Field equivalence principle for electromagnetic waves in a heterogeneous generally bianisotropic medium

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Summary

We derive the field equivalence principle for electromagnetic waves propagating in a heterogeneous generally bianisotropic medium. For the given medium, we define the complementary medium corresponding to the transposed constitutive matrix. We define the frequency–domain complementary Green function as the frequency–domain Green function in the complementary medium but with the opposite sign at the time derivative. We then derive the provisional field equivalence principle as the relation between the frequency–domain wave field in the given medium and the frequency–domain complementary Green function. This provisional field equivalence principle yields the reciprocity relation between the frequency–domain Green function and the frequency–domain complementary Green function. The final version of the field equivalence principle is then obtained by inserting the reciprocity relation into the provisional field equivalence principle.

Keywords

Electromagnetic waves, heterogeneous media, bianisotropic media, constitutive tensor, magnetic vector potential, electric potential, Green function, field equivalence principle, reciprocity relation.

1. Introduction

In this paper, we derive the field equivalence principle for electromagnetic waves propagating in a heterogeneous linear dielectric medium which is generally bianisotropic.

We consider the linear constitutive relations for bianisotropic media in the Boys–Post representation without spatial dispersion (Lakhtakia, 2000; Post, 2003; Weiglhofer, 2003; Strunc, 2007). The Boys–Post representation $\mathbf{D}=\mathbf{D}(\mathbf{E},\mathbf{B})$, $\mathbf{H}=\mathbf{H}(\mathbf{E},\mathbf{B})$ of the constitutive relations is more natural than the Tellegen representation $\mathbf{D}=\mathbf{D}(\mathbf{E},\mathbf{H})$, $\mathbf{B}=\mathbf{B}(\mathbf{E},\mathbf{H})$, and is best suited for the formulation in terms of the magnetic vector potential $\mathbf{A}$. For the sake of simplicity, we assume that the media are static (do not change with time). In this case we can work in the frequency domain.

We consider the Maxwell equations with linear constitutive relations in the Boys–Post representation for 4–vector potential (A_1, A_2, A_3, A_4), where $\mathbf{A} = (A_1, A_2, A_3)$ is the magnetic vector potential and $A_4 = -\varphi$ is minus the electric potential, and define the corresponding 4×4 tensorial Green function.

For the given medium, we define the complementary medium corresponding to the transposed constitutive matrix. We define the frequency–domain complementary Green function as the frequency–domain Green function in the complementary medium but with the opposite sign at the time derivative. We then derive the provisional field equivalence principle as the relation between the frequency–domain wave field in the given

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medium and the frequency–domain complementary Green function. This provisional field equivalence principle yields the reciprocity relation between the frequency–domain Green function and the frequency–domain complementary Green function. The final version of the field equivalence principle is then obtained by inserting the reciprocity relation into the provisional field equivalence principle.

The lower–case Roman indices take values 1, 2 and 3. The lower–case Greek indices take values 1, 2, 3 and 4. The Einstein summation over repetitive indices is used throughout the paper.

2. Frequency–domain Maxwell equations with linear constitutive relations

In order to avoid coefficient $(2\pi)^{-\frac{1}{2}}$ at the electric current density and electric charge density in the definition of the frequency–domain Green function, we consider here Fourier transform (Červený, 2001, eq. A.1.2)

$$f(\omega) = \int_{-\infty}^{+\infty} \mathrm{d}t \; f(t) \; \exp(\mathrm{i}\omega t) \tag{1}$$

from time t to circular frequency ω without coefficient $(2\pi)^{-\frac{1}{2}}$.

