Transformation of amplitudes at an interface between two inhomogeneous weakly anisotropic media

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Summary

We perform an important step to extend the applicability of the first-order ray tracing (FORT) and dynamic ray tracing (FODRT) procedures to laterally varying layered, weakly anisotropic media. We present formulae for transformation of the amplitude of an incident P or coupled S wave into the amplitudes of reflected and transmitted P and coupled S waves.

Keywords: Body waves; Seismic anisotropy; Wave propagation.

1 Introduction

In our previous papers, we studied the first-order ray tracing and dynamic ray tracing (FORT and FODRT) of seismic body P waves and coupled S waves propagating in *smoothly varying*, weakly anisotropic media without interfaces (Pšenčík & Farra, 2005, 2007; Farra & Pšenčík, 2008, 2009). For derivation of the FORT and FODRT equations, we use the perturbation theory in which deviations of anisotropy from isotropy are considered to be first-order perturbations. In this paper, we make an important step to generalize the above-mentioned procedures for *layered* media. Specifically, we concentrate on the transformation of ray amplitudes, computed within the FORT and FODRT concept, at a curved interface separating two inhomogeneous, weakly anisotropic media. As in the standard problem of reflection/transmission (R/T), the incident and generated waves satisfy boundary conditions corresponding to the given configuration.

In the high frequency approximation, the problem of reflection/transmission of an arbitrary body wave at a curved interface separating two anisotropic media reduces to the problem of reflection/transmission of a plane wave at a plane interface separating two

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homogeneous halfspaces. Wavefront of the plane wave is tangent to the wavefront of the studied wave and the plane interface is tangent to the studied interface, both at the point of incidence of the wave at the interface. The properties of the homogeneous halfspaces correspond to the properties of the studied medium at the point of incidence on both sides of the interface.

Study of the problem of reflection/transmission (R/T) in anisotropic media has rather long history. The problem of reflection/transmission of plane waves at a plane interface between two homogeneous anisotropic halfspaces was studied, for example, by Fedorov (1968), Musgrave (1970), Daley & Hron (1977), Chapman (2004). For more references see Červený (2001). Gajewski & Pšenčík (1987) used the plane-wave R/T coefficients in the ray-theory computations of seismic wavefields in laterally varying layered anisotropic media. Considerable attention has been paid to various simplifications of R/T coefficients based, for example, on the assumption of weak-contrast interface, with anisotropy of the surrounding media of arbitrary strength (Ursin & Haugen, 1996; Klimeš, 2003) or on the assumption of weak-contrast interface and weak anisotropy of the surrounding media (Rueger, 1997, 2002; Vavryčuk & Pšenčík, 1998; Zillmer et al., 1998; Vavryčuk, 1999, Jílek, 2002).

In this contribution, we make no weak-contrast interface assumption. We only assume that media on both sides of the interface are weakly, but generally anisotropic. Use of the FORT and FODRT concepts leads to important transformation formulae applicable to coupled shear waves. Slowness vectors of generated waves are determined separately for P waves and for coupled S waves (like in isotropic media). The R/T coefficients are determined by numerical solution of the system of six inhomogeneous linear algebraic equations (like in anisotropic media). For media with anisotropy of higher symmetry, with specific orientation of symmetry elements with respect to the interface, it might be possible to find explicit expressions for the R/T coefficients. Here, however, we consider the case of most general anisotropy.

In Sec.2, we present expressions for displacement vector and traction in the first-order approximation. In Sec.3, we describe procedures of the determination of slowness vectors of generated waves and of the determination of amplitudes of generated waves. We end with several concluding remarks.

The lower-case indices $i, j, k, l, ...$ take the values of 1,2,3, the upper-case indices $I, J, K, L, ...$ take the values of 1,2. The Einstein summation convention over repeated indices is used. The upper index $[\mathcal{M}]$ is used to denote quantities related to the S-wave common ray.

