Constructing model P1I for reflection studies

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Summary

The algorithm of constructing smooth velocity models suitable for ray tracing by inversion of given data with minimization of the Sobolev norm composed of the second derivatives of velocity is applied to the 2-D P and S-wave velocity model called P1I. The P and S-wave velocity data available for the model construction are quite smooth, and thus construction of a smooth one-block model called P1 is possible. Because the model is constructed for reflection studies, and the data for smooth interfaces are available, we construct the model P1I composed of several smooth velocity blocks separated by smooth interfaces. We then calculate synthetic seismograms in the constructed model, which simulate a reflection measurement in the modelled locality.

Keywords

Ray tracing, velocity model, smoothing, Sobolev norm, synthetic seismograms.

1. Introduction

The construction of the velocity model of a geological structure is the first step in any calculation based on the application of ray methods. If the discrete values of velocity are given, we need to fit the data by a velocity model. In order to successfully perform ray tracing, proper smoothing of the velocity model is a key issue. The recommended method is the construction of a velocity model by data fitting with minimization of the Sobolev norm composed of the second velocity derivatives. Several papers on this topic were published by the researchers of the SW3D consortium. The first paper on this topic by Klimeš (2000a) is devoted to the detailed theoretical description of the smoothing algorithm. The papers by Bulant (2002) and Žáček (2002) contain a shortened overview of the theory and show numerical application to 2-D and 3-D models; preliminary versions of the papers were published by Bulant (2000), Žáček (2000), and Bulant (2001). The paper by Klimeš (2000b) contains the description of application of the smoothing algorithm to the Marmousi model, and contains detailed description of the history file used to smooth the model using the SW3D software. The paper by Bulant (2010) shows an example of construction of a smooth 1-D model suitable for calculation of Green functions for moment tensor inversion in (micro)seismic monitoring of natural earthquakes in locality Dobrá Voda.

In this paper, we apply the algorithm to the 2-D P and S-wave velocity model called P1I. Using the P and S-wave velocity data available for the model, we first construct the smooth one-block version of the model called P1. We then use the available data for interfaces, and construct model P1I composed of several smooth velocity blocks separated by smooth interfaces. We then calculate synthetic seismograms in the constructed model, which simulate a reflection measurement in the modelled locality.

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2. Input information for velocity model smoothing

2.1. Data for the P and S-wave velocity

The data for the model consist of a 2-D velocity profile of the length of 47.3 km and the depth of 6 km. The P-wave velocity data for the locality are available on the 2-D grid of 474 × 61 points with equal horizontal and vertical spacing of 0.1 km. For the construction of the velocity model, we convert the velocity to slowness. The S-wave slowness is obtained from the available ratio of P and S-wave velocity given on the same grid. See Figure 1 for the velocity data.

