriate, because the resulting drift curve does not reflect the systematic shift caused by possible error of the VSP travel time at the initial point $\mathbf{x}^{\mathrm{Rselect}}$.

3. Error of sonic–log travel time due to assumption of "vertical propagation"

Under "vertical propagation" we understand the assumption of

$$p_i(\mathbf{x}) = n_i(\mathbf{x}) u(\mathbf{x}) \, \mathrm{sgn}[n_i(\mathbf{x}) p_i^0(\mathbf{x})] \quad , \tag{22}$$

i.e., the assumption that all the rays are always parallel with vector n_i. As already mentioned in Section 2.1, vector n_i will be in most cases vertical, and the assumption (22) will then be the assumption of vertical propagation.

Inserting assumption (22) into the trapezoidal quadrature (18) along the well, we obtain approximation of the sonic–log travel time for "vertical propagation" in the form of

$$\tilde{\tau}(\mathbf{x}^{\mathrm{new}}) = \tilde{\tau}(\mathbf{x}) + \frac{1}{2} [u(\mathbf{x}^{\mathrm{new}}) + u(\mathbf{x})] \; n_i(\mathbf{x}) [x_i^{\mathrm{new}} - x_i] \; \mathrm{sgn}[n_i(\mathbf{x}) p_i^0(\mathbf{x})] \quad . \tag{23}$$

We denote by $t_i(\mathbf{x})$ the tangent vector to the well, by

$$\Delta\tau(\mathbf{x}) = p_i(\mathbf{x}) \, t_i(\mathbf{x}) \tag{24}$$

the increment of the sonic–log travel time in the direction along the well, and by

$$\Delta\tilde{\tau}(\mathbf{x}) = n_i(\mathbf{x}) \, t_i(\mathbf{x}) \; u(\mathbf{x}) \; \mathrm{sgn}[n_i(\mathbf{x}) p_i^0(\mathbf{x})] \tag{25}$$

the increment of the approximate sonic–log travel time in the direction along the well under assumption of "vertical propagation". Then the local relative sonic–log travel–time error is

$$\varrho\tau = \frac{\Delta\tilde{\tau} - \Delta\tau}{\Delta\tau} \quad . \tag{26}$$

We may calculate and inspect this relative error along the part of the well where the slowness $u(\mathbf{x})$ has been determined by a sonic log.

Note that the calculation of sonic–log travel time described in Sections 2.1 to 2.5 excludes this error due to the assumption of "vertical propagation", which is the reason why we consider this method of calculation of sonic–log travel time to be superior to the method of Gretener (1961).

Let us now approximate the error (26). We decompose the slowness vector into the vector parallel with n_i and into the vector $p_i^{\perp}$ perpendicular to n_i,

$$p_i = n_i p^{\parallel} + p_i^{\perp} \quad . \tag{27}$$

Then

$$\Delta\tau = n_i t_i p^{\parallel} + p_i^{\perp} t_i \tag{28}$$

and

$$\Delta\tilde{\tau} = n_i t_i p^{\parallel} \sqrt{1 + p_i^{\perp} p_i^{\perp} (p^{\parallel})^{-2}} \quad . \tag{29}$$

We see that $\varrho\tau = 0$ if $p_i^{\perp} = 0$. Note that vector $p_i^{\perp} = p_i^{0\perp}$, unlike $p^{\parallel}$, varies smoothly. If we insert (28) and (29) into (26) and neglect $p_i^{\perp} t_i$ in the denominator, we may decompose relative error (26) into two terms,

$$\varrho\tau = \left[\sqrt{1 + \frac{p_i^{\perp}}{p^{\parallel}} \frac{p_i^{\perp}}{p^{\parallel}}} - 1 \right] - \frac{p_i^{\perp}}{p^{\parallel}} \frac{t_i}{n_k t_k} \quad . \tag{30}$$

For small $|p_i^{\perp}|/p^{\parallel}$, the first term may further be approximated. Finally, introducing the decomposition

$$t_i = n_i t^{\parallel} + t_i^{\perp} \quad , \tag{31}$$

we arrive at

$$\varrho\tau = \frac{1}{2} \frac{p_i^{\perp}}{p^{\parallel}} \frac{p_i^{\perp}}{p^{\parallel}} - \frac{p_i^{\perp}}{p^{\parallel}} \frac{t_i^{\perp}}{t^{\parallel}} \quad . \tag{32}$$

The first error term is independent of the well inclination, depends on the "horizontal" offset of the VSP source from the studied part of the well only. The second error term depends also on the well inclination and may be much larger than the first term for wells deviated from the "vertical". Equation (32) provides better understanding of the sonic–log travel–time error caused by neglecting the non–verticality of the well and by not taking into account the non–verticality of the VSP slowness vector in a heterogeneous structure. The equation allowed as, for example, to estimate the 2.2% error mentioned in the Introduction for the case of 7° non–vertical deviation of the well and slowness.

