Kirchhoff prestack depth migration in velocity models with and without rotation of the tensor of elastic moduli: Orthorhombic and triclinic anisotropy

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Summary

We use the Kirchhoff prestack depth migration to calculate migrated sections in simple anisotropic homogeneous velocity models in order to demonstrate the impact of rotation of the tensor of elastic moduli on migrated images. The recorded wave field is generated in models composed of two homogeneous layers separated by a non-inclined curved interface. The anisotropy of the upper layer is orthorhombic or triclinic with the rotation of the tensor of elastic moduli. We apply the Kirchhoff prestack depth migration to single-layer velocity models with different types of anisotropy: orthorhombic and triclinic anisotropy with and without the rotation of the tensor of elastic moduli. We show the errors of the migrated interface caused by incorrect velocity models used for migration. The study is limited to P-waves.

Keywords

3-D Kirchhoff prestack depth migration, anisotropic velocity model, rotation of the tensor of elastic moduli

1. Introduction

The dimensions of the velocity model, shot-receiver configuration, methods for calculation of the recorded wave field and the migration are the same as in the paper by Bucha (2013a). Our aim is to study the effect of the rotation of the tensor of elastic moduli in models with orthorhombic and triclinic anisotropy on migrated images.

To compute the synthetic recorded wave field, we use simple anisotropic velocity models composed of two homogeneous layers separated by one curved interface that is non-inclined. The anisotropy in the upper layer is orthorhombic or triclinic with the rotation of the tensor of elastic moduli. The angles of rotation are equal to 15, 30 and 45 degrees.

We migrate in correct and incorrect single-layer models with orthorhombic and triclinic anisotropy. The distribution of elastic moduli in each correct model corresponds to the upper layer of the velocity model in which the corresponding synthetic data have been calculated. Incorrect models have orthorhombic and triclinic anisotropy without the rotation of the tensor of elastic moduli.

We show mispositioning, distortion and defocusing of the migrated interface caused by incorrect velocity models used for migration.

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2. Anisotropic velocity models

The dimensions of the velocity models are the same as in the paper by Bucha (2013a). The horizontal dimensions of the velocity model are 9.2 km x 10 km ($x_1 \times x_2$ coordinate axes) and the depth is 3 km (x_3 axis).

2.1. Velocity models for the recorded wave field

The recorded wave field is computed in the velocity models composed of two layers. We use six velocity models. Three of them have the orthorhombic anisotropy with the rotation of the tensor of elastic moduli in the upper layer. Additional three models have the triclinic anisotropy with the rotation of the tensor of elastic moduli in the upper layer. The angles of the rotation are 15, 30 and 45 degrees around axis x_2 (see Figure 1).

2.1.1 Orthorhombic anisotropy with the rotation of elastic moduli

The medium in the upper layer of these three velocity models is orthorhombic (Schoenberg & Helbig, 1997) with the rotation of the tensor of elastic moduli around axis x_2. The matrix of density-reduced elastic moduli A_{ij} in km^2/s^2 for the rotation angle equal to 15 degrees (OA-15) reads

$$\begin{pmatrix} 8.54 & 3.52 & 2.50 & 0.00 & -0.82 & 0.00 \\ & 9.84 & 2.48 & 0.00 & -0.30 & 0.00 \\ & & 5.89 & 0.00 & 0.05 & 0.00 \\ & & & 2.01 & 0.00 & -0.05 \\ & & & & 1.85 & 0.00 \\ & & & & & 2.17 \end{pmatrix} . \tag{1}$$

The matrix of density-reduced elastic moduli A_{ij} in km^2/s^2 for the rotation angle equal to 30 degrees (OA-30) reads

$$\begin{pmatrix} 7.48 & 3.30 & 3.01 & 0.00 & -1.10 & 0.00 \\ & 9.84 & 2.70 & 0.00 & -0.52 & 0.00 \\ & & 5.95 & 0.00 & -0.23 & 0.00 \\ & & & 2.05 & 0.00 & -0.08 \\ & & & & 2.36 & 0.00 \\ & & & & & 2.14 \end{pmatrix} . \tag{2}$$

The matrix of density-reduced elastic moduli A_{ij} in km^2/s^2 for the rotation angle equal to 45 degrees (OA-45) reads

$$\begin{pmatrix} 6.46 & 3.00 & 3.26 & 0.00 & -0.77 & 0.00 \\ & 9.84 & 3.00 & 0.00 & -0.60 & 0.00 \\ & & 6.46 & 0.00 & -0.77 & 0.00 \\ & & & 2.09 & 0.00 & -0.09 \\ & & & & 2.61 & 0.00 \\ & & & & & 2.09 \end{pmatrix} . \tag{3}$$

The bottom layer in all three velocity models is isotropic and the P-wave velocity in the layer is $V_p = 3.6$ km/s.

