Smoothing the Marmousi model

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Summary

The only way to make an excessively complex velocity model suitable for application of ray-based methods, such as the Gaussian beam or Gaussian packet methods, is to smooth it.

We have smoothed the Marmousi model (Versteeg & Grau 1991) by minimizing the second spatial derivatives of the slowness. This was done by minimizing the relevant Sobolev norm of slowness.

We show that minimizing the relevant Sobolev norm of slowness is a suitable technique for preparing the optimum models for asymptotic ray theory methods. However, the price we pay for a model suitable for ray tracing is an increase of the difference between the smoothed and original model. Similarly, the estimated error in the travel time also increases due to the difference between the models. In smoothing the Marmousi model, we have found the estimated error of travel times at the verge of acceptability.

1 Introduction

The computation of rays is extremely sensitive to the smoothness of the model. In rough models, the behaviour of rays becomes chaotic and geometrical spreading and the number of arrivals increase with travel time rapidly (Abdullaev 1993, Tappert & Tang 1996, Keers et al. 1997). Moreover, a large number of two-point rays to each receiver makes calculation of two-point travel times slow and expensive. Often, two-point rays cannot be found within the numerical accuracy.

We need a reasonably smooth velocity model for a depth migration technique based on Gaussian packets. In Gaussian packet method, we do not need to find two-point rays, but a sufficiently dense set of rays must be calculated. Thus, the desired model should be suitable for ray tracing.

A suitable way to smooth a complex velocity model, is to minimize the appropriate Sobolev norm of the velocity or slowness. We show that minimizing the Sobolev norm may be used for efficiently controlling the behaviour of rays in complex structures.

2 Basic ideas about the desired model

The desired smoothed model has to fulfil two main and, unfortunately, contradictive requirements:
(a) to be in “good agreement” with the original Marmousi model, and
(b) to be “sufficiently smooth” for ray tracing and Gaussian packet computations.

Under the term “good agreement”, we understand a small difference between the smoothed and original model, expressed in terms of the standard L2 Lebesgue norm.

The meaning of the term “sufficiently smooth” is more complicated. In a complex model, the geometrical spreading and number of arrivals exponentially increase with increasing travel time. The exponential increment is controlled by the Lyapunov exponent (Lyapunov 1949). Since the Lyapunov exponent depends on the second spatial derivatives of the velocity or slowness (Klimeš 2000), the second derivatives should be minimized.

By minimizing the square of the Sobolev norm of slowness we may minimize the corresponding partial derivatives. The Sobolev scalar product is a linear combination of the L2 Lebesgue scalar products of the zero, first, second or higher partial derivatives (Tarantola 1987).

The vague terms “good agreement” and “sufficiently smooth” cannot be easily quantified before a detailed study of the behaviour of rays in smoothed models is made.

3 Algorithm

The original Marmousi velocity model consists of discrete values of velocity uniformly spaced on a dense grid. In obtaining a smoothed model, we
(a) choose a coarser data grid to reduce the amount of data to fit,
(b) arithmetically average the densely sampled slowness of the Marmousi model over cells centred at the grid points of the coarser data grid,
(c) choose a coarse B-spline model grid (we need to interpolate the discrete values of slowness on a coarse model grid for ray tracing; we have chosen bicubic B-splines as the interpolating functions, benefiting from the continuity of the second derivatives) and
(d) fit the averaged slowness values by the smoothed model (inversion).

In the inversion, the objective function being minimized is composed of two terms. The first term is a difference between the smoothed and original model, expressed in terms of the standard L2 Lebesgue norm. The second term is the square of the Sobolev norm of slowness, multiplied by a multiplication factor. After each linear minimization, we iteratively adjust the multiplication factor to find the model of desired smoothness, or to find several models of various degrees of smoothness.

4 Numerical results

We calculate several smoothed models, the corresponding values of the relative root-mean-square (RMS) difference of slowness between the smoothed and original model, the angular dependence of the Lyapunov exponents, the values of the "average Lyapunov exponents", rays and numbers of arrivals.

Figure 1 shows two smoothed models with various values of the Sobolev norm of slowness. Figure 2 displays rays computed in these smoothed models. We can see the dependence of the behaviour of rays on the Sobolev norm of slowness. We move the source along the whole profile and test ray tracing. The behaviour of rays is always of the same kind as in these illustrative figures.

In Figure 2a, we can see rays trapped in the low velocity channels. This model (with the Sobolev norm of slowness of $0.37 \cdot 10^{-8}\mathrm{sm}^{-3}$) is probably not suitable for ray-based methods. Let us mention maximum number of arrivals of 19, see Table 1. In Figure 2b (model with the Sobolev norm of slowness of $0.39 \cdot 10^{-9}\mathrm{sm}^{-3}$), the behaviour of rays is much more regular. Maximum number of arrivals is only 5, see Table 1. This model is probably suitable for ray-based methods.

