Smoothing SEG/EAGE Salt Model for ray tracing using Sobolev scalar products

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Summary

The efficiency of ray tracing in a seismic model can be evaluated in terms of the "average Lyapunov exponent" for the model. The "average Lyapunov exponent" may be approximated by the square root of the corresponding Sobolev norm of the functions describing the model. We may thus construct models optimally smooth for ray tracing by minimizing the Sobolev norm during the inversion of seismic data.

The SEG/EAGE 3-D Salt Model is given in the form of 3-D velocity cube and several triangulated interfaces. A smooth version of the model parameterized by tricubic B–splines is obtained by velocity inversion with minimization of the Sobolev norm. A version containing B-spline–parameterized interfaces is obtained by including the data for the interfaces into the inversion.

1 Introduction

The Sobolev scalar product (see, e.g., Tarantola 1987) of two functions is a linear combination of the L_2 Lebesgue scalar products of the zero, first, second or higher derivatives of the functions. The linear combination should preserve the properties required of scalar products (symmetry and positiveness).

The minimization of the Sobolev norm during linearized inversion of given data enables to control the model parameters unresolved by the data being fitted. Even if a reasonably looking model can be obtained without minimizing the Sobolev norm, it may be too rough for some computational methods. The errors of many computational methods may be approximated by a function dependent on the Sobolev norm of functions describing the model. We may thus construct models optimally smooth for given computational methods (Klimeš 2000a).

Probably the smoothest models are required by the ray methods. Unfortunately, we still have no quantitative criteria of applicability and accuracy of ray methods or their extensions. On the other hand, the numerical efficiency of ray tracing can be evaluated in terms of the "average Lyapunov exponent", introduced by Klimeš (1999, 2000c, 2002) for 2-D models without structural interfaces. The "average Lyapunov exponent" may be approximated by the square root of the corresponding Sobolev norm of the model. This enables us to determine the quantitative criteria on the models, so as to be suitable for ray tracing, in terms of Sobolev scalar products. The criteria then allow us to construct the optimum models of geological structures for ray tracing.

2 Constructing seismic models for ray tracing and Kirchhoff migrations

In complex models, the behaviour of rays becomes chaotic and the geometrical spreading, number of arrivals and density of caustic surfaces increase exponentially with travel time. The exponential increment may be quantitatively described in terms of the "average Lyapunov exponent" for the model. The "average Lyapunov exponent" may roughly be approximated by the square root of the Sobolev norm composed of the second partial derivatives of the functions describing the slowness or velocity field in the model (Klimeš 2000a). This enables us to determine the quantitative criteria on models, so as to be suitable for ray tracing, in terms of Sobolev scalar products. We choose the maximum "average Lyapunov exponent" for the model, we estimate the maximum Sobolev norm of the model, and we estimate the value of weighting factor S of the Sobolev norm in the objective function. We then perform the inversion, compute the values of the "average Lyapunov exponent" and the Sobolev norm in the inverted model, re-estimate the value of S, and continue iteratively with the inversion until we obtain the model of the required smoothness.

For example, to construct a model suitable for ray–based migrations, Klimeš (2000b) chose the value of the "average Lyapunov exponent" $\bar{\lambda}$ for the model as

$$\bar{\lambda} \le \frac{2}{T_{\max}} \quad , \tag{1}$$

where $T_{\max} \ge T_S + T_R$ is the maximum sum of travel times from the source and from the receiver. The product of the average numbers of arrivals from a source and a receiver then probably does not exceed 1.8, and the product of maximum numbers of arrivals from a source and a receiver then probably does not exceed 7.

If we choose coefficients $b_{ij|ab}$ of the Sobolev norm according to Klimeš (2000b), the Sobolev norm $\|u\|$ of the model satisfying equation (1) should obey equation (Klimeš 2000b)

$$\|u\| \le \sqrt{\frac{8}{3}} \left[\frac{4 + 2\,K_{\rm osc} \ln 2}{T_{\max}} \right]^2 \frac{1}{\langle v \rangle^3} \quad , \tag{2}$$

where $\langle v \rangle$ is the average velocity in the model, and K_{osc} is the average number of velocity oscillations along rays of length corresponding to $T_{\max}$.

3 Numerical example: smoothing SEG/EAGE 3-D Salt Model for Kirchhoff migrations

Routines for application of the above described minimization of the Sobolev norm during the seismic inversion have been coded in Fortran 77. All the computations are carried out on a PC Pentium with 64 Mbytes of memory. The software used for the computations is available at http://sw3d.mff.cuni.cz .

