C-23 SMOOTHING 2-D MODEL PLUTO 1.5 FOR RAY TRACING USING SOBOLEV SCALAR PRODUCTS

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Summary

The efficiency of ray tracing in a seismic model can be evaluated in terms of the "average Lyapunov exponent" for the model. The exponent may be approximated by the square root of the corresponding Sobolev norm of the functions describing the model. We may thus construct models optimally smooth for ray tracing by minimizing the Sobolev norm during the inversion of seismic data.

The model Pluto 1.5 is given in the form of 2-D grid of values of the velocity. If the ray method is to be used, a continuous and smooth version of the model must be prepared. Such representation of the model parameterized by tricubic B–splines is obtained by velocity inversion with minimization of the Sobolev norm. Smoothness of the resulting model and its suitability for ray methods is controlled by weighting factor of the Sobolev norm.

1 Introduction

The Sobolev scalar product (see, e.g., Tarantola 1987) of two functions is a linear combination of the L_2 Lebesgue scalar products of the zero, first, second or higher derivatives of the functions. The linear combination should preserve the properties required of scalar products (symmetry and positiveness).

The minimization of the Sobolev norm during linearized inversion of given data enables to control the model parameters unresolved by the data being fitted. Even if a reasonably looking model can be obtained without minimizing the Sobolev norm, it may be too rough for some computational methods. The errors of many computational methods may be approximated by a function dependent on the Sobolev norm of functions describing the model. We may thus construct models optimally smooth for given computational methods (Klimeš 2000a).

Probably the smoothest models are required by the ray methods. Unfortunately, we still have no quantitative criteria of applicability and accuracy of ray methods or their extensions. On the other hand, the numerical efficiency of ray tracing can be evaluated in terms of the "average Lyapunov exponent", introduced by Klimeš (1999, 2000c) for 2-D models without structural interfaces. The "average Lyapunov exponent" may be approximated by the square root of the corresponding Sobolev norm of the model. This enables us to determine the quantitative criteria on the models, so as to be suitable for ray tracing, in terms of Sobolev scalar products. The criteria then allow us to construct the optimum models of geological structures for ray tracing.

2 Constructing seismic models for ray tracing and Kirchhoff migrations

In complex models, the behaviour of rays becomes chaotic and the geometrical spreading, number of arrivals and density of caustic surfaces increase exponentially with travel time. The exponential increment may be quantitatively described in terms of the "average Lyapunov exponent" for the model. The "average Lyapunov exponent" may roughly be approximated by the square root of the Sobolev norm composed of the second partial derivatives of the functions describing the slowness or velocity field in the model (Klimeš 2000a). This enables us to determine the quantitative criteria on models, so as to be suitable for ray tracing, in terms of Sobolev scalar products. We choose the maximum "average Lyapunov exponent" for the model, we estimate the maximum Sobolev norm of the model, and we estimate the value of weighting factor S of the Sobolev norm in the objective function. We then perform the inversion, compute the values of the "average Lyapunov exponent" and the Sobolev norm in the inverted model, re-estimate the value of S, and continue iteratively with the inversion until we obtain the model of the required smoothness.

For example, to construct a model suitable for ray–based migrations, Klimeš (2000b) chose the value of the "average Lyapunov exponent" $\bar{\lambda}$ for the model as

$$\bar{\lambda} \leq \frac{2}{T_{\max}} \quad , \tag{1}$$

where $T_{\max} \geq T_S + T_R$ is the maximum sum of travel times from the source and from the receiver. The product of the average numbers of arrivals from a source and a receiver then probably does not exceed

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1.8, and the product of maximum numbers of arrivals from a source and a receiver then probably does not exceed 7.

If we choose coefficients $b_{ij|ab}$ of the Sobolev norm according to Klimeš (2000b), the Sobolev norm $\|v\|$ of the velocity field in the model satisfying equation (1) should obey equation (Klimeš 2000b)

$$\|v\| \leq \sqrt{\frac{8}{3}} \left[\frac{4 + 2\, K_{\mathrm{osc}} \ln 2}{T_{\mathrm{max}}} \right]^2 \frac{1}{\langle v \rangle} \quad , \tag{2}$$

where $\langle v \rangle$ is the average velocity in the model, and K_{osc} is the average number of velocity oscillations along rays of length corresponding to T_{max}. This knowledge allows us to roughly estimate the value of weighting factor S_v of the Sobolev norm to be used for the velocity inversion, see Chapter 3.4.2.

3 Numerical example: smoothing model Pluto 1.5 for ray tracing

Routines for application of the above described minimization of the Sobolev norm during the seismic inversion have been coded in Fortran 77. All the computations are carried out on a PC Pentium with 64 Mbytes of memory. The software used for the computations is available at http://sw3d.mff.cuni.cz .