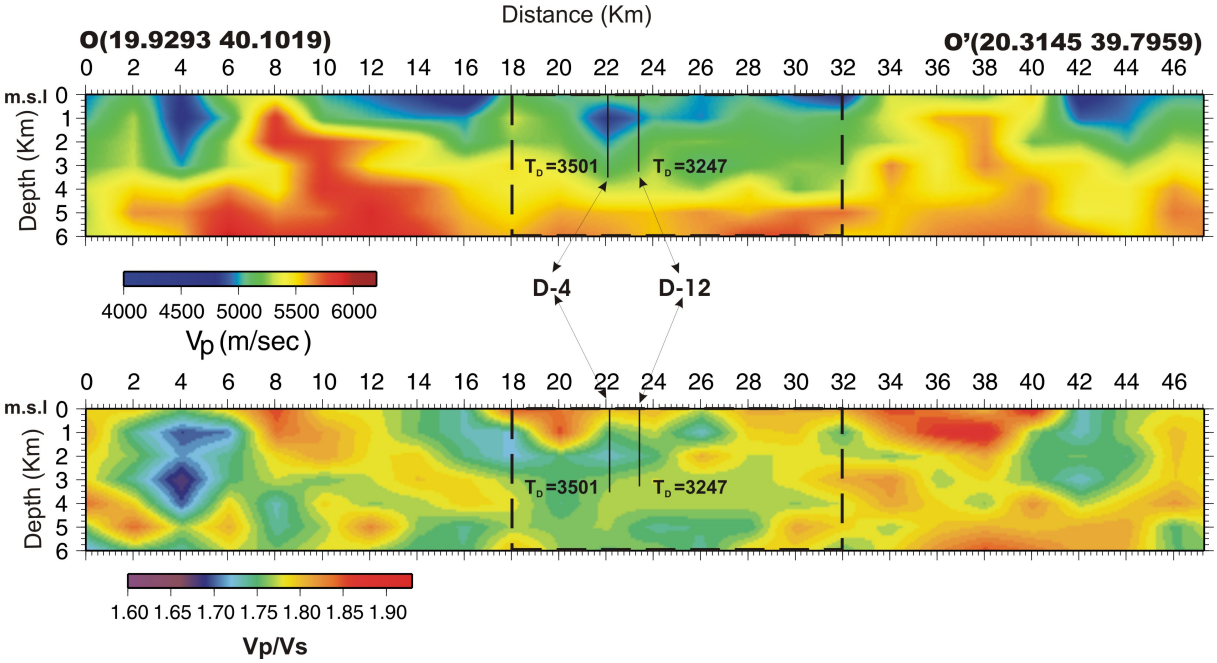

Figure 1: P-wave velocity and P/S velocity ratio used as the data for construction of model P1.

2.2. Data for the structural interfaces

The data for the structural interfaces are available in the form of digitized values, see Figure 2. For the construction of the velocity model we use only the interfaces located in the central part of the profile depicted as the "Most important study area" in Figure 2.

2.3. Model parameterization, amount of smoothing

The slowness in the velocity model is interpolated by bicubic splines. As an input for slowness smoothing, the parameterization of the model, i.e. the number and the coordinates of the spline points must be manually specified. The values of slowness in the prescribed spline points are calculated during smoothing. Smoothing is based on fitting the values of slowness at the data points and on minimizing the square of the Sobolev norm composed of the second slowness derivatives. The coefficient of the Sobolev norm is also prescribed manually.

The structural interfaces in the model are fitted by cubic splines, similarly as the velocities. We again prescribe manually the number and positions of the spline points used to model the interfaces, and we also prescribe the amount of smoothing applied to the interfaces.

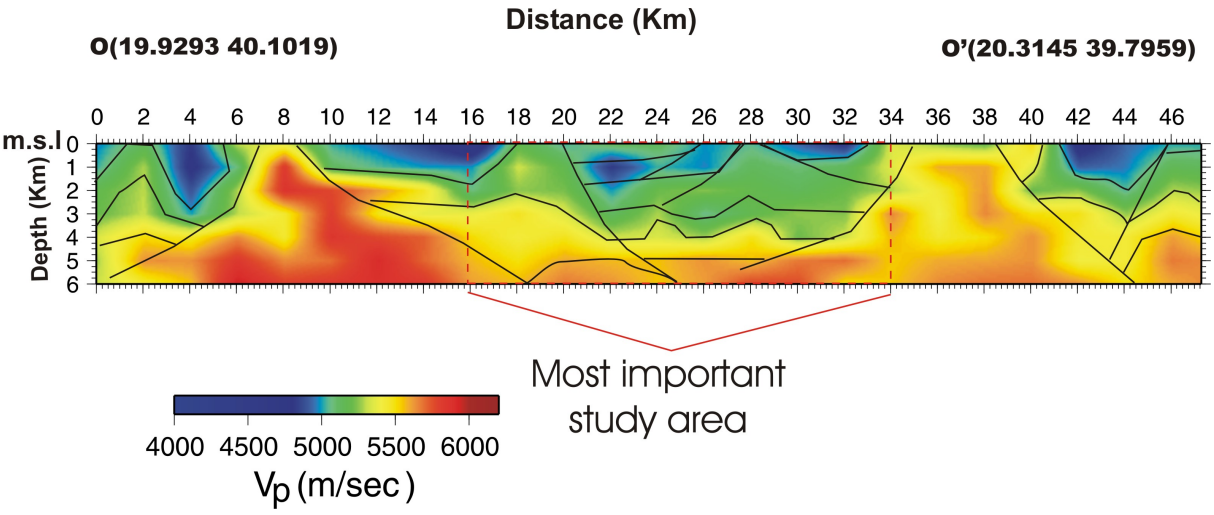

Figure 2: The data available for construction of the structural interfaces in model P1I.