Sobolev norm	**av. Lyap. exp.**	**max. no. arrivals**	**rel. RMS diff.**
$0.37 \cdot 10^{-8}\mathrm{sm}^{-3}$	$1.31\mathrm{s}^{-1}$	19	0.111
$0.39 \cdot 10^{-9}\mathrm{sm}^{-3}$	$0.56\mathrm{s}^{-1}$	5	0.143

Table 1. The Sobolev norm of slowness, the value of "average Lyapunov exponent", maximum number of arrivals and the relative RMS difference between the smoothed and the Marmousi model.

The relative RMS difference of slowness between two smoothed models and the original Marmousi model are shown in Table 1. We can see that the price for a model suitable for ray tracing is a massive increment of the relative RMS difference of slowness between the smoothed and original model. We can also see that this difference is almost the same for both presented smoothed models. This is caused by the complexity of the original Marmousi model.

5 Conclusions

The minimization of the relevant Sobolev norm of slowness is a powerful tool for preparing the optimum models for the asymptotic ray theory methods. As we have demonstrated in numerical examples, it can be used for smoothing

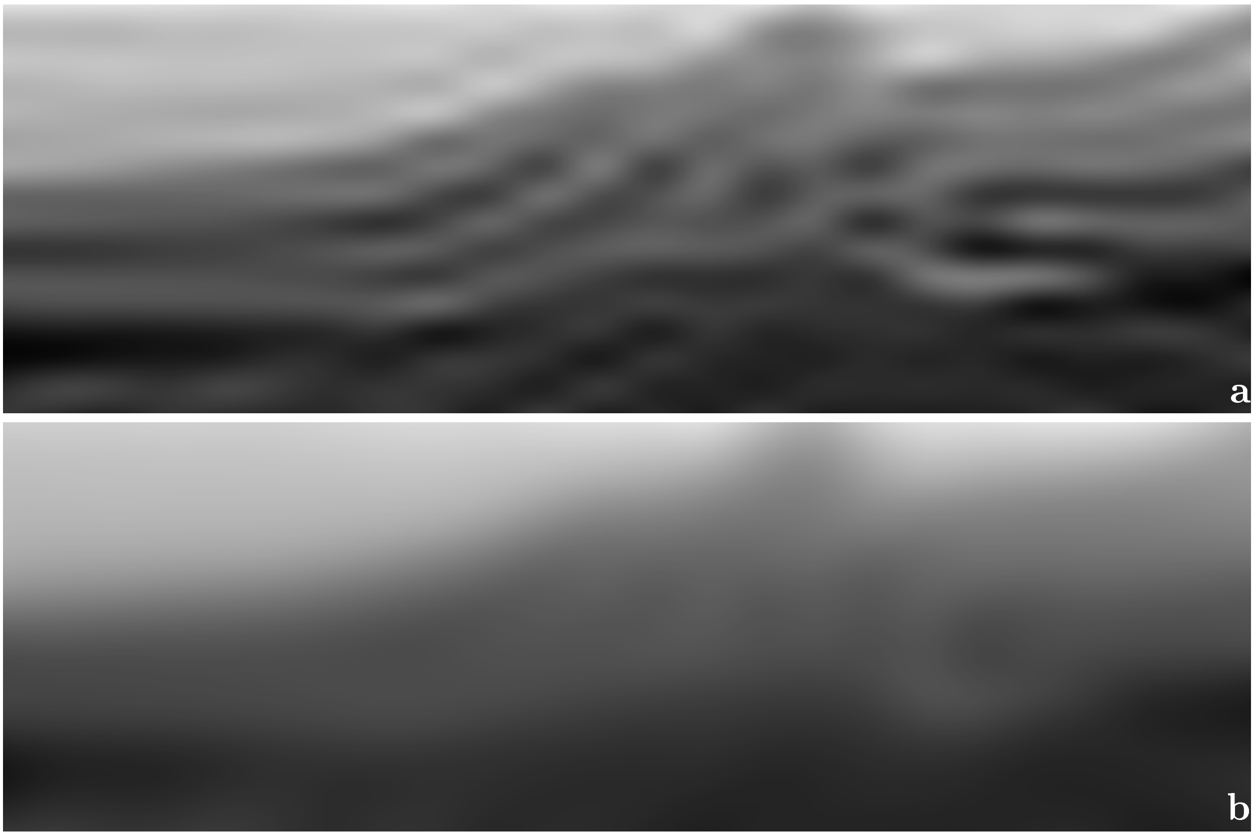

Figure 1. The smoothed models with the Sobolev norm of slowness of (a) $0.37 \cdot 10^{-8}\mathrm{sm}^{-3}$ and (b) $0.39 \cdot 10^{-9}\mathrm{sm}^{-3}$.

very complex models. However, the difference of slowness between the smoothed and the original model then increases rapidly. Also, the error of the travel time then increases.

We must keep in mind that there exists a natural relation between the complexity of the original model and the resulting difference between the sufficiently smoothed model and the original model. The more complex the original model, the more change it requires. Thus, the decision is up to the user, whether or not the model is too complex for smoothing. The required maximum error of travel time is then a key argument.

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References

Abdullaev, S.S. (1993): Chaos and Dynamics of Rays in Waveguide Media. Gordon and Breach, Amsterdam.