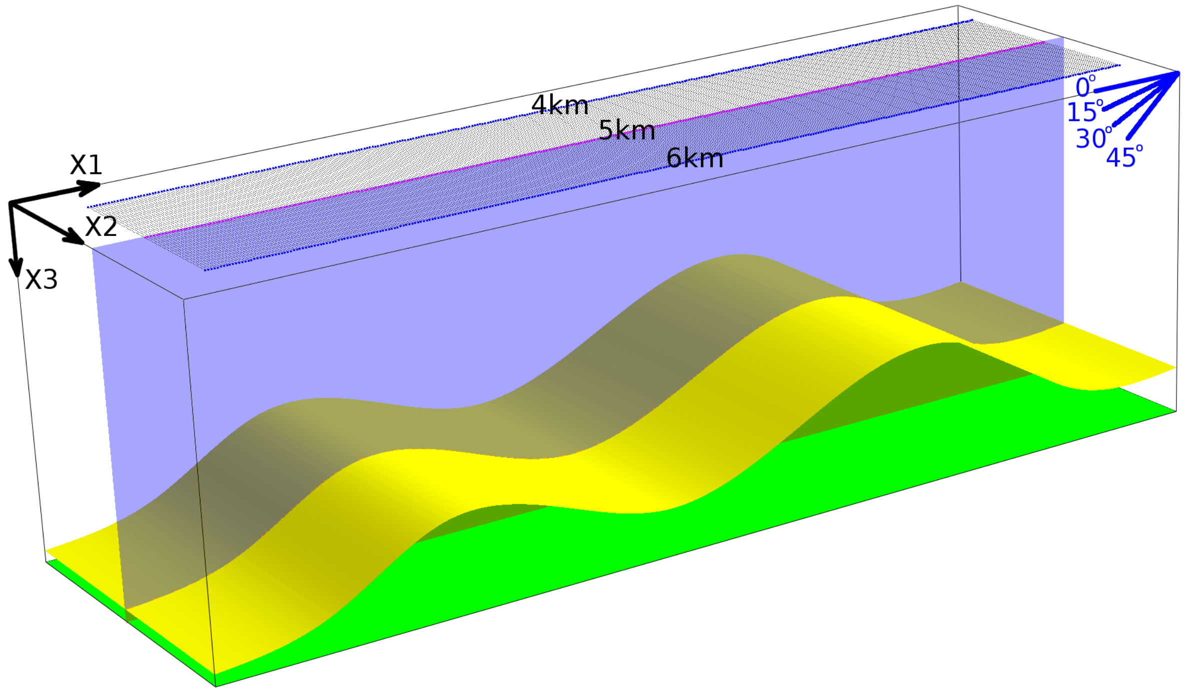


Figure 1. Part of the velocity model with 81 parallel profile lines, the non-inclined curved interface and the bottom velocity model plane. The horizontal dimensions of the depicted part of the velocity model are 9.2 km x 3 km, the depth is 3 km. We compute and stack migrated sections in the 2-D plane located in the middle of the shot-receiver configuration, at horizontal coordinate x_2=5 km. The rotation of the tensor of elastic moduli around axis x_2 is equal to 15, 30 and 45 degrees.

2.1.2 Triclinic anisotropy with the rotation of elastic moduli

The medium in the upper layer of these three velocity models is triclinic (Mensch & Rasolofosaon, 1997) with the rotation of the tensor of elastic moduli around axis x_2. The matrix of density-reduced elastic moduli A_{ij} in km^2/s^2 for the rotation angle equal to 15 degrees (TA-15) reads

$$\begin{pmatrix} 11.68 & 0.88 & 0.47 & 1.09 & 1.37 & 0.99 \\ & 10.60 & 2.12 & 0.35 & 0.13 & -0.53 \\ & & 14.38 & 0.32 & 0.10 & -0.83 \\ & & & 4.99 & -0.22 & 0.22 \\ & & & & 5.17 & -0.53 \\ & & & & & 5.01 \end{pmatrix} . \tag{4}$$