The memory requirements of the method are $M(D+M)$ storage locations, where M is the number of model parameters, and D is the number of data points. The requirements disallow to use all the available data and to reach the minimum error in inversion of 3-D SEG/EAGE Salt Model.

3.1 Data for the model

The SEG/EAGE 3-D Salt Model (Aminzadeh *et al.* 1997) is specified in the form suitable for the `GOCAD` model visualisation software. The dimensions of the model are 13500 times 13500 times 4200 metres. Several interfaces in the model are stored in the form of triangulated surfaces, and the 3-D velocity cube is given. We will take the data and build a 3-D model by inversion. In the first example, we shall construct a smooth model without interfaces, and in the second example a model containing the salt body and some other selected interfaces.

The P-wave velocity is discretized on a grid of $676 \times 676 \times 210$ points with a spacing of $20 \times 20 \times 20\,\mathrm{m}$. This is a very large amount of data, approximately 300 Mbytes. We immediately see that, due to the limited memory of the computer being used, a much sparser grid must be used for the data for the velocity inversion. We thus resample the velocities to several much sparser grids. On these grids we convert the velocity into the slowness. The sparse slowness grids are then used as the data for the inversion.

3.2 Choosing the maximum Lyapunov exponent and the maximum Sobolev norm

To estimate the maximum sum of travel times from a source and a receiver $T_{\max}$, we compute the average slowness of the model $\langle u \rangle = 0.428 \quad 10^{-3}\,\mathrm{s\,m^{-1}}$. If we consider the maximum sum of the ray paths to be about 14500 m, we arrive at the estimation of $T_{\max} = 6.21\,\mathrm{s}$. According to equation (1) we choose the maximum "average Lyapunov exponent" $\bar{\lambda}_{\max}$ of the model as $\bar{\lambda}_{\max} = 0.322\,\mathrm{s^{-1}}$. The maximum Sobolev norm $\|u\|_{\max}$ of the slowness field in the model chosen according to equation (2) is $\|u\|_{\max} = 0.152 \quad 10^{-9}\,\mathrm{s\,m^{-3}}$.

We have no quantitative criteria on the smoothness of the structural interfaces, and we thus first try to apply the minimum reasonable smoothing of the interfaces. Whether the interfaces are sufficiently smooth will be tested by ray tracing.

3.3 Smooth version of the Salt Model

We try to fit the data by a smooth model, composed of one block with three–dimensionally varying velocity. The data for the inversion are the slownesses on a coarser grid, as mentioned above. In the model, we are interpolating the slowness by tricubic B-splines. The inversion is linear, with objective function

$$y = \left(\frac{\widehat{|u - u_0|}}{\sigma_u} \right)^2 + (S_u \|u\|)^2 \quad , \tag{3}$$

where $\widehat{|u - u_0|}$ is the standard slowness deviation of the model from the gridded slownesses, $\|u\|$ is the 3-D Sobolev norm of the slowness field in the model, and $\sigma_u = 1\,\mathrm{s\,m^{-1}}$ is the given unit slowness deviation. We first perform the inversion with weighting factor $S_u = 0\,\mathrm{m^3\,s^{-1}}$ and search for the best combination of data grid and model spline grid, which would yield the lowest standard relative velocity deviation $|\widehat{\frac{v-v_0}{v_0}}|$ of the velocity in the inverted model from the velocity in the full data cube.

For the best model spline grid of $12 \times 12 \times 14$ points with a spacing of $1200 \times 1200 \times 300\,\mathrm{m}$ and the corresponding data grid of $18 \times 18 \times 17$ points with a spacing of $800 \times 800 \times 250\,\mathrm{m}$ we estimate the value of $S_u = \frac{\widehat{|u-u_0|}}{\sigma_u \|u\|_{\max}} = \frac{0.322}{0.152} \frac{10^{-4}}{10^{-9}}\,\mathrm{m^3\,s^{-1}} \approx 200000\,\mathrm{m^3\,s^{-1}}$. We then choose several values of S_u for subsequent iterations in order to find the best model which meets our requirements on the maximum "average Lyapunov exponent" $\bar{\lambda}_{\max}$.