2 Basic equations

Let us consider two inhomogeneous weakly anisotropic media in a welded contact, separated by a curved interface Σ with unit normal $\mathbf{N}$ at an arbitrarily chosen point on Σ, in which we wish to study the R/T process. We choose the orientation of the normal $\mathbf{N}$ so that it points into the medium, in which the incident wave propagates. This medium is specified by the density $\rho^{(1)}$ and the density-normalized elastic parameters

$a^{(1)}_{ijkl}$. The medium on the other side of the interface is specified by $\rho^{(2)}$ and $a^{(2)}_{ijkl}$. An incident wave generates P and coupled S waves in weakly anisotropic media on both sides of the interface. The incident and four generated waves satisfy the boundary conditions, which involve their displacement vectors $\mathbf{u}$ and tractions $\mathbf{T}$. We, therefore, specify these quantities first.

2.1 Displacement vector

In the frequency domain, the displacement vector $\mathbf{u}$ of an incident wave at the point of incidence of the wave at the interface Σ can be expressed as:

$$\mathbf{u}(x_m,\omega) = \mathbf{U}\exp[\mathrm{i}\omega\tau(x_m)] \ . \tag{1}$$

Here i is the imaginary unit, ω is the circular frequency, $\tau(x_m)$ is the eikonal, $p_m = \partial\tau/\partial x_m$ being the components of the corresponding first-order slowness vector $\mathbf{p}$, and $\mathbf{U}$ is the vectorial amplitude coefficient. Let us emphasize again that $\mathbf{p}$ and $\mathbf{U}$ are values of the slowness vector and the vectorial amplitude in the zero-order ray approximation at the point of incidence of the wave at the interface Σ.

The vectorial amplitude coefficient $\mathbf{U}$ of a P wave can be expressed in the following way:

$$\mathbf{U} = \mathcal{C}\mathbf{f}^{[3]} = \frac{\mathcal{C}_0\mathbf{f}^{[3]}}{(\rho c^{[3]})^{1/2}\mathcal{L}^{[3]}} \ . \tag{2}$$

The term $\mathcal{C}$ is the P-wave scalar amplitude factor, ρ is the density, $c^{[3]} = c(p^{[3]}_m)$ and $\mathcal{L}^{[3]} = \mathcal{L}(p^{[3]}_m)$ are the first-order P-wave phase velocity and geometrical spreading, respectively; see Pšenčík & Farra (2007). All quantities are related to the first-order slowness vector $\mathbf{p} = \mathbf{p}^{[3]}$. This is also true for the first-order P-wave polarization vector $\mathbf{f}^{[3]} = \mathbf{f}^{[3]}(p^{[3]}_m)$:

$$\mathbf{f}^{[3]}(p_m) = c^2(p_m)\frac{B_{13}(p_m)}{V_P^2 - V_S^2}\mathbf{e}^{[1]}(p_m) + c^2(p_m)\frac{B_{23}(p_m)}{V_P^2 - V_S^2}\mathbf{e}^{[2]}(p_m) + \mathbf{e}^{[3]}(p_m) \ . \tag{3}$$

The vectorial amplitude coefficient $\mathbf{U}$ of a coupled S wave propagating in a weakly anisotropic medium can be expressed in the following way (Farra & Pšenčík, 2009):

$$\mathbf{U} = \mathcal{A}\mathbf{f}^{[1]} + \mathcal{B}\mathbf{f}^{[2]} = \frac{\mathcal{A}_0\mathbf{f}^{[1]} + \mathcal{B}_0\mathbf{f}^{[2]}}{(\rho c^{[\mathcal{M}]})^{1/2}\mathcal{L}^{[\mathcal{M}]}} \ . \tag{4}$$