3. Models P1 and P1I

3.1. Smooth one-block model called P1

First we construct a smooth one-block version of the model in order to estimate the amount of smoothing to be applied to the velocity field. We parameterize both the P and S-wave velocities by 474 × 61 spline points located at the same positions as the velocity data points. As the data we use the values of P and S-wave slowness, and in the model we interpolate the slownesses as well. We then apply the procedure of simultaneous least-square fitting of the discrete data and minimizing the square of the Sobolev norm composed of second derivatives of the slownesses. We then perform the inversion with different amounts of smoothing, check the resulting models, and we select the optimum value of smoothing parameter. In this case the amount of smoothing applied is relatively small. Refer to Figure 3 for the resulting one-block smooth P-wave velocity model, Figure 4 for the P-wave velocity data shown in the same colour scale as the model, and Figure 5 for the differences between the data and the model. See also corresponding Figures 6 to 8 for the S-wave velocities.

3.2. Model with interfaces called P1I

In the next step, we construct the model composed of several blocks separated by smooth interfaces. We choose only the data for the interfaces located in the "study area" of the model (see Figure 2), we specify the numbers and the positions of the spline points of individual interfaces. We then apply the procedure of simultaneous least-square fitting of the discrete data for interfaces and minimizing the square of the Sobolev norm composed of second derivatives, i.e. we minimize the curvature of the interfaces. We then perform the inversion with different amounts of smoothing, check the resulting models, and we select the optimum value of smoothing parameter. As we increase the smoothing, we first influence the behavior of the interfaces between the given data points. If we further increase the smoothing, then the interfaces become smoother and smoother and they start to deviate from the given data. We usually prefer to have the interfaces as smooth as possible, but still reasonably fitting the data points. See Figure 9 for the picture of the resulting interfaces and individual blocks of the model P1I.

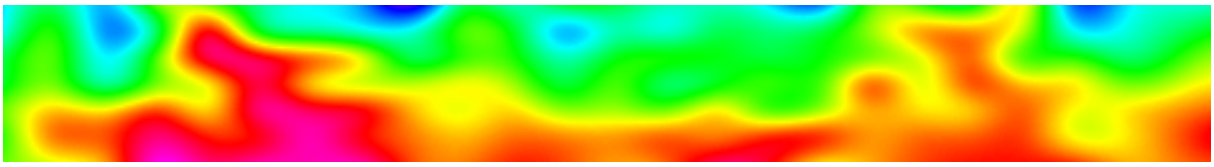

Figure 3: P-wave velocity in the smooth one-block model P1. The P-wave velocity ranges from 4.64 km/s shown in blue to 5.93 km/s shown in magenta. The whole colour circle corresponds to the interval of 1.50 $\mathrm{km\,s^{-1}}$.

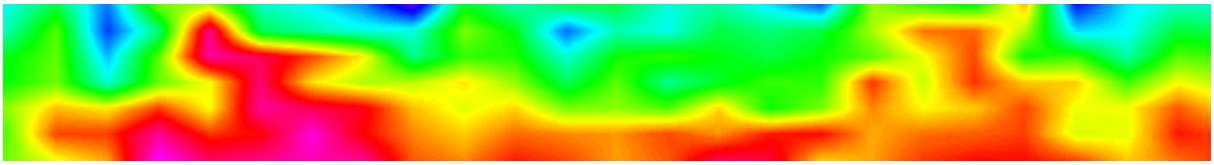

Figure 4: P-wave velocity data for the model P1. The data are the same as in Figure 1, but are shown here in the same colour scale as the resulting smooth model showen above in Figure 3.

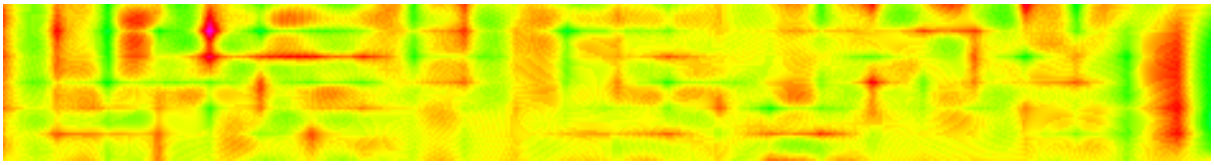

Figure 5: Differences between the P-wave velocity in the smooth one-block model P1 and the data used for the model construction. The colour scale ranges from green for the highest difference of 0.08 km/s, then through yellow color for zero differences, and then through red to magenta for the lowest difference of -0.13 km/s. The whole colour circle corresponds to the interval of 0.40 $\mathrm{km\,s^{-1}}$.