Smoothing SEG/EAGE Salt Model

We consider the model obtained after the inversion with the value of $S_u = 50000\,\mathrm{m}^3\,\mathrm{s}^{-1}$ as the best smooth model, which meets our requirements. The "average Lyapunov exponent" of the inverted model is $\bar{\lambda} = 0.308\,\mathrm{s}^{-1}$, its standard relative velocity deviation is $|\widehat{\frac{v-v_0}{v_0}}| = 8.22\%$

3.4 Salt Model - version with interfaces

We try to fit the data by a model containing the most important interfaces, which are characterized by the highest steps in the velocity field. These interfaces are the ocean bottom, the interface limiting the salt body, the geopressure horizon, and the bottom flat interface below the salt. For the interfaces we use the triangulated surfaces in the GOCAD representation as the data for inversion. Concerning the velocity field, we again use the slownesses on coarser grids as the data, and we interpolate the slowness by tricubic B-splines in the model. The inversion is again linear with objective function

$$y = \left(\frac{\widehat{|x - x_0|}}{\sigma_x}\right)^2 + (S_x\|x\|)^2 + \left(\frac{\widehat{|u - u_0|}}{\sigma_u}\right)^2 + (S_u\|u\|)^2 \quad , \tag{4}$$

where $\widehat{|x - x_0|}$ is the standard deviation of the interfaces in the model from the triangulated GOCAD surfaces, $\|x\|$ is the 3-D Sobolev norm of the interfaces, and $\sigma_x = 1\,\mathrm{m}$ is the given unit interfaces deviation. The inversion of the interfaces and the inversion of the velocity field are mutually independent.

We choose the starting parametrization of the interfaces manually, and we first invert the interfaces with $S_x = 0\,\mathrm{m}$. We then increase the value of S_x until we reach the minimum reasonable smoothing of the interfaces, characterized by acceptable values of standard interface deviations. We also optimize the B-spline grids for the interfaces. The velocity inversion is analogous to the velocity inversion in the smooth model.

The best model we obtained has the standard relative deviation of velocity $|\widehat{\frac{v-v_0}{v_0}}| = 5.09\%$. The "average Lyapunov exponent" of the velocity field in the model is $0.181\,\mathrm{s}^{-1}$. Note that the exponent does not take into account the velocity steps at the interfaces.

3.5 Tests of ray tracing in the inverted models

We performed several tests of ray tracing in the models obtained after the inversions. As it is difficult to clearly display the results of 3-D ray tracing, we decided to shoot the rays only in the direction of the $x_1 - x_3$ plane (one–parametric shooting in a 3-D model). We then projected the computed rays onto the x_1–x_3 plane and onto the x_2–x_3 plane. The projected rays together with the velocity sections corresponding to the x_2 coordinates of the sources are displayed in Figures 1 to 4.

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Acknowledgments

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Smoothing SEG/EAGE Salt Model

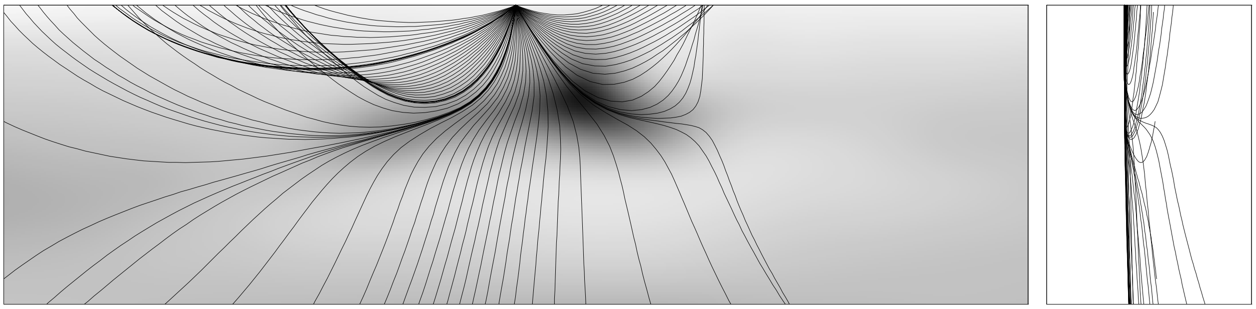

Figure 1: Rays computed by one–parametric shooting of a refracted wave in the x_1-x_3 plane in the smooth version of the Salt Model. The rays are projected onto the x_1-x_3 and onto the x_2-x_3 planes. The coordinates of the point source are [7000,5250,0] m. The x_1-x_3 plane is shaded according to the P-wave velocity in the x_1-x_3 section containing the point source. The grey scale ranges from white for 1500 $\mathrm{m\,s^{-1}}$ in the water to dark grey for 4450 $\mathrm{m\,s^{-1}}$ in the salt body. The standard relative deviation of the velocity in the model from the full data cube is 8.22%.