We denote the 3–D spatial Levi–Civita symbol by ϵ^{ijk}. Frequency–domain Maxwell equations

$$\epsilon^{ijk} E_{k,j} - \mathrm{i}\omega \, B^i = 0 \tag{2}$$

and

$$B^k_{,k} = 0 \tag{3}$$

for *electric field strength* $E_j = E_j(x^m, \omega)$ and *magnetic induction* $B^j = B^j(x^m, \omega)$ are satisfied if we put

$$E_k = A_{4,k} + \mathrm{i}\omega \, A_k \tag{4}$$

and

$$B^k = \epsilon^{klm} A_{m,l} \quad , \tag{5}$$

where $A_i = A_i(x^m, \omega)$ is the *magnetic vector potential* and $A_4 = A_4(x^m, \omega)$ represents *minus electric potential* $\varphi = \varphi(x^m, \omega)$,

$$A_4 = -\varphi \quad . \tag{6}$$

The Frequency–domain Maxwell equations for *electric displacement* $D^j = D^j(x^m, \omega)$ and *magnetic field strength* $H_j = H_j(x^m, \omega)$ read

$$\epsilon^{ijk} H_{k,j} + \mathrm{i}\omega \, D^i = J^i \tag{7}$$

and

$$D^k_{,k} = J^4 \quad , \tag{8}$$

where $J^4 = J^4(x^m, \omega)$ represents *electric charge density* $\rho = \rho(x^m, \omega)$,

$$J^4 = \rho \quad , \tag{9}$$

and $J^i = J^i(x^m, \omega)$ is the *electric current density.*

We thus need the *constitutive relations* which express the mutual dependence between the above mentioned quantities E^k, B^k, D^j, H_j, J^j and J^4.

In this paper, we consider *dielectric media* in which the electric current density and electric charge density vanish outside the source region,

$$J^{\gamma} = 0 \quad , \tag{10}$$

and 4–vector J^γ represents just the source term.

We assume the constitutive relations in the Boys–Post representation which express the dependence of the electric displacement D^j and magnetic field strength H_j on electric field strength E_j and magnetic induction B^j. In this paper, we consider just the *linear* constitutive relations in the Boys–Post representation.

The linear point constitutive relations without spatial dispersion but with possible time dispersion can be expressed in the frequency domain as (Weiglhofer, 2000, eq. 1.12; 2003, eq. 57)

$$D^i = \epsilon^{ij} E_j + \alpha^i{}_j B^j \quad , \tag{11}$$

and (Weiglhofer, 2000, eq. 1.13, 2003, eq. 58)

$$H_i = \beta_i{}^j E_j + \mu^{-1}_{ij} B^j \quad . \tag{12}$$

Electric field strength $E_j = E_j(x^m, \omega)$, magnetic induction $B^j = B^j(x^m, \omega)$, electric displacement $D^j = D^j(x^m, \omega)$, magnetic field strength $H_j = H_j(x^m, \omega)$, *permittivity tensor* $\epsilon^{ij} = \epsilon^{ij}(x^m, \omega)$, *inverse permeability tensor* $\mu^{-1}_{ij} = \mu^{-1}_{ij}(x^m, \omega)$, and *magneto-electric tensors* $\alpha^i{}_j = \alpha^i{}_j(x^m, \omega)$ and $\beta_i{}^j = \beta_i{}^j(x^m, \omega)$ depend on spatial coordinates x^m and circular frequency ω.

We define $4\times4\times4\times4$ *constitutive tensor* $\chi^{\alpha\beta\gamma\delta}$ (Post, 1962, eq. 6.12; 2003, eq. 27; Hehl & Obukhov, 2003, eq. D.1.9) by relations

$$\chi^{4i4j} = -\chi^{i44j} = -\chi^{4ij4} = \chi^{i4j4} = -\epsilon^{ij} \quad , \tag{13}$$

$$\chi^{ij4k} = -\chi^{ijk4} = \epsilon^{ijr} \beta_r{}^k \quad , \tag{14}$$

$$\chi^{4ikl} = -\chi^{i4kl} = -\alpha^i{}_s \epsilon^{skl} \tag{15}$$

and

$$\chi^{ijkl} = \epsilon^{ijr} \mu^{-1}_{rs} \epsilon^{skl} \quad . \tag{16}$$

The $4\times4\times4\times4$ frequency–domain constitutive tensor $\chi^{\alpha\beta\gamma\delta} = \chi^{\alpha\beta\gamma\delta}(x^m, \omega)$ is skew with respect to the first pair of indices

$$\chi^{\alpha\beta\gamma\delta} = -\chi^{\beta\alpha\gamma\delta} \quad , \tag{17}$$

and with respect to the second pair of indices

$$\chi^{\alpha\beta\gamma\delta} = -\chi^{\alpha\beta\delta\gamma} \quad , \tag{18}$$

and thus has 36 independent components.