The matrix of density-reduced elastic moduli A_{ij} in km^2/s^2 for the rotation angle equal to 30 degrees (TA-30) reads

$$\begin{pmatrix} 12.87 & 1.03 & 0.19 & 0.73 & 0.81 & 0.83 \\ & 10.60 & 1.97 & 0.47 & 0.42 & -0.42 \\ & & 13.74 & 0.58 & 1.13 & -0.30 \\ & & & 4.88 & -0.14 & 0.19 \\ & & & & 4.89 & -0.98 \\ & & & & & 5.12 \end{pmatrix} . \tag{5}$$

The matrix of density-reduced elastic moduli A_{ij} in km^2/s^2 for the rotation angle equal to 45 degrees (TA-45) reads

$$\begin{pmatrix} 13.35 & 1.30 & 0.75 & 0.57 & 0.15 & 0.42 \\ & 10.60 & 1.70 & 0.57 & 0.60 & -0.28 \\ & & 12.15 & 0.57 & 1.75 & 0.42 \\ & & & 4.80 & 0.14 & 0.10 \\ & & & & 5.45 & -1.13 \\ & & & & & 5.20 \end{pmatrix} . \quad (6)$$

The bottom layer in all three velocity models is isotropic and the P-wave velocity in the layer is $V_p = 3.6$ km/s.

2.2. Velocity models for the migration

We migrate in correct single-layer models with orthorhombic and triclinic anisotropy with the rotation of the tensor of elastic moduli (matrices (1)-(6)). The distribution of elastic moduli in correct models corresponds to the upper layer of the velocity models in which the synthetic data have been calculated.

Additionally we migrate in incorrect single-layer models with orthorhombic and triclinic anisotropy without the rotation of the tensor of elastic moduli in order to simulate situations in which we have made an incorrect guess of the anisotropic velocity model for migration.

2.2.1 Orthorhombic anisotropy without the rotation of elastic moduli

Orthorhombic anisotropy without rotation (OA) is specified by Schoenberg & Helbig (1997). The matrix of density-reduced elastic moduli in km^2/s^2 reads

$$\begin{pmatrix} 9.00 & 3.60 & 2.25 & 0.00 & 0.00 & 0.00 \\ & 9.84 & 2.40 & 0.00 & 0.00 & 0.00 \\ & & 5.94 & 0.00 & 0.00 & 0.00 \\ & & & 2.00 & 0.00 & 0.00 \\ & & & & 1.60 & 0.00 \\ & & & & & 2.18 \end{pmatrix} . \quad (7)$$

2.2.2 Triclinic anisotropy without the rotation of elastic moduli

Triclinic anisotropy without rotation (TA) is specified by Mensch & Rasolofosaon (1997). The matrix of density-reduced elastic moduli in km^2/s^2 reads

$$\begin{pmatrix} 10.3 & 0.9 & 1.3 & 1.4 & 1.1 & 0.8 \\ & 10.6 & 2.1 & 0.2 & -0.2 & -0.6 \\ & & 14.1 & 0.0 & -0.5 & -1.0 \\ & & & 5.1 & 0.0 & 0.2 \\ & & & & 6.0 & 0.0 \\ & & & & & 4.9 \end{pmatrix} . \quad (8)$$

3. Kirchhoff prestack depth migration

The measurement configuration, calculation of the recorded wave field and the Kirchhoff prestack depth migration are the same as in the paper by Bucha (2013a).

3.1 Migration using the correct velocity models

The anisotropy in the upper layer of each velocity model used to compute the recorded wave field is the same as the anisotropy in the homogeneous single-layer velocity model used for migration.

3.1.1 Orthorhombic anisotropy with the rotation of elastic moduli

Figure 2 shows three stacked migrated sections calculated in the correct single-layer velocity models with the orthorhombic anisotropy with the rotation of the tensor of elastic moduli around the axis x_2 for three angles 15, 30 and 45 degrees. The distribution of elastic moduli in the single-layer velocity model for migration is the same as the distribution in the upper layer of the velocity model used to calculate the recorded wave field (matrices (1), (2) and (3)).

The migrated interface is clear and coincides nearly perfectly with the interface in the velocity model used to compute the recorded wave field.

3.1.2 Triclinic anisotropy with the rotation of elastic moduli

Similarly, Figure 3 shows three stacked migrated sections calculated in the correct single-layer velocity models with the triclinic anisotropy with the rotation of the tensor of elastic moduli around the axis x_2 for three angles 15, 30 and 45 degrees. The distribution of elastic moduli in the velocity models is specified by matrices (4), (5) and (6).