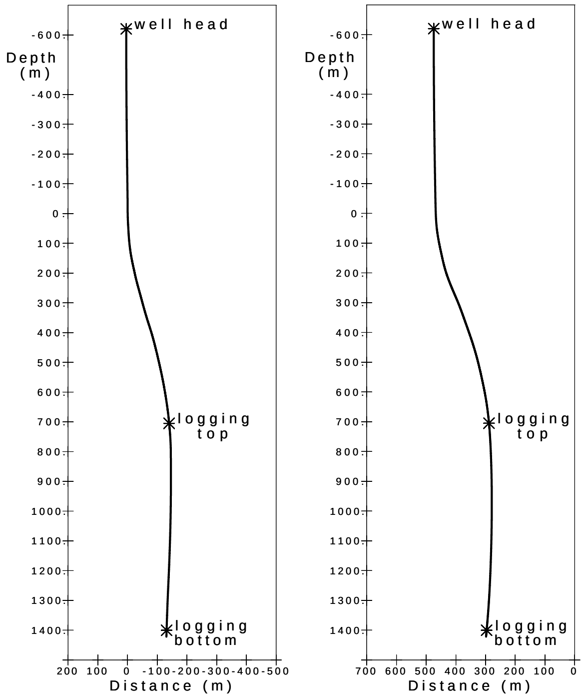

Figure 2. The trajectory of the well in which the measurements have been carried out. Left plot shows the projection onto x-z plane, the right plot the projection onto y-z plane.

4. Numerical example

The authors have been motivated to carry out this study when they faced the task to compare the results of sonic logging and of VSP in a well which considerably deviated from the vertical, see Figure 2. The depth scale of Figure 2 is in metres with zero depth at the sea level, the well head is in the depth of -620.27 m, the bottom of the well is in the depth of 1421.76 m. Sonic logging has been performed within the depth range from 712.24 m to 1419.39 m with the step of 0.1528 m (0.5 ft) along the well. There were 64 VSP receivers in the depth range from -498.33 m to 1370.37 m with the step of 30.24 m (100 ft) along the well.

The well is located in Texas, North America, where the geological structure is dominated by approximately horizontal sedimentary layers. Thus the reference smooth velocity model has been chosen to be 1-D with the vertical velocity variations only, see Figure 3. The model has been constructed from sonic–log velocities and from VSP travel times using the method of minimization of the Sobolev norm composed of the second velocity derivatives according to Bulant (2002).

The difference (19) of the sonic–log travel time from the reference travel time in the smooth velocity model has been calculated according to equation (18), and is plotted by the solid line in Figure 4. We have also calculated the approximate sonic–log travel time under assumption of vertical propagation using equation (23), and plotted its difference from the reference travel time by the dotted line in Figure 4.

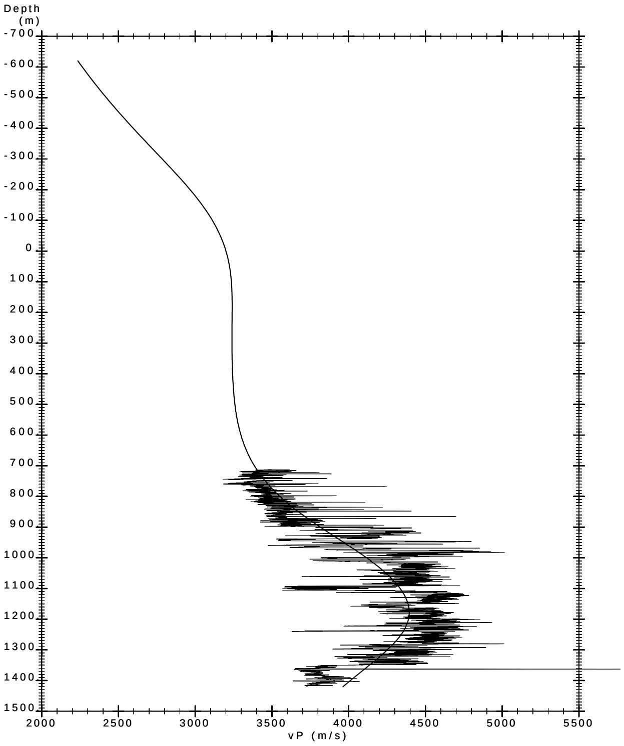

Figure 3. The P-wave velocity measured by sonic logging (thin line) in the well, and the P-wave velocity (thick line) in the smooth reference velocity model.