Analogously to Voigt notation in elasticity, the constitutive tensor can be expressed as the 6×6 constitutive matrix

$$\chi^{\alpha\beta\gamma\delta} = \begin{array}{c} \\ 41 \\ 42 \\ 43 \\ 23 \\ 31 \\ 12 \end{array} \begin{array}{c} \begin{array}{cccccc} 41 & 42 & 43 & 23 & 31 & 12 \end{array} \\ \begin{pmatrix} -\epsilon^{11} & -\epsilon^{12} & -\epsilon^{13} & -\alpha^1{}_1 & -\alpha^1{}_2 & -\alpha^1{}_3 \\ -\epsilon^{21} & -\epsilon^{22} & -\epsilon^{23} & -\alpha^2{}_1 & -\alpha^2{}_2 & -\alpha^2{}_3 \\ -\epsilon^{31} & -\epsilon^{32} & -\epsilon^{33} & -\alpha^3{}_1 & -\alpha^3{}_2 & -\alpha^3{}_3 \\ \beta_1{}^1 & \beta_1{}^2 & \beta_1{}^3 & \mu^{-1}_{11} & \mu^{-1}_{12} & \mu^{-1}_{13} \\ \beta_2{}^1 & \beta_2{}^2 & \beta_2{}^3 & \mu^{-1}_{21} & \mu^{-1}_{22} & \mu^{-1}_{23} \\ \beta_3{}^1 & \beta_3{}^2 & \beta_3{}^3 & \mu^{-1}_{31} & \mu^{-1}_{32} & \mu^{-1}_{33} \end{pmatrix} \end{array} \quad , \tag{19}$$

which lines correspond to the first pair of indices and columns to the second pair of indices, see Post (1962, eq. 6.21). The constitutive matrix need not be symmetric,

$$\chi^{\alpha\beta\gamma\delta} \neq \chi^{\gamma\delta\alpha\beta} \quad . \tag{20}$$

Constitutive relations (11) and (12) can be expressed as

$$\begin{pmatrix} -D^1 \\ -D^2 \\ -D^3 \\ H_1 \\ H_2 \\ H_3 \end{pmatrix} = \begin{matrix} \\ 41 \\ 42 \\ 43 \\ 23 \\ 31 \\ 12 \end{matrix} \begin{matrix} \begin{matrix} 41 & 42 & 43 & 23 & 31 & 12 \end{matrix} \\ \begin{pmatrix} \chi^{4141} & \chi^{4142} & \chi^{4143} & \chi^{4123} & \chi^{4131} & \chi^{4112} \\ \chi^{4241} & \chi^{4242} & \chi^{4243} & \chi^{4223} & \chi^{4231} & \chi^{4212} \\ \chi^{4341} & \chi^{4342} & \chi^{4343} & \chi^{4323} & \chi^{4331} & \chi^{4312} \\ \chi^{2341} & \chi^{2342} & \chi^{2343} & \chi^{2323} & \chi^{2331} & \chi^{2312} \\ \chi^{3141} & \chi^{3142} & \chi^{3143} & \chi^{3123} & \chi^{3131} & \chi^{3112} \\ \chi^{1241} & \chi^{1242} & \chi^{1243} & \chi^{1223} & \chi^{1231} & \chi^{1212} \end{pmatrix} \end{matrix} \begin{pmatrix} E_1 \\ E_2 \\ E_3 \\ B^1 \\ B^2 \\ B^3 \end{pmatrix} \quad , \qquad (21)$$

see Post (1962, eq. 6.21). In the frequency domain, Maxwell equations (7) and (8) with linear constitutive relations in the Boys–Post representation for $A_\alpha = A_\alpha(x^m, \omega)$ read

$$(\chi^{\alpha jk\delta} A_{\delta,k})_{,j} - \mathrm{i}\omega(\chi^{\alpha j4\delta} A_\delta)_{,j} - \mathrm{i}\omega\chi^{\alpha 4k\delta} A_{\delta,k} - \omega^2\chi^{\alpha 44\delta} A_\delta - J^\alpha = 0 \quad , \qquad (22)$$

see Post (1962, eq. 6.28; 2003, eq. 26). If the definition volume for Maxwell equations (22) is not infinite, we assume homogeneous boundary conditions (Aki & Richards, 1980, box 2.4).