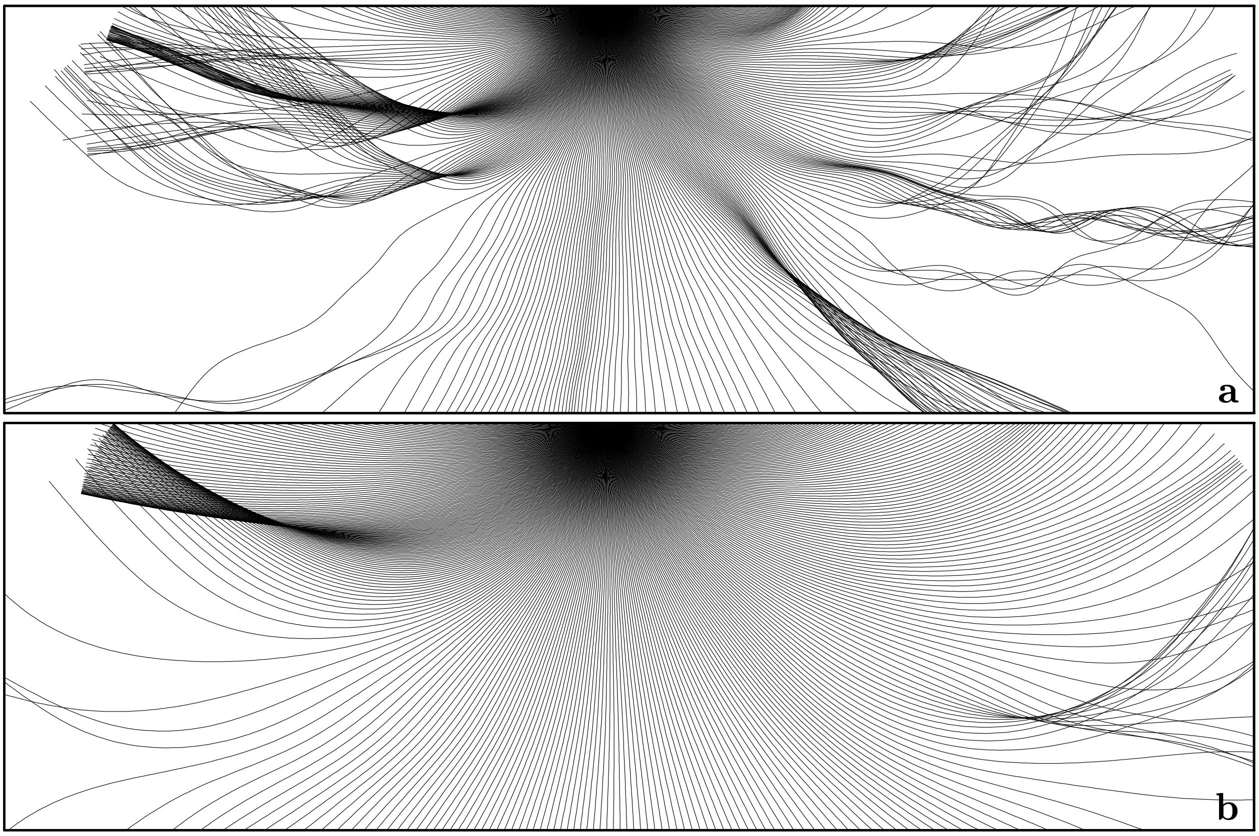

Figure 2. Rays in models with the Sobolev norm of slowness of (a) $0.37 \cdot 10^{-8}\mathrm{sm}^{-3}$ and (b) $0.39 \cdot 10^{-9}\mathrm{sm}^{-3}$. The maximum travel time is of 2.3 seconds.

Keers, H., Dahlen, F.A. & Nolet,G. (1997): Chaotic ray behaviour in regional seismology. Geophys. J. int., **131**, pp. 361–380.

Klimeš, L. (2000): Lyapunov exponents for 2-D ray tracing without interfaces. Expanded Abstracts of 70th Annual Meeting (Calgary), Soc. Explor. Geophysicists, Tulsa, pp. 2293–2296.

Lyapunov, A.M. (1949): Problème Général de la Stabilité du Mouvement. Annals of Mathematical Studies, Vol. 17, Princeton Univ. Press.

Oseledec, V.I. (1968): A multiplicative ergodic theorem: Lyapunov characteristic numbers for dynamical systems. Trans. Moscow Math. Soc., **19**, 179–210 in Russian, 197–231 in English translation.

Tappert, F.D. & Tang, X. (1996): Ray chaos and eigenrays. J. Acoust. Soc. Am., **99**, 185–195.

Tarantola, A. (1987): Inverse Problem Theory. Elsevier, Amsterdam.

Versteeg, R.J. & Grau, G. (eds.) (1991): The Marmousi experience. Proc. EAGE workshop on Practical Aspects of Seismic Data Inversion (Copenhagen, 1990), Eur. Assoc. Explor. Geophysicists, Zeist.