The terms $\mathcal{A}$ and $\mathcal{B}$ are S-wave scalar amplitude factors, $c^{[\mathcal{M}]} = c(p^{[\mathcal{M}]}_m)$ and $\mathcal{L}^{[\mathcal{M}]} = \mathcal{L}(p^{[\mathcal{M}]}_m)$ are the first-order common S-wave phase velocity and geometrical spreading, respectively; see Farra & Pšenčík (2008, 2009). The vectors $\mathbf{f}^{[K]} = \mathbf{f}^{[K]}(p^{[\mathcal{M}]}_m)$ are two mutually perpendicular vectors, to which the amplitude factors $\mathcal{A}$ and $\mathcal{B}$ are related. The vectors are situated in the plane, which we call S-wave polarization plane, which is perpendicular to the vector $\mathbf{f}^{[3]} = \mathbf{f}^{[3]}(p^{[\mathcal{M}]}_m)$. All quantities are related to the first-order slowness vector $\mathbf{p} = \mathbf{p}^{[\mathcal{M}]}$. The vectors $\mathbf{f}^{[K]}$ are given by the following expressions:

$$\mathbf{f}^{[K]}(p_m) = \mathbf{e}^{[K]}(p_m) - c^2(p_m)\frac{B_{K3}(p_m)}{V_P^2 - V_S^2}\mathbf{e}^{[3]}(p_m) \ . \tag{5}$$

Note that vectors $\mathbf{f}^{[i]}$ are generally non-unit and are different for $\mathbf{p}^{[3]}$ and $\mathbf{p}^{[\mathcal{M}]}$. Symbols B_{13} and B_{23} in eqs (3) and (5) are elements of the symmetric matrix $\mathbf{B}(p_m)$:

$$B_{jl}(p_m) = \Gamma_{ik}(p_m) e_i^{[j]} e_k^{[l]} \ . \tag{6}$$

The terms $\Gamma_{ik}(p_m)$ are elements of the generalized Christoffel matrix $\mathbf{\Gamma}$:

$$\Gamma_{ik}(p_m) = a_{ijkl} p_j p_l \ . \tag{7}$$

Symbols p_m denote again components of the first-order slowness vector $\mathbf{p}$ of the considered wave, i.e., of P wave ($p_m^{[3]}$) in eq.(3) and of S wave ($p_m^{[\mathcal{M}]}$) in eq.(5). Symbols $e_i^{[j]}$ denote components of three mutually perpendicular unit vectors $\mathbf{e}^{[j]}$. The vector $\mathbf{e}^{[3]}$ is chosen so that $\mathbf{e}^{[3]} = \mathbf{n}$. Here $\mathbf{n}$ is a unit vector specifying the direction of the first-order slowness vector $\mathbf{p}$ of the corresponding wave. The slowness vector $\mathbf{p}$ ($\mathbf{p} = \mathbf{p}^{[3]}$ for P waves, $\mathbf{p} = \mathbf{p}^{[\mathcal{M}]}$ for coupled S waves) must satisfy the corresponding first-order eikonal equation

$$G(p_m) = 1 \ . \tag{8}$$

Here G represents either the first-order approximation of the eigenvalue of the Christoffel matrix (7), corresponding to P wave, or an average of first-order eigenvalues of the Christoffel matrix (7), corresponding to coupled S waves. The explicit form of the first-order eikonal equations for P and coupled S waves can be found in Pšenčík & Farra (2005) and Farra & Pšenčík (2008). The two mutually perpendicular unit vectors $\mathbf{e}^{[1]}$ and $\mathbf{e}^{[2]}$ can be chosen arbitrarily in the plane perpendicular to the vector $\mathbf{e}^{[3]} = \mathbf{n}$.

Symbols V_P and V_S in eqs (3) and (5) denote the P- and S-wave velocities corresponding to the reference isotropic medium closely approximating the studied weakly anisotropic medium at the point of incidence.