3. Green function

The 4×4 tensorial frequency–domain Green function $G_{\alpha\mu} = G_{\alpha\mu}(\mathbf{x}, \mathbf{x}', \omega)$ corresponding to Maxwell equations (22) is the solution of equation

$$\big[\chi^{\alpha jk\delta}(\mathbf{x},\omega)\, G_{\delta\mu,k}(\mathbf{x},\mathbf{x}',\omega)\big]_{,j} - \mathrm{i}\omega\big[\chi^{\alpha j4\delta}(\mathbf{x},\omega)\, G_{\delta\mu}(\mathbf{x},\mathbf{x}',\omega)\big]_{,j}$$
$$- \mathrm{i}\omega\chi^{\alpha 4k\delta}(\mathbf{x},\omega)\, G_{\delta\mu,k}(\mathbf{x},\mathbf{x}',\omega) - \omega^2\chi^{\alpha 44\delta}(\mathbf{x},\omega)\, G_{\delta\mu}(\mathbf{x},\mathbf{x}',\omega) - \delta^\alpha_\mu\, \delta(\mathbf{x}-\mathbf{x}') = 0 \qquad (23)$$

analytical with respect to the inverse Fourier transform. The partial derivatives are related to variables $\mathbf{x}$ and t.

Taking scalar product of the definition (23) of the frequency–domain Green function with $J^\mu(\mathbf{x}', \omega)$ and integrating over the subset V of the definition volume for Maxwell equations (22) containing the support of $J^\mu(\mathbf{x}', \omega)$, we see in comparison with (22) that

$$A_\alpha(\mathbf{x},\omega) = \int_V \mathrm{d}^3\mathbf{x}'\; G_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\, J^\mu(\mathbf{x}',\omega) \qquad (24)$$

is the solution of the frequency–domain Maxwell equations.

4. Field equivalence principle

Analogously to Kamenetskii (2001. eq. 12), we define *complementary medium* $\tilde{\chi}^{\alpha\beta\gamma\delta}$ as

$$\tilde{\chi}^{\alpha\beta\gamma\delta}(\mathbf{x},\omega) = \chi^{\delta\gamma\beta\alpha}(\mathbf{x},\omega) \quad . \qquad (25)$$

We define the frequency–domain *complementary Green function* $\tilde{G}_{\alpha\mu}(\mathbf{x}, \mathbf{x}', \omega)$ as the frequency–domain Green function in the complementary medium but with the opposite sign corresponding to the time derivative,

$$\big[\chi^{\delta kj\alpha}(\mathbf{x},\omega)\, \tilde{G}_{\delta\mu,k}(\mathbf{x},\mathbf{x}',\omega)\big]_{,j} + \mathrm{i}\omega\chi^{\delta k4\alpha}(\mathbf{x},\omega)\, \tilde{G}_{\delta\mu,k}(\mathbf{x},\mathbf{x}',\omega)$$
$$+ \mathrm{i}\omega\big[\chi^{\delta 4j\alpha}(\mathbf{x},\omega)\, \tilde{G}_{\delta\mu}(\mathbf{x},\mathbf{x}',\omega)\big]_{,j} - \omega^2\chi^{\delta 44\alpha}(\mathbf{x},\omega)\, \tilde{G}_{\delta\mu}(\mathbf{x},\mathbf{x}',\omega) - \delta^\alpha_\mu\, \delta(\mathbf{x}-\mathbf{x}') = 0 \; . \quad (26)$$