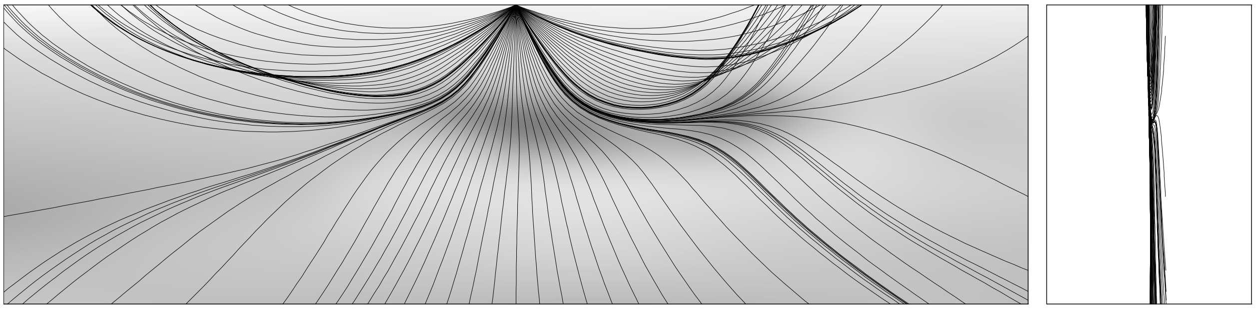

Figure 2: Rays computed by one–parametric shooting of a refracted wave in the x_1-x_3 plane in the smooth version of the Salt Model. The x_2 coordinate of the point source and of the velocity section is now 7000 m. The grey scale is the same as in Fig. 1.

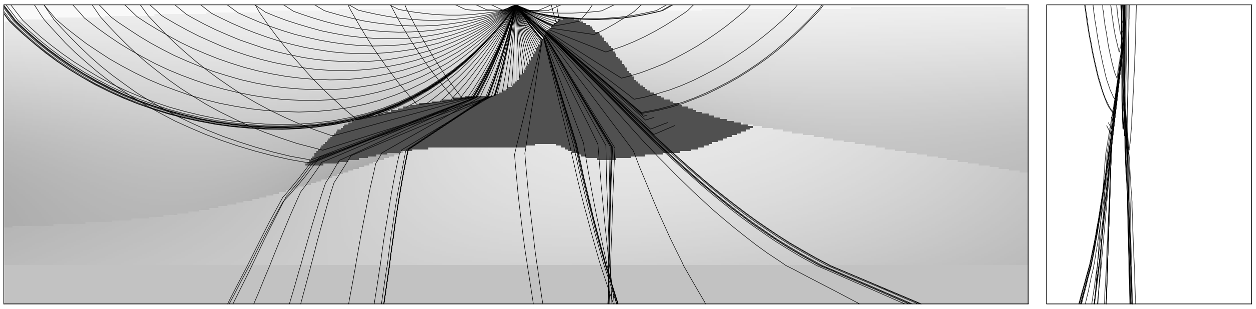

Figure 3: Rays computed by one–parametric shooting of a refracted wave in the x_1-x_3 plane in the Salt Model, the version with interfaces. The x_2 coordinate of the point source and of the velocity section is 5250 m. The grey scale is the same as in Fig. 1. The standard relative deviation of the velocity in the model from the full data cube is 5.09%.

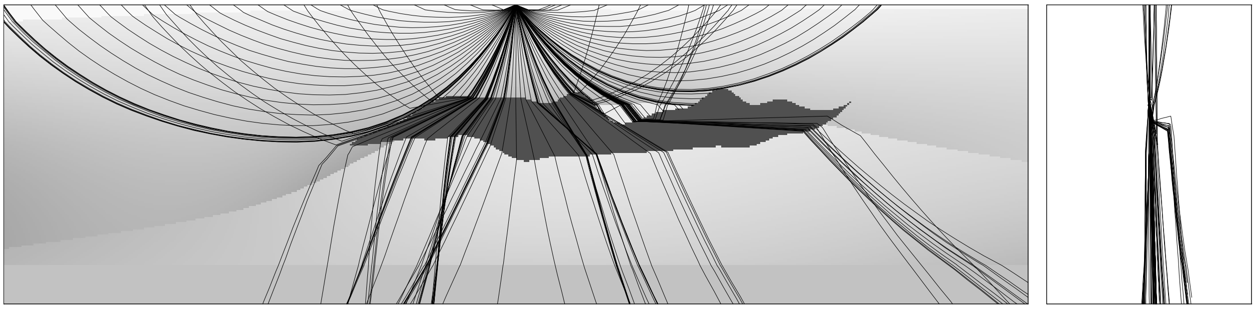

Figure 4: Rays computed by one–parametric shooting of a refracted wave in the x_1-x_3 plane in the Salt Model, the version with interfaces. The x_2 coordinate of the point source and of the velocity section is 7000 m. The grey scale is the same as in Fig. 1.