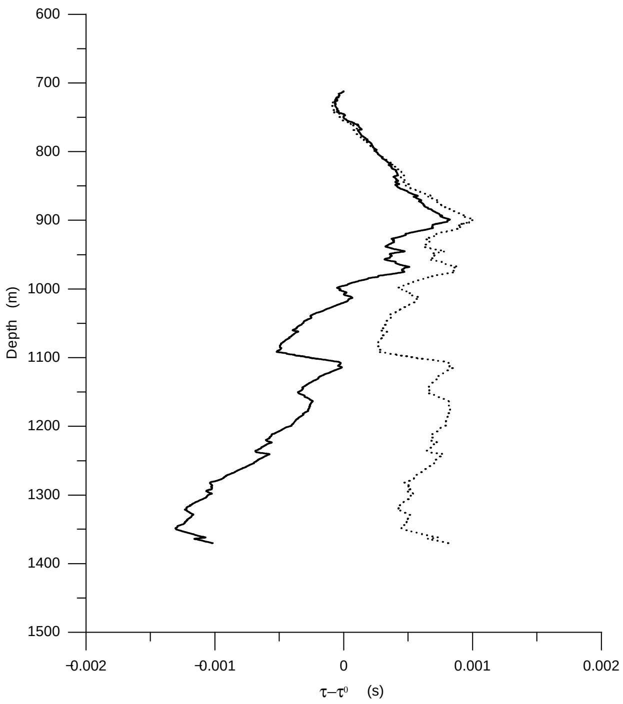

Figure 4. The difference of the sonic–log travel time from the reference travel time (solid line), and the same difference calculated under the assumption of vertical propagation (dotted line).

5. Conclusions

We have proposed the method for calculating sonic–log travel time in the geological structure from the values of velocity measured by sonic logging, while taking into account the effects of the non–vertical propagation of seismic waves due to the source offset and due to the heterogeneous velocity in the structure, together with the effects of well trajectory differing from strictly vertical. We also estimated the errors due to the commonly used assumption of vertical propagation.

In the numerical example, the difference of the sonic–log travel time from the reference travel time is approximately −1 milisecond over the 700 m of the depth interval of the measurements. The same travel–time difference under assumption of vertical propagation is almost +1 milisecond. The error caused by the assumption of vertical propagation for this particular well is thus almost twice as large as the value of the difference of the sonic–log travel time from the reference travel time.

Acknowledgements

The authors are grateful to Phil Christie for explaining the problem and for his assistance.

The research has been supported by the Grant Agency of the Czech Republic under Contract 205/07/0032, by the Ministry of Education of the Czech Republic within research project MSM0021620860, by the European Commission under Contract MTKI-CT-2004-517242 (project IMAGES), and by the members of the consortium "Seismic Waves in Complex 3–D Structures" (see "http://sw3d.mff.cuni.cz").

References

Bulant, P. (2002): Sobolev scalar products in the construction of velocity models: Application to model Hess and to SEG/EAGE Salt Model. *Pure and appl. Geophys.*, **159**, 1487–1506.

Bulant, P. & Klimeš, L. (1999): Interpolation of ray theory traveltimes within ray cells. *Geophys. J. int.*, **139**, 273–282.

Červený, V., Klimeš, L. & Pšenčík, I. (1988): Complete seismic–ray tracing in three–dimensional structures. In: Doornbos, D.J. (ed.): *Seismological Algorithms*, pp. 89–168, Academic Press, New York.

Gretener, P. E. F. (1961): An analysis of the observed time discrepancies between continuous and conventional well velocity surveys. *Geophysics*, **26**, 1-11.

Stewart, R. R., Huddleston, P. D. & Kan, T. K. (1984): Seismic versus sonic velocities. *Geophysics*, **49**, 1153-1168.

Vinje, V., Iversen, E. & Gjøystdal, H. (1993): Traveltime and amplitude estimation using wavefront construction. *Geophysics*, **58**, 1157–1166.