We consider volume V which is the subset of the definition volume for Maxwell equations (22) and need not contain the support of current density $J^\alpha(\mathbf{x}, \omega)$. We multiply equation (26) for the frequency–domain complementary Green function by $A_\alpha(\mathbf{x}, \omega)$, subtract the product of frequency–domain Maxwell equations (22) with $\tilde{G}_{\alpha\mu}(\mathbf{x}, \mathbf{x}', \omega)$, and integrate over volume V,

$$
\begin{aligned}
A_\mu(\mathbf{x}',\omega) = \int_V \mathrm{d}^3\mathbf{x} \Big\{ & J^\alpha(\mathbf{x},\omega)\,\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) + \big[\chi^{\delta kj\alpha}(\mathbf{x},\omega)\,\tilde{G}_{\delta\mu,k}(\mathbf{x},\mathbf{x}',\omega)\big]_{,j} A_\alpha(\mathbf{x},\omega) \\
& + \mathrm{i}\omega\chi^{\delta k4\alpha}(\mathbf{x},\omega)\,\tilde{G}_{\delta\mu,k}(\mathbf{x},\mathbf{x}',\omega) A_\alpha(\mathbf{x},\omega) \\
& + \mathrm{i}\omega\big[\chi^{\delta 4j\alpha}(\mathbf{x},\omega)\,\tilde{G}_{\delta\mu}(\mathbf{x},\mathbf{x}',\omega)\big]_{,j} A_\alpha(\mathbf{x},\omega) \\
& - \omega^2\chi^{\delta 44\alpha}(\mathbf{x},\omega)\,\tilde{G}_{\delta\mu}(\mathbf{x},\mathbf{x}',\omega) A_\alpha(\mathbf{x},\omega) \\
& - \big[\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,A_{\delta,k}(\mathbf{x},\omega)\big]_{,j}\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) \\
& + \mathrm{i}\omega\big[\chi^{\alpha j4\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\big]_{,j}\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) \\
& + \mathrm{i}\omega\chi^{\alpha 4k\delta}(\mathbf{x},\omega)\,A_{\delta,k(\mathbf{x},\omega)}\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) \\
& + \omega^2\chi^{\alpha 44\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\Big\}\,, \qquad (27)
\end{aligned}
$$

which can be expressed as

$$
\begin{aligned}
A_\mu(\mathbf{x}',\omega) = \int_V \mathrm{d}^3\mathbf{x} \Big\{ & \tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,J^\alpha(\mathbf{x},\omega) + \big[\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,\tilde{G}_{\alpha\mu,j}(\mathbf{x},\mathbf{x}',\omega)\big]_{,k} A_\delta(\mathbf{x},\omega) \\
& + \mathrm{i}\omega\chi^{\alpha j4\delta}(\mathbf{x},\omega)\,\tilde{G}_{\alpha\mu,j}(\mathbf{x},\mathbf{x}',\omega) A_\delta(\mathbf{x},\omega) \\
& + \mathrm{i}\omega\big[\chi^{\alpha 4k\delta}(\mathbf{x},\omega)\,\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\big]_{,k} A_\delta(\mathbf{x},\omega) \\
& - \omega^2\chi^{\alpha 44\delta}(\mathbf{x},\omega)\,\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) A_\delta(\mathbf{x},\omega) \\
& - \big[\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,A_{\delta,k}(\mathbf{x},\omega)\big]_{,j}\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) \\
& + \mathrm{i}\omega\big[\chi^{\alpha j4\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\big]_{,j}\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) \\
& + \mathrm{i}\omega\chi^{\alpha 4k\delta}(\mathbf{x},\omega)\,A_{\delta,k}(\mathbf{x},\omega)\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega) \\
& + \omega^2\chi^{\alpha 44\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\Big\}\,, \qquad (28)
\end{aligned}
$$

and finally as

$$
\begin{aligned}
A_\mu(\mathbf{x}',\omega) = \int_V \mathrm{d}^3\mathbf{x} \Big\{ & \tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,J^\alpha(\mathbf{x},\omega) + \big[\tilde{G}_{\alpha\mu,j}(\mathbf{x},\mathbf{x}',\omega)\,\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\big]_{,k} \\
& - \big[\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,A_{\delta,k}(\mathbf{x},\omega)\big]_{,j} \\
& + \mathrm{i}\omega\big[\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,\chi^{\alpha j4\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\big]_{,j} \\
& + \mathrm{i}\omega\big[\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,\chi^{\alpha 4k\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\big]_{,k}\Big\}\,. \qquad (29)
\end{aligned}
$$