Note the poorly displayed migrated interface in the horizontal range of approximately 4−6 km. We explained the nearly vanishing interface for the angle of rotation 15 degrees in the paper by Bucha (2013b). The poorly displayed part of the migrated interface is worse for angles of rotation 30 and 45 degrees. We suppose that the cause of nearly vanishing part of interface is analogous as the case with the angle of rotation 15 degrees.

3.2 Migration using the incorrect velocity models

In this test we use the Kirchhoff prestack depth migration to calculate migrated sections in incorrect homogeneous velocity models. We simulate situations in which we have made an incorrect guess of the rotation of the tensor of elastic moduli around axis x_2.

3.2.1 Orthorhombic anisotropy without the rotation of elastic moduli

Here we migrate in incorrect single-layer velocity models with orthorhombic anisotropy without the rotation of the tensor of elastic moduli defined by matrix (7) (OA medium). The errors of the migrated interface increase with the angle of the rotation (see Figure 4). Note the nearly correctly migrated interface in the horizontal range of approximately 4−6 km for all angles of the rotation.

3.2.2 Triclinic anisotropy without the rotation of elastic moduli

In this case we migrate in incorrect single-layer velocity models with triclinic anisotropy without the rotation of the tensor of elastic moduli defined by matrix (8) (TA medium). The errors of the migrated interface again increase with the angle of the rotation (see Figure 5). Note the mispositioning of parts of the migrated interface is in the opposite direction in comparison with orthorhombic case. In addition, note the nearly correctly migrated interface in the horizontal range of approximately 4−6 km for all angles of the rotation.