3.1 Data for the model

The grided velocities for 2-D model Pluto 1.5 were created by Smaart Joint Venture and are available at http://www.smaartjv.com . The P-wave velocities are discretized on a grid of 6960 × 1201 points with a spacing of 25 ft × 25 ft. The grid has thus horizontal dimension 173975 feet and vertical dimension 30000 feet. The grid contains three salt bodies of constant velocity. See Figure 1.

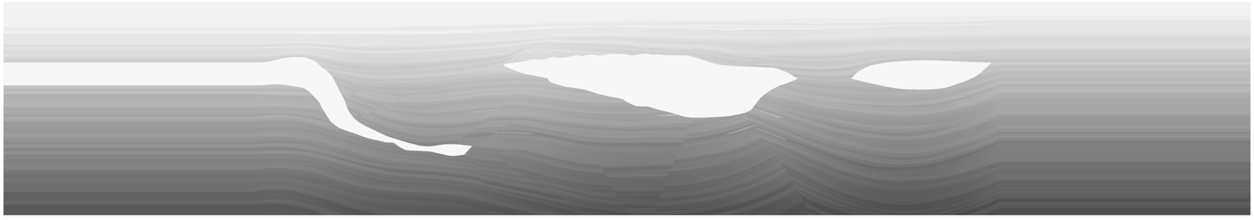

Figure 1: P-wave velocity in the original data grid of 6960×1201 points with a spacing of 25 ft×25 ft. The background velocity varies from 4920 ft s^{-1} at the top of the model to 13580 ft s^{-1} at the bottom. The velocity of the salt bodies is constant and equals 14800 ft s^{-1}. The horizontal dimension of the model is 173975 ft, its vertical dimension is 30000 ft.

We decided to create a model containing the three salt bodies separated by structural interfaces from the continuous background. We calculate the data for the interfaces from the grided velocities by searching for two neighbouring gridpoints, one of them displaying the velocity of the background, and the second one the velocity of the salt. The coordinates of a point of the interface are then taken between such two gridpoints.

We cannot use all the identified points at interfaces and all the gridpoints of the velocity grid as the data for inversion due to the limited memory of our computer. We thus have to resample the data to a sparser grid, and we have to use the data on the sparser grid as the data for the inversion. We know from previous experiments (see, e.g., Bulant 2001) that the model spline grid should be about 1.5 sparser than the corresponding data grid. We also know that the memory requirements of the current coding of the method are $M(D + M)$ storage locations, where M is the number of model parameters, and D is the number of data points. We thus take $D \approx 1.5\ M$ and directly estimate the optimum dimensions of the data and model spline grids which are the maximum grids which may be used on our computer. Figure 2 shows the values of P-wave velocity and the points at interfaces used as the data for the inversion described below in Chapters 3.4.1 and 3.4.2.

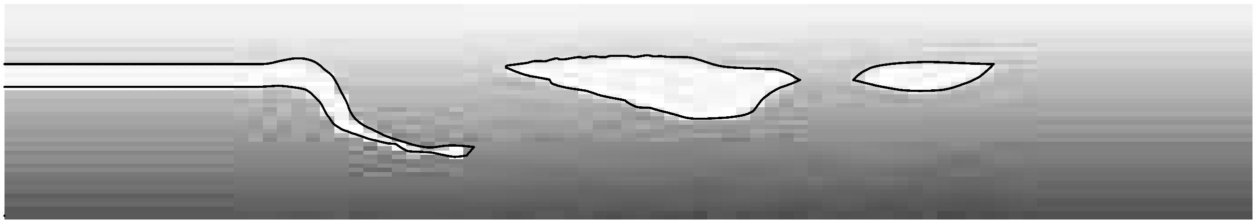

Figure 2: P-wave velocity in the data grid of 50 × 87 points with a spacing of 600 ft × 2000 ft, plotted together with the points at interfaces identified with the grid spacing of 75 ft. The gridded velocity and the points at interfaces are used as the data for the inversion described in Chapters 3.4.1 and 3.4.2. The background velocity varies from 4920 ft s^{-1} at the top of the model to 13580 ft s^{-1} at the bottom. The velocity of the salt bodies is constant and equals 14800 ft s^{-1}. The points at interfaces are plotted by black dots appearing as lines.

3.2 Model parametrization

We choose the parameterization of the model as follows: for the interfaces we take B–spline grid with spline points regularly spaced in the region of the salt bodies and with a single point at each side of the model. We use one surface to model the top of the left salt, second surface for the bottom of the left salt, third surface for the top of the middle and of the right salt, and fourth surface for the bottom of the middle and of the right salt.

For the background velocity, we take a a regular grid of 38×60 B–spline points with a spacing of $800\,\mathrm{ft} \times 3000\,\mathrm{ft}$.