2.2 Traction

Traction $\mathbf{T}$ is given by the expression:

$$T_i = \tau_{ij} N_j = \rho a_{ijkl} N_j u_{k,l} \ . \tag{9}$$

See, for example Gajewski and Pšenčík (1987), Červený (2001). Inserting the expression (1) for the displacement vector into eq. (9) leads to

$$T_i = \mathrm{i}\omega\rho a_{ijkl} N_j U_k p_l \ \exp[\mathrm{i}\omega\tau(x_m)] \ . \tag{10}$$

2.3 Boundary conditions

The incident and four generated waves satisfy the boundary conditions, which in case of interface separating two solid media, consist in the requirements of the continuity of displacement vectors $\mathbf{u}$ and tractions $\mathbf{T}$ across the interface. In order to distinguish quantities related to reflected and transmitted waves, we use superscripts R and T, respectively. Quantities related to the incident wave have no superscript. Sometimes, when we discuss properties of all generated waves, we use the superscript G.

3 Reflection/transmission

The above boundary conditions lead to two sets of equations. The first set, resulting from the continuity of the traveltime of all involved waves across the interface, represents equations for the determination of slowness vectors of generated waves. The second set, resulting from the boundary conditions themselves, represents equations for the determination of scalar amplitude factors of generated waves. In the following, we deal successively with both sets of equations.

3.1 Transformation of slowness vectors across an interface

The continuity of traveltime along the interface Σ implies continuity of the spatial traveltime derivatives taken along the interface. This can be expressed in the following way:

$$p_i^G - (p_k^G N_k)N_i = p_i - (p_k N_k)N_i \ . \tag{11}$$

Here, p_i and p_i^G are components of the first-order slowness vectors of the incident and generated (G) waves, N_i are components of the unit normal to the interface Σ. Eq.(11) is an alternative expression of the Snell law for anisotropic media. From equation (11) we can determine the components of slowness vectors of generated waves, tangential to the interface. It remains to determine their components to the normal $\mathbf{N}$ to the interface. We can write the slowness vectors of generated waves in the form

$$p_i^G = b_i + \xi^G N_i = p_i - (p_k N_k)N_i + \xi^G N_i \ , \tag{12}$$

where ξ^G represents the component of $\mathbf{p}^G$ to $\mathbf{N}$. The components ξ^G are the sought parameters. They can be found from the first-order eikonal equations satisfied by generated waves on corresponding sides of the interface

$$G(b_i + \xi^G N_i) = 1 \ . \tag{13}$$

Eikonal equation (13) can be rewritten as a polynomial equation of the fourth degree in ξ. It has four roots, two of which are non-physical. They can be identified as two conjugate roots, whose imaginary parts are larger than imaginary parts of remaining two roots. From remaining two roots, we accept the one, whose first-order ray-velocity vector $\mathbf{v}^G$ points into the medium, in which the generated wave should propagate (in case of real roots) or which satisfies the radiation condition (in case of complex conjugate roots). Explicitly it means that $N_i v_i^G \geq 0$ for reflected and $N_i v_i^G \leq 0$ for transmitted waves in case of real roots and $\mathrm{Im}\xi^G \geq 0$ for reflected and $\mathrm{Im}\xi^G \leq 0$ for transmitted waves in case of complex conjugate roots. Symbols v_i^G denote components of the ray-velocity vector $\mathbf{v}^G$. The waves corresponding to real roots of polynomial equation are called homogeneous while those related to complex roots are called inhomogeneous waves.

The above described procedure should be used when we use a solver of the polynomial equation, which provides all four roots. We can, however, also use an alternative, and, perhaps, more efficient, procedure, already used by Dehghan, Farra & Nicolétis (2007), see also Sec.4.3 of Jech & Pšenčík (1989). In weakly anisotropic media, it is reasonable to assume that the sought root of eq. (13) is close to the root of similar equation corresponding to a reference isotropic medium. We can thus use the root for the isotropic case as a guess of the sought root, and use the Newton-Raphson iterative method to find it. The iterative formula derived from the expansion of the eigenvalue G in eq. (13) with respect to ξ^G reads:

$$\mathbf{p}^{G\{j\}} = \mathbf{b} + \xi^{G\{j\}}\mathbf{N} \ , \tag{14}$$

where j is the iteration number and

$$\xi^{G\{j\}} = \xi^{G\{j-1\}} - \frac{G(p_m^{G\{j-1\}}) - 1}{N_k \frac{\partial G}{\partial p_k}(p_m^{G\{j-1\}})} \ . \tag{15}$$

The solution in a reference isotropic medium can be used as the initial guess $\xi^{G\{0\}}$. The explicit expressions for $\frac{\partial G}{\partial p_k}$ for P waves in media of arbitrary anisotropy and for coupled S waves in media of orthorhombic and TI symmetries can be found in Pšenčík & Farra (2007) and Farra & Pšenčík (2008), respectively.