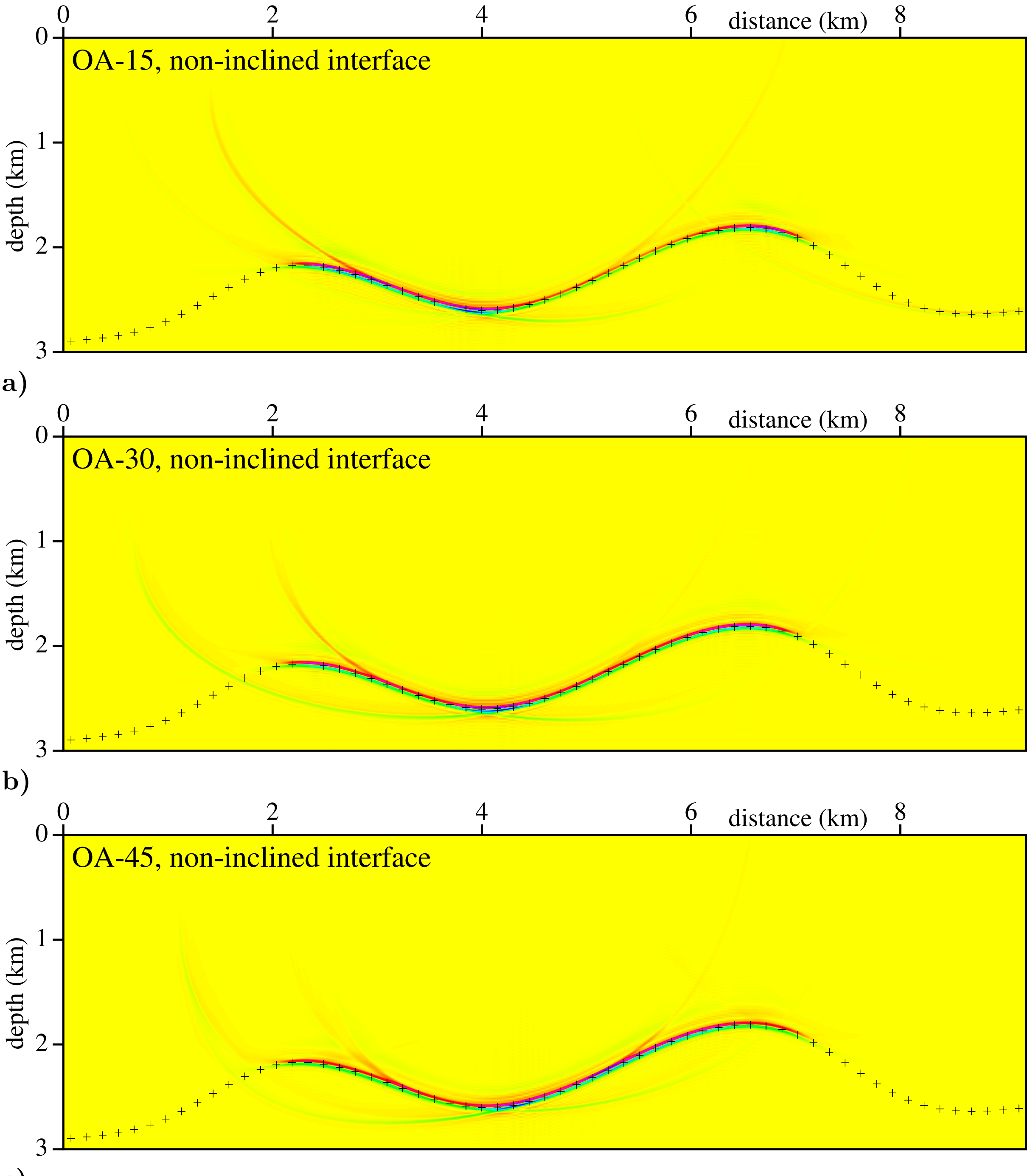


Figure 2. Stacked migrated sections calculated in the correct velocity models without interfaces, specified by orthorhombic anisotropy with **a)** 15 degree rotation of the tensor of elastic moduli around the x_2 axis (OA-15), **b)** 30 degree rotation of the tensor of elastic moduli around the x_2 axis (OA-30) and **c)** 45 degree rotation of the tensor of elastic moduli around the x_2 axis (OA-45). The distribution of elastic moduli in the single-layer velocity models for migration is the same as the distribution in the upper layer of the velocity models used to calculate the recorded wave field. 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The crosses denote the interface in the velocity models used to compute the recorded wave field.