3.3 Choosing the maximum Lyapunov exponent and the maximum Sobolev norm

The average velocity of the model is $\langle v \rangle = 9902.58\,\mathrm{ft\,s}^{-1}$, the average slowness of the model is $\langle u \rangle = 0.128\ 10^{-3}\,\mathrm{s\,ft}^{-1}$. If we consider the maximum sum of the ray paths to be about 100000 ft, we arrive at the estimation that the maximum sum of the travel times is limited by $T_{\max} = 12.8\,\mathrm{s}$. According to equation (1) we choose the maximum "average Lyapunov exponent" $\bar{\lambda}_{\max}$ of the model as $\bar{\lambda}_{\max} = \frac{2}{T_{\max}} = 0.156\,\mathrm{s}^{-1}$. The maximum Sobolev norm $\|v\|_{\max}$ of the velocity field in the model chosen according to equation (2) is $\|v\|_{\max} = 0.462\ 10^{-4}\,(\mathrm{ft\,s})^{-1}$.

Unfortunatelly, we still have no quantitative criteria on the smoothness of the structural interfaces. We thus do not prescribe the maximum Sobolev norm of the interfaces, and we try to apply the minimum reasonable smoothing of the interfaces. Whether the interfaces are sufficiently smooth will be tested by ray tracing.

3.4 Inversion and smoothing

The objective function is taken in the form

$$y = \left(\frac{|x - x_0|}{\sigma_x}\right)^2 + (S_x\|x\|)^2 + \left(\frac{|v - v_0|}{\sigma_v}\right)^2 + (S_v\|v\|)^2 \quad , \tag{3}$$

where $|x - x_0|$ is the standard deviation of the position of the interfaces in the model from the data, $\|x\|$ is the 2-D Sobolev norm of the curvature of the interfaces in the model, $|v - v_0|$ is the standard velocity deviation of the model from the data, and $\|v\|$ is the 2-D Sobolev norm of the velocity field in the model being constructed. S_x and S_v are numerical parameters which control the smoothing during the inversion. We use unit given velocity deviation $\sigma_v = 1\,\mathrm{ft\,s}^{-1}$ and unit given interface deviation $\sigma_x = 1\,\mathrm{ft}$ in the selected objective function.

The inversion of the interfaces and the inversion of the velocity field are mutually independent in this example. We can thus perform the inversion of the interfaces in the first step, and the inversion of the velocity in the second step.

3.4.1 Inversion of the interfaces

In this example, we try to apply the minimum reasonable smoothing of the interfaces. We thus start the inversion without smoothing of the interfaces, with $S_x = 0\,\mathrm{ft}$ (the interfaces are smoothed just by the projection onto the B–splines). We then increase the value of S_x several times, each time we perform the same velocity inversion, and finally we choose the value of S_x which provides the model with minimum standard velocity deviation. The smoothness of the interfaces will be tested by ray tracing.

3.4.2 Inversion of the velocity

We decided to use the values of velocity resampled to the grid of 50×87 points with a spacing of $600\,\mathrm{ft} \times 2000\,\mathrm{ft}$ as the data for the velocity inversion. We use the model B-spline parametrization of 38×60 spline points with a spacing of $800\,\mathrm{ft} \times 3000\,\mathrm{ft}$. This configuration of the data and model spline grids requires 15116400 storage locations in computer memory, which is almost the maximum amount available on our PC with 64 Mbytes of memory.

The inversion without minimization of the Sobolev norm is ill–conditioned.

The inversion with very small weighting factor S_v of the Sobolev norm yields the velocity model which differs from the given velocities by 3.261 %. The "average Lyapunov exponent" of the model is $0.497\,\mathrm{s}^{-1}$, and the behavior of the rays in the model is too chaotic, see Figure 3. We can now very roughly estimate the weighting factor S_v

$$S_v = \frac{|v - v_0|}{\sigma_v \|v\|_{\max}} = 5693809.5\,\mathrm{ft\,s} \quad , \tag{4}$$

similarly as in the paper by Klimeš (2000b, eq. 20).

When we perform the inversion with the weighting factor $S_v = 1000000\,\mathrm{ft\,s}$, we obtain very smooth model with the standard relative velocity deviation 2.706 % and the "average Lyapunov exponent" $0.075\,\mathrm{s}^{-1}$. Figure 5 shows, how the smoothing of the velocity field by including the Sobolev norm in the velocity inversion regularizes the chaotic behaviour of rays. The results of the inversion for different values of S_v are shown in Table 1. For the model with the lowest relative velocity deviation see Figure 4.

Note that in this example the velocity deviation is probably caused rather by insufficient B-spline sampling then by smoothing.