We apply the divergence theorem to the integral of the gradient and obtain the field equivalence principle in its provisional form

$$
\begin{aligned}
A_\mu(\mathbf{x}',\omega) = & \int_V \mathrm{d}^3\mathbf{x}\,\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,J^\alpha(\mathbf{x},\omega) \\
& + \oint_{\partial V} \mathrm{d}^2\mathbf{x}\,\big\{\tilde{G}_{\alpha\mu,j}(\mathbf{x},\mathbf{x}',\omega)\,\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,A_\delta(\mathbf{x},\omega)\,n_k(\mathbf{x}) \\
& \qquad - \tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\,n_j(\mathbf{x})\,\chi^{\alpha jk\delta}(\mathbf{x},\omega)\,A_{\delta,k}(\mathbf{x},\omega) \\
& \qquad + \mathrm{i}\omega\,\tilde{G}_{\alpha\mu}(\mathbf{x},\mathbf{x}',\omega)\big[\chi^{\alpha j4\delta}(\mathbf{x},\omega)+\chi^{\alpha 4j\delta}(\mathbf{x},\omega)\big]A_\delta(\mathbf{x},\omega)\,n_j(\mathbf{x})\big\}\,, \qquad (30)
\end{aligned}
$$

where $n_i(\mathbf{x})$ is the unit normal to the surface ∂V of volume V pointing outside volume V.

For $J^\alpha(\mathbf{x},\omega) = \delta^\alpha_\nu\,\delta(\mathbf{x}-\mathbf{x}'')$, equation (30) should yield $A_\mu(\mathbf{x}',\omega) = G_{\mu\nu}(\mathbf{x}',\mathbf{x}'',\omega)$, see Maxwell equations (22) and (23). Integrating provisional field equivalence principle

(30) over the whole space, we obtain *reciprocity relation*

$$G_{\mu\nu}(\mathbf{x}',\mathbf{x}'',\omega) = \tilde{G}_{\nu\mu}(\mathbf{x}'',\mathbf{x}',\omega) \quad . \tag{31}$$

We insert this reciprocity relation into the above provisional form (30) of the field equivalence principle and obtain the final version of the *field equivalence principle*

$$\begin{aligned} A_\mu(\mathbf{x}',\omega) = &\int_V \mathrm{d}^3\mathbf{x}\; G_{\mu\alpha}(\mathbf{x}',\mathbf{x},\omega)\, J^\alpha(\mathbf{x},\omega) \\ &+ \oint_{\partial V} \mathrm{d}^2\mathbf{x}\, \big\{ G_{\mu\alpha,j}(\mathbf{x}',\mathbf{x},\omega)\, \chi^{\alpha jk\delta}(\mathbf{x},\omega)\, A_\delta(\mathbf{x},\omega)\, n_k(\mathbf{x}) \\ &\qquad - G_{\mu\alpha}(\mathbf{x}',\mathbf{x},\omega)\, n_j(\mathbf{x})\, \chi^{\alpha jk\delta}(\mathbf{x},\omega)\, A_{\delta,k}(\mathbf{x},\omega) \\ &\qquad + \mathrm{i}\omega\, G_{\mu\alpha}(\mathbf{x}',\mathbf{x},\omega) \big[\chi^{\alpha j4\delta}(\mathbf{x},\omega) + \chi^{\alpha 4j\delta}(\mathbf{x},\omega)\big] A_\delta(\mathbf{x},\omega)\, n_j(\mathbf{x}) \big\} \; . \end{aligned} \tag{32}$$

The integral over volume V represents the wave field corresponding to the sources situated inside volume V. The integral over the surface ∂V of volume V represents the wave field corresponding to the sources situated outside volume V, and is zero if all sources are situated inside volume V.

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