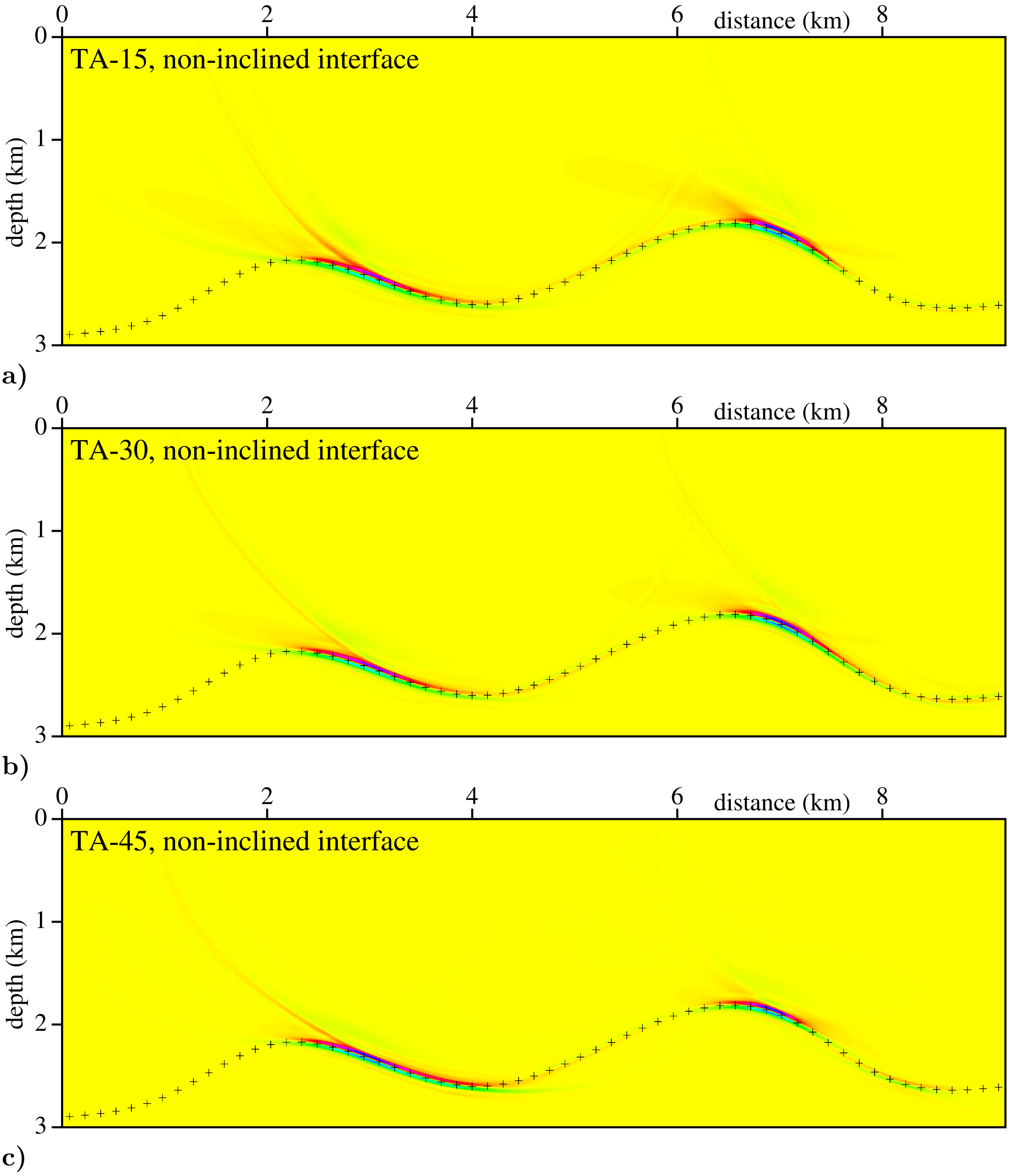
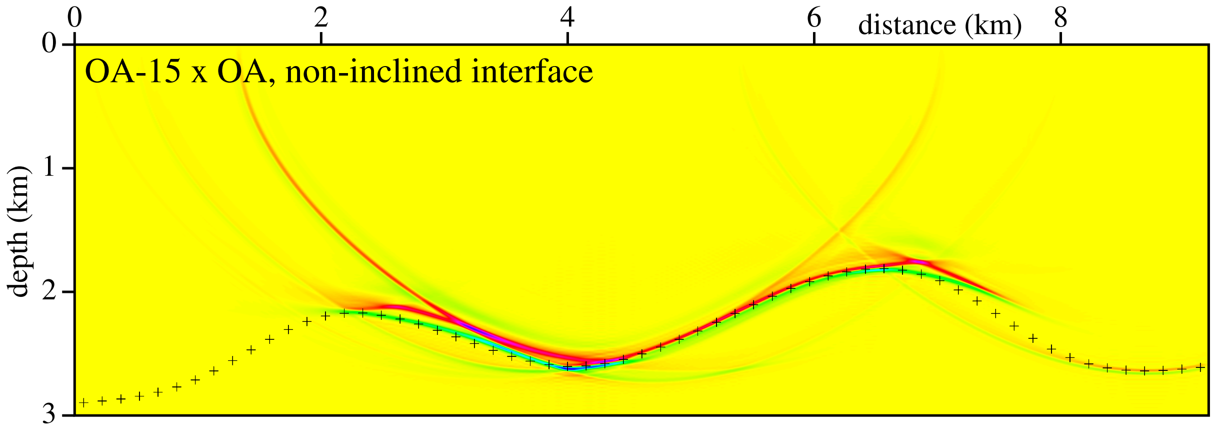
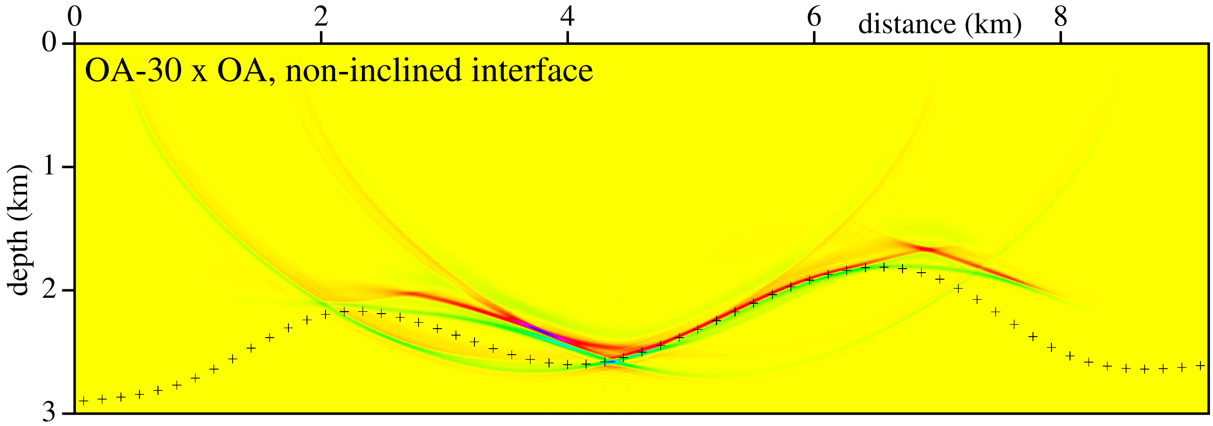