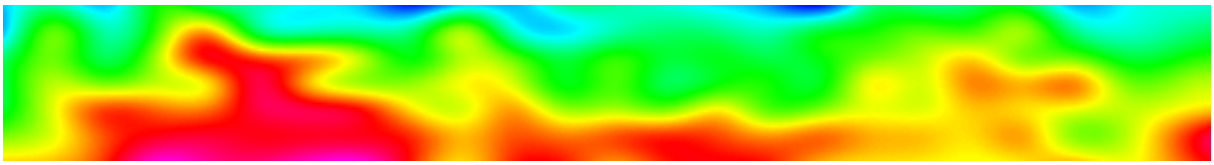

Figure 6: S-wave velocity in the smooth one-block model P1. The S-wave velocity ranges from 2.66 km/s shown in blue to 3.35 km/s shown in magenta. The whole colour circle corresponds to the interval of $1.5\,\mathrm{km\,s^{-1}}/\sqrt{3} = 0.866\,\mathrm{km\,s^{-1}}$.

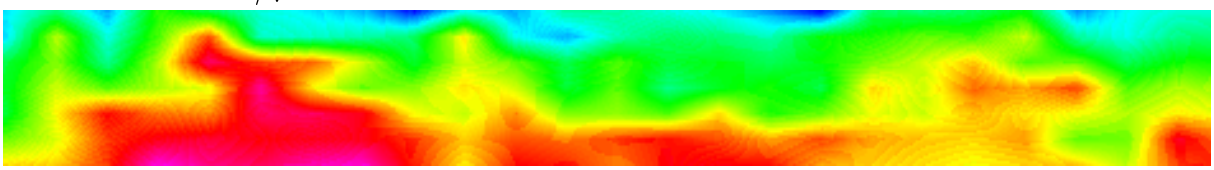

Figure 7: S-wave velocity data for the model P1. The colour scale is the same as in Figure 6.

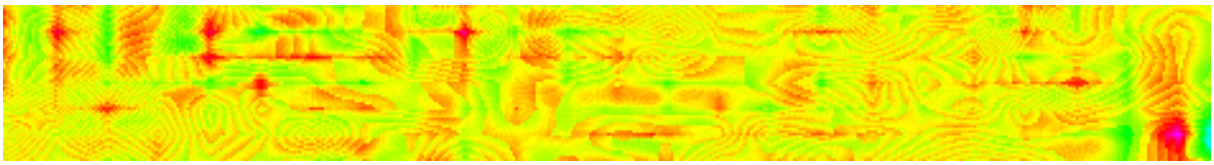

Figure 8: Differences between the S-wave velocity in the smooth one-block model P1 and the data used for the model construction. The colour scale ranges from cyan for the highest difference of 0.08 km/s, through green to yellow color for zero differences, and then through red to magenta for the lowest difference of -0.08 km/s. The whole colour circle corresponds to the interval of $0.40\,\mathrm{km\,s^{-1}}/\sqrt{3} = 0.23\,\mathrm{km\,s^{-1}}$.

Once we have constructed the structural interfaces in the model, we can perform the slowness fitting in the individual blocks. We use the same coefficient of the Sobolev norm as we used for the model P1, but we use coarser spline grids covering individual model blocks then the spline grid in the model P1. We finally construct the P and

Figure 9: Structural interfaces and color-coded individual structural blocks of the model P1I. The stars indicate the positions of the data points used for the construction of the interfaces. Compare with Figure 2.

S-wave velocity model P1I with interfaces. See Figures 10 and 13 for the P-wave and S-wave velocity field, and see also Figure 11 for the P-wave velocity differences between the model with interfaces and the data, and Figure 12 for the P-wave velocity differences between the smooth one-block model P1 and the model P1I with interfaces. See also Figure 14 for the S-wave velocity differences between the model P1I and the data, and Figure 15 for the S-wave velocity differences between the models P1 and P1I.

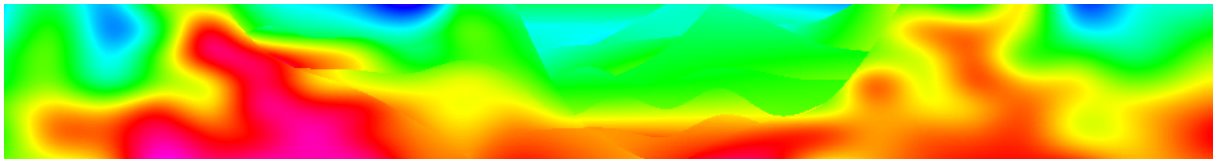

Figure 10: P-wave velocity in the model P1I with interfaces. The P-wave velocity ranges from 4.66 km/s shown in blue to 5.93 km/s shown in magenta. The colour scale is the same as in Figure 3.

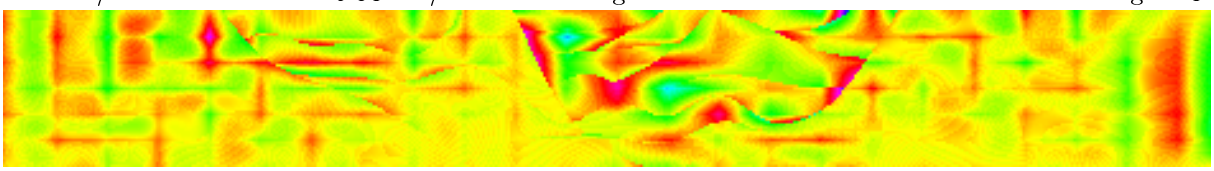

Figure 11: Differences between the P-wave velocity in the model P1I with interfaces, and the velocity data used for the model construction. The colour scale ranges from cyan for the highest difference of 0.15 km/s, through green to yellow color for zero differences, and then through red to dark blue for the lowest difference of -0.21 km/s. The colour scale is the same as in Figure 5.

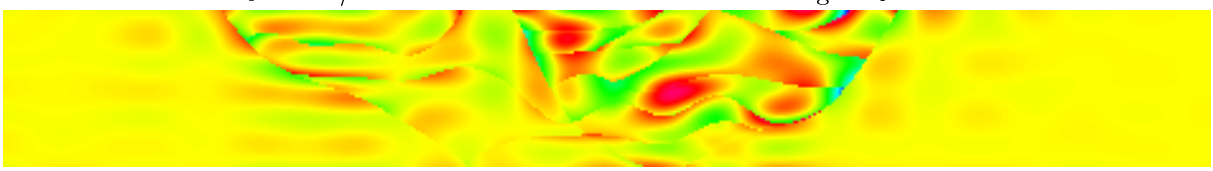

Figure 12: Differences between the P-wave velocity in the smooth one-block model P1 and the model P1I with interfaces. The colour scale ranges from blue for the highest difference of 0.19 km/s, through green to yellow color for zero differences, and then through red to magenta for the lowest difference of -0.10 km/s. The colour scale is the same as in Figure 5.

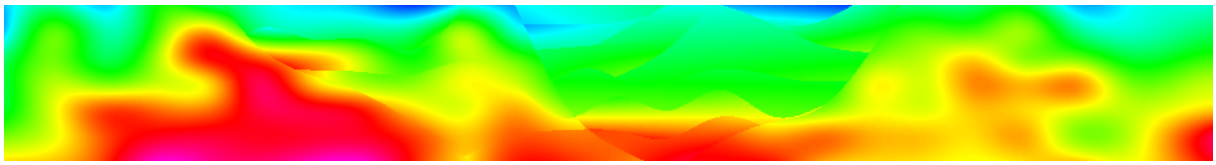

Figure 13: S-wave velocity in the model P1I with interfaces. The S-wave velocity ranges from 2.69 km/s shown in red to 3.42 km/s shown in cyan, the colour scale is the same as in Figure 6.

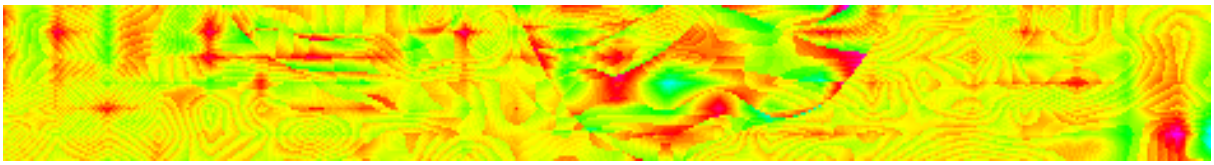

Figure 14: Differences between the S-wave velocity in the model P1I with interfaces, and the velocity data used for the model construction. The colour scale ranges from cyan for the highest difference of 0.08 km/s, then through green to yellow color for zero differences, and through red to magenta for the lowest difference of -0.08 km/s. The colour scale is the same as in Figure 8.