3.2 Transformation of amplitudes across an interface

The continuity of traveltime along the interface Σ leads to equality of exponential factors of displacement vectors of incident and generated waves. Taking this into account, we can write the boundary conditions in the following form:

$$\mathcal{A}^R f_i^{[1]R} + \mathcal{B}^R f_i^{[2]R} + \mathcal{C}^R f_i^{[3]R} - \mathcal{A}^T f_i^{[1]T} - \mathcal{B}^T f_i^{[2]T} - \mathcal{C}^T f_i^{[3]T} = -U_i \ ,$$

$$\mathcal{A}^R X_i^{[1]R} + \mathcal{B}^R X_i^{[2]R} + \mathcal{C}^R X_i^{[3]R} - \mathcal{A}^T X_i^{[1]T} - \mathcal{B}^T X_i^{[2]T} - \mathcal{C}^T X_i^{[3]T} = -X_i \ , \tag{16}$$

where

$$X_i = \rho^{(1)} a_{ijkl}^{(1)} N_j U_k p_l \ ,$$

$$X_i^{[3]R} = \rho^{(1)} a_{ijkl}^{(1)} N_j f_k^{[3]R} p_l^{[3]R} \ , \quad X_i^{[3]T} = \rho^{(2)} a_{ijkl}^{(2)} N_j f_k^{[3]T} p_l^{[3]T} \ ,$$

$$X_i^{[N]R} = \rho^{(1)} a_{ijkl}^{(1)} N_j f_k^{[N]R} p_l^{[\mathcal{M}]R} \ , \quad X_i^{[N]T} = \rho^{(2)} a_{ijkl}^{(2)} N_j f_k^{[N]T} p_l^{[\mathcal{M}]T} \ . \tag{17}$$

The symbols X_i in eq. (17) correspond to the incident wave, symbols $X_i^{[3]G}$ to generated P waves and $X_i^{[N]G}$, $N = 1,2$, to generated coupled S waves. The slowness vectors of generated waves are determined by the procedure described in the preceding section. The vectors $\mathbf{f}^{[i]G}$ are determined from eqs (3) or (5). If the incident wave is the P wave, then the quantities U_i and X_i on the right-hand side of eq. (16) follow from (2) and from the first equation in (17), in which U_k follows again from (2) and p_l are components of the P-wave first-order slowness vector $\mathbf{p}^{[3]}$. In case of the incident S wave, the quantities U_i and X_i follow from eq.(4) and from the first equation in (17), in which U_k follows from (4) and p_l are components of the first-order slowness vector $\mathbf{p}^{[\mathcal{M}]}$ of the common S wave.

Eqs (16) represent a set of six inhomogeneous linear algebraic equations for six unknowns $\mathcal{A}^R$, $\mathcal{B}^R$, $\mathcal{C}^R$, $\mathcal{A}^T$, $\mathcal{B}^T$ and $\mathcal{C}^T$, scalar wave amplitudes of four waves generated by incidence of the wave with vectorial amplitude $\mathbf{U}$. These amplitudes together with corresponding vectors $\mathbf{f}^{[m]}$ can be used in expressions like (2) and (4) to specify vectorial amplitudes along the rays of generated waves. These rays may be rays of reflected or transmitted P waves or common rays of reflected or transmitted coupled S waves.

4 Concluding remarks

In this contribution, we showed how to compute the first-order amplitudes of P or coupled S waves generated at an interface between two inhomogeneous, weakly anisotropic