Figure 3. Stacked migrated sections calculated in the correct velocity models without interfaces, specified by triclinic anisotropy with **a)** 15 degree rotation of the tensor of elastic moduli around the x_2 axis (TA-15), **b)** 30 degree rotation of the tensor of elastic moduli around the x_2 axis (TA-30) and **c)** 45 degree rotation of the tensor of elastic moduli around the x_2 axis (TA-45). The distribution of elastic moduli in the single-layer velocity models for migration is the same as the distribution in the upper layer of the velocity models used to calculate the recorded wave field. 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The crosses denote the interface in the velocity models used to compute the recorded wave field.



a)



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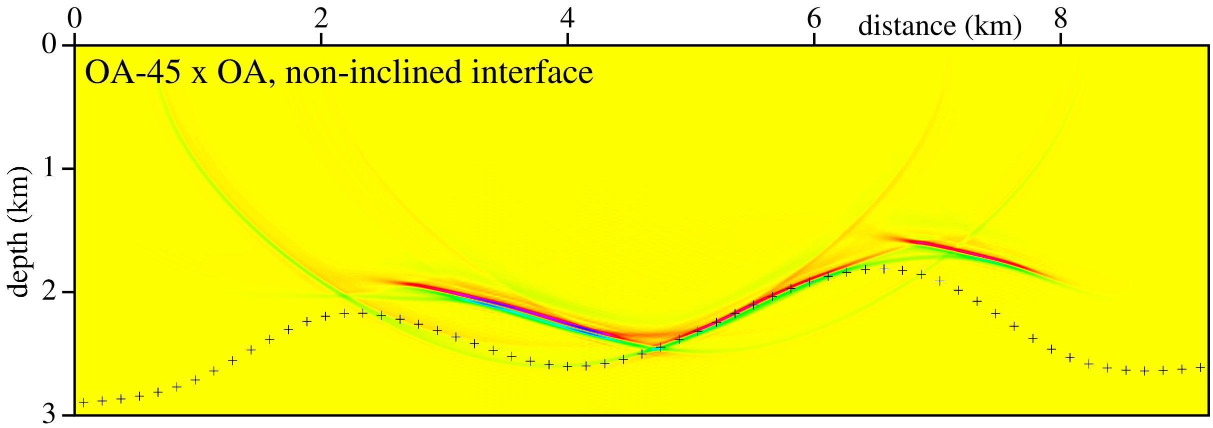

c)

Figure 4. Stacked migrated sections calculated in the incorrect velocity models with orthorhombic anisotropy (OA) without the rotation of the tensor of elastic moduli. The correct anisotropy is orthorhombic with **a)** 15 degree rotation of the tensor of elastic moduli around the x_2 axis (OA-15), **b)** 30 degree rotation of the tensor of elastic moduli around the x_2 axis (OA-30) and **c)** 45 degree rotation of the tensor of elastic moduli around the x_2 axis (OA-45). 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The crosses denote the interface in the velocity models used to compute the recorded wave field.