$S_v/(\mathrm{ft\,s})$	$Reldev/(\%)$	$Absdev/(\mathrm{ft\,s^{-1}})$	$\bar{\lambda}/(\mathrm{s^{-1}})$
1000	3.261	263.054	0.497
125000	2.611	205.600	0.258
200000	2.627	206.767	0.199
500000	2.672	212.113	0.116
1000000	2.706	214.784	0.075

Table 1: Results of the inversion of the velocity field in model Pluto 1.5 in dependence on weighting factor S_v of the Sobolev norm used. *Reldev* stands for the relative deviation of the velocity, *Absdev* stands for the deviation of the velocity, and $\bar{\lambda}$ is the "average Lyapunov exponent". The velocity field is parametrized by 38×60 spline points with a spacing of $800\,\mathrm{ft} \times 3000\,\mathrm{ft}$. The dimensions of the data grid are 50×87 points with a spacing of $600\,\mathrm{ft} \times 2000\,\mathrm{ft}$.

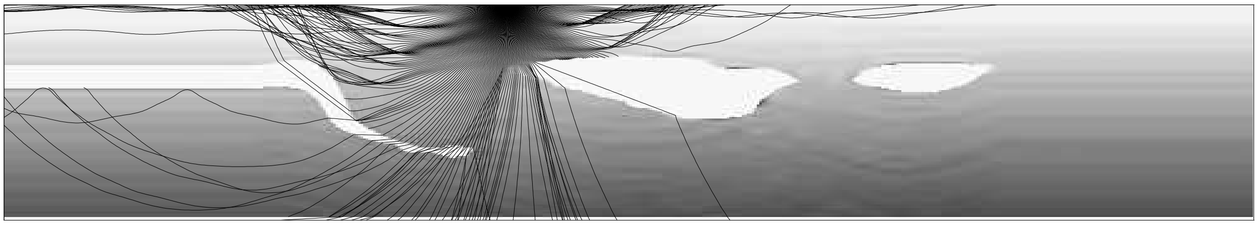

Figure 3: Rays computed by one–parametric shooting in the model Pluto 1.5 obtained after the inversion with the weighting factor $S_v = 1000\,\mathrm{ft\,s}$. The colour scale is the same as in Figure 1. The standard relative deviation of the velocity in the model from the full data grid is 3.261%.

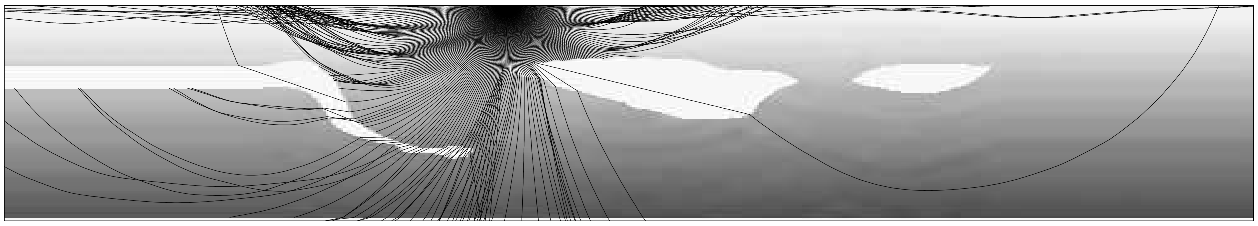

Figure 4: Rays computed by one–parametric shooting in the model Pluto 1.5 obtained after the inversion with $S_v = 125000\,\mathrm{ft\,s}$. The standard relative deviation of the velocity in the model from the full data grid is 2.611%.

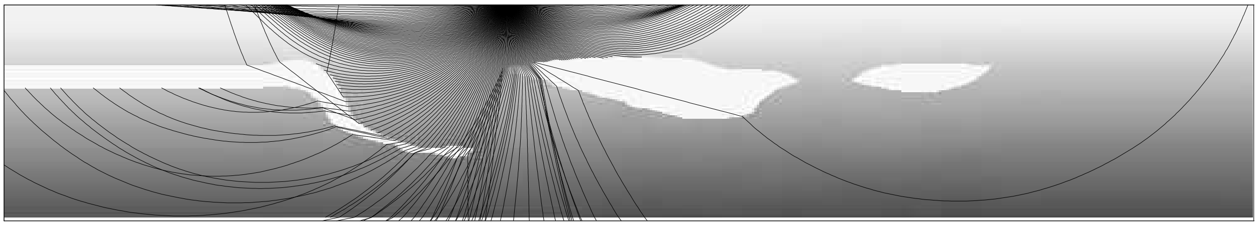

Figure 5: Rays computed by one–parametric shooting in the model Pluto 1.5 obtained after the inversion with $S_v = 1000000\,\mathrm{ft\,s}$. The standard relative deviation of the velocity in the model from the full data grid is 2.706%.

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