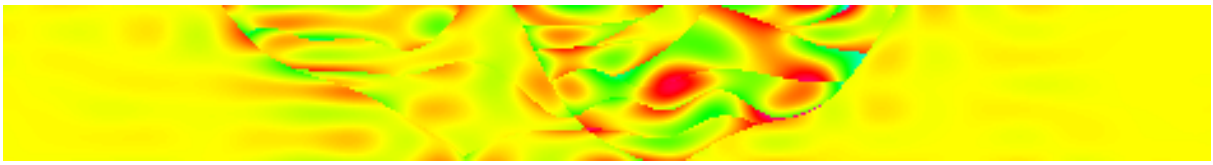

Figure 15: Differences between the S-wave velocity in the smooth one-block model P1 and the model P1I with interfaces. The colour scale ranges from cyan for the highest difference of 0.08 km/s, and then through green to yellow color for zero differences, and to red for the lowest difference of -0.07 km/s. The colour scale is the same as in Figure 8.

4. Synthetic seismograms

In the resulting model P1I with interfaces, we calculate synthetic seismograms to simulate a reflection measurement in the modelled locality. We perform the calculation for the point source located at the surface at 25 km, and for the profile of 37 receivers placed from 16 km to 34 km with the step of 0.5 km. We have calculated all the primary reflected waves, i.e. the P waves reflected at the first interface as P waves, then SS waves, and then waves converted after the reflection, i.e. SP and PS waves. The same set of the four waves was computed for reflection at the second interface (i.e. PPPP, SSSS, PPSS, and SSPP waves), then at the third interface, at the fourth interface, and so on. As an illustration of the ray tracing calculations, see Figures 16 to 23 for the P-wave rays reflected at the first to eighth interface.

The source is a vertical force, the source time function is Gabor signal with reference frequency 100 Hz, bandpass filtered by a cosine filter given by frequencies 4, 20, 184 and 200 Hz. The receivers record the vertical and radial (along the profile) components of the wave field. The seismograms are shown with no differential scaling between components and traces, so that true relative amplitudes are shown. See Figure 24 for the synthetic seismograms calculated along the profile.

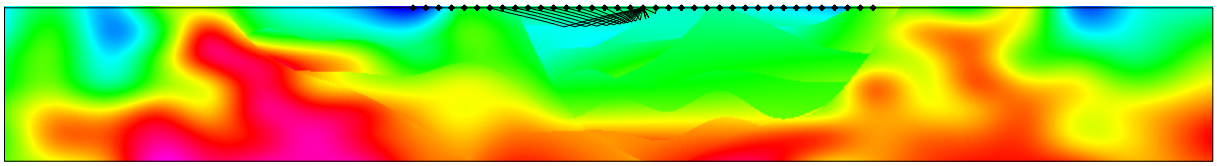

Figure 16: P-wave rays shot from the point source at the surface of the model, reflected at the first interface, and arriving to the profile of receivers.

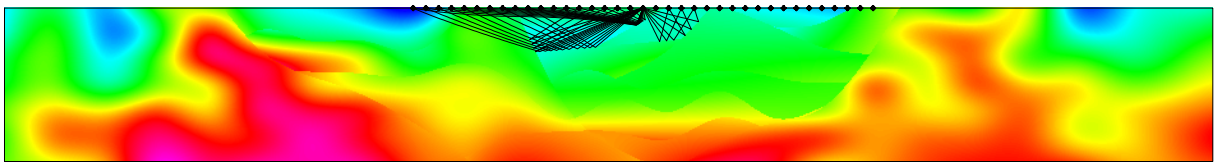

Figure 17: P-wave rays reflected at the second interface.

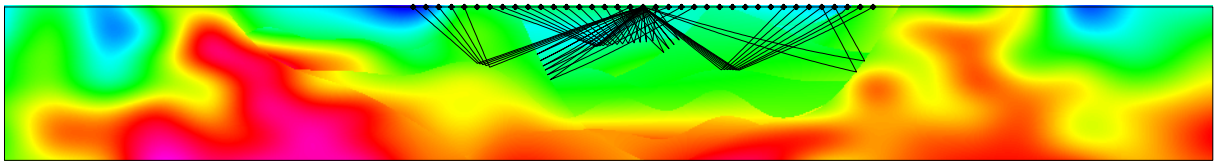

Figure 18: P-wave rays reflected at the third interface.

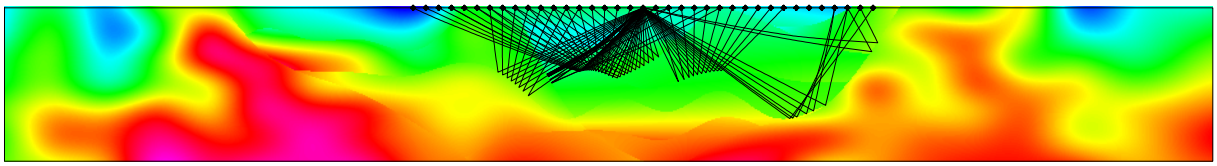

Figure 19: P-wave rays reflected at the fourth interface.

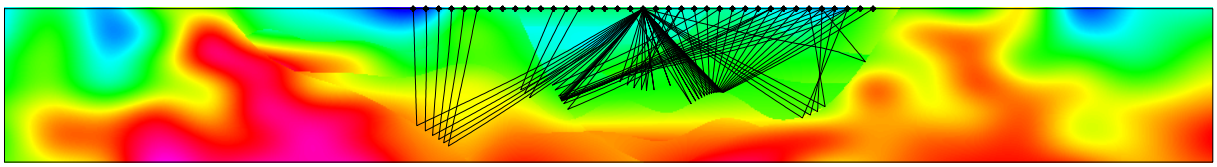

Figure 20: P-wave rays reflected at the fifth interface.

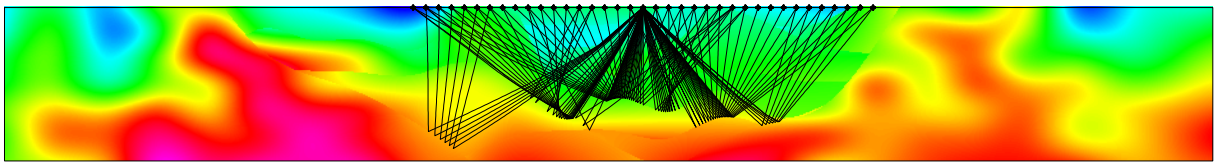

Figure 21: P-wave rays reflected at the sixth interface.

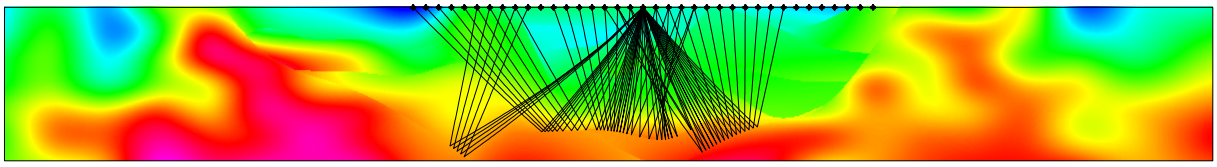

Figure 22: P-wave rays reflected at the seventh interface.

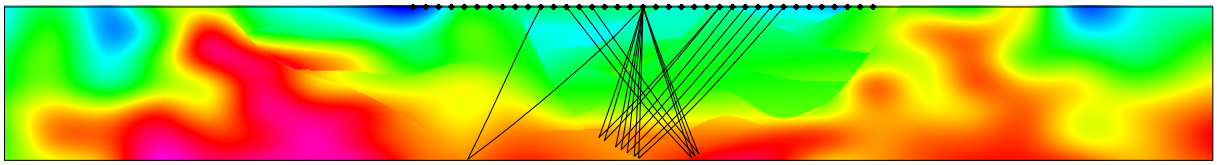

Figure 23: P-wave rays reflected at the eighth interface.

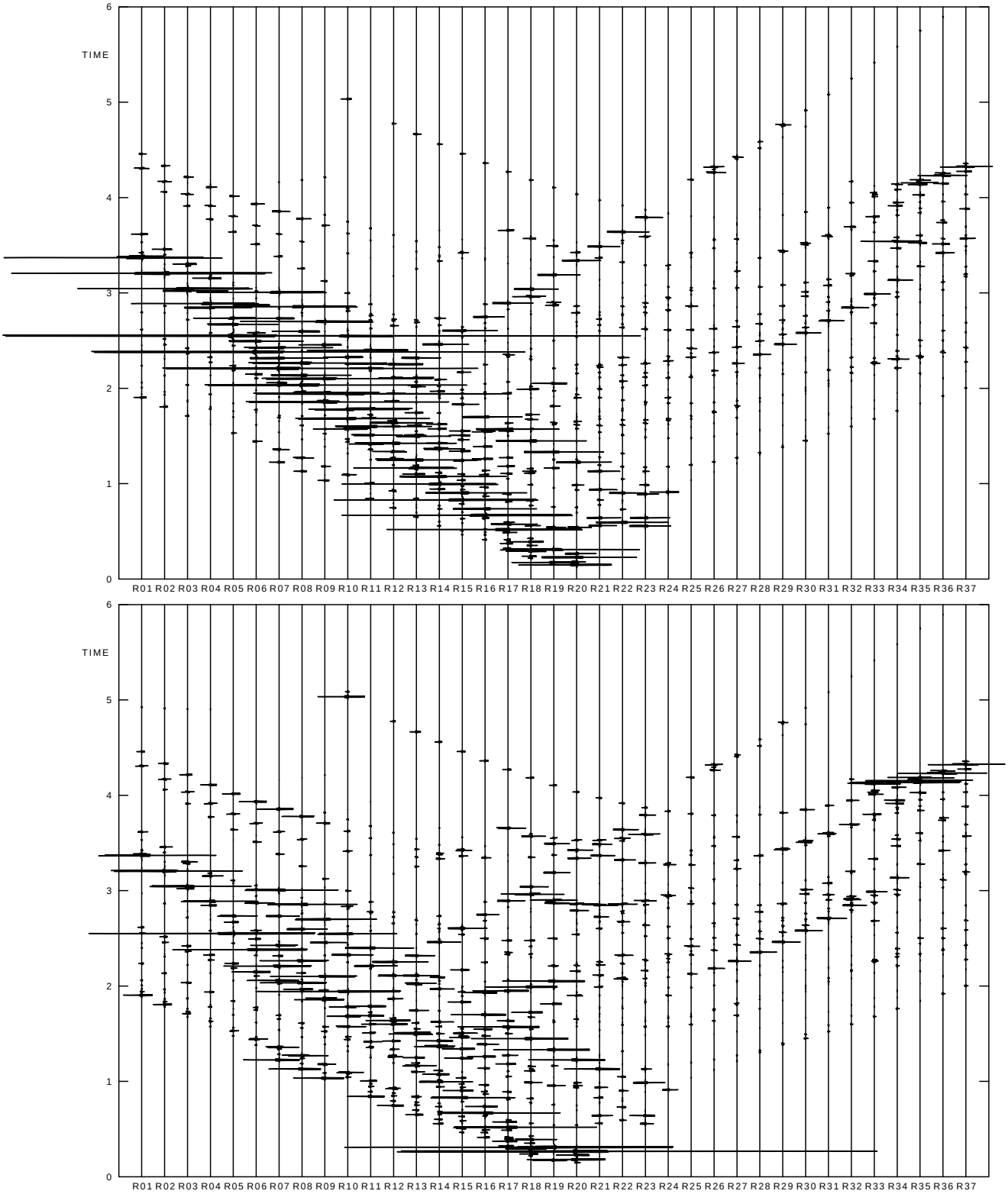

Figure 24: Vertical and radial components of the synthetic seismograms calculated for the profile shown on Figures 16 to 23. The source is a vertical force, the source time function is Gabor signal with reference frequency 100 Hz. The seismograms are shown with no differential scaling between components and traces.

Conclusions

The given P and S-wave velocity data are relatively smooth. The amount of smoothing required to create a smooth one-block model P1 was relatively small, and the relative differences between the smooth model and the given data are in the order of $(0.1\,\mathrm{km\,s^{-1}}/5\,\mathrm{km\,s^{-1}}) = 2\%$.

The data given for the structural interfaces allow to construct the model P1I with interfaces, which is suitable for ray tracing. The relative differences between the model P1I with interfaces and the data are again relatively small, they remain in the order of $(0.2\,\mathrm{km\,s^{-1}}/5\,\mathrm{km\,s^{-1}}) = 4\%$. The calculation of P and S waves primary reflected at the interfaces provides synthetic seismograms which simulate a reflection study along the given profile.

Acknowledgements

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