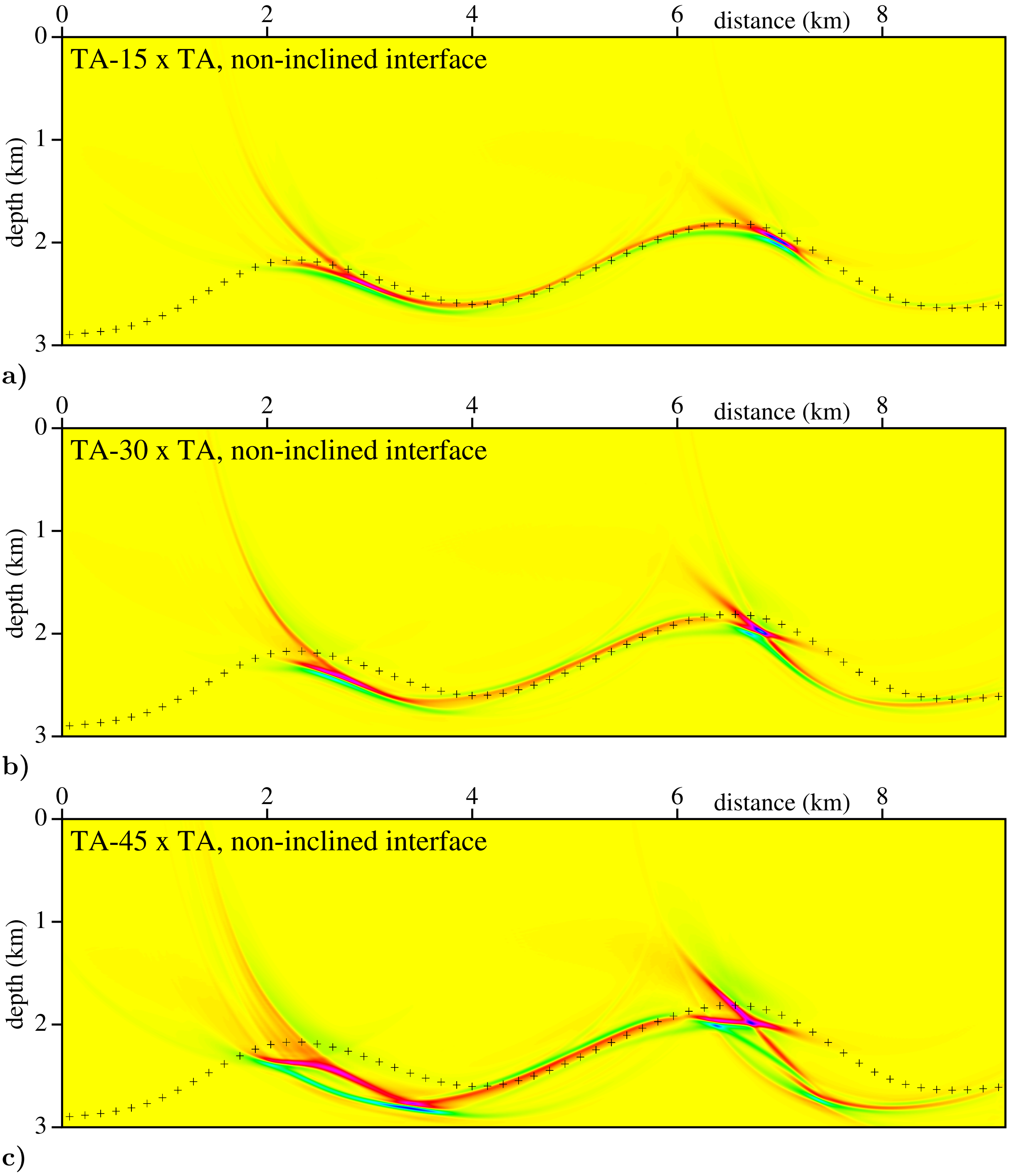

Figure 5. Stacked migrated sections calculated in the incorrect velocity models with triclinic anisotropy (TA) without the rotation of the tensor of elastic moduli. The correct anisotropy is triclinic with **a)** 15 degree rotation of the tensor of elastic moduli around the x_2 axis (TA-15), **b)** 30 degree rotation of the tensor of elastic moduli around the x_2 axis (TA-30) and **c)** 45 degree rotation of the tensor of elastic moduli around the x_2 axis (TA-45). 81×240 common-shot prestack depth migrated sections, corresponding to 81 profile lines and 240 sources along each profile line, have been stacked. The crosses denote the interface in the velocity models used to compute the recorded wave field.

4. Conclusions

We have generated synthetic data using the ray theory in simple velocity models of orthorhombic and triclinic anisotropy with and without the rotation of the tensor of elastic moduli. We have applied the Kirchhoff prestack depth migration to generate migrated sections in anisotropic homogeneous velocity models

- with the correct orthorhombic and triclinic anisotropy with rotation of the tensor of elastic moduli around the x_2 axis,
- with the incorrect orthorhombic and triclinic anisotropy without rotation of the tensor of elastic moduli around the x_2 axis.

In the case of correct orthorhombic anisotropy with rotation of the tensor of elastic moduli, the migrated interface in the final stacked image coincides nearly perfectly with the interface in the model used to compute the recorded wave field.

In the case of correct triclinic anisotropy with rotation of the tensor of elastic moduli, we have observed unexpected nearly vanishing part of the migrated interface. We have found out two causes of poorly displayed part of the migrated interface (for details see Bucha, 2013b). The first cause is zero reflection coefficient and phase change decreasing amplitudes of synthetic seismograms, the second cause is worse illumination of the interface by rays.

In the case of incorrect orthorhombic and triclinic anisotropy without the rotation of the tensor of elastic moduli, the errors of the migrated interface increase with the angle of the rotation. Approximately the same part of migrated interface is displayed nearly correctly for all angles of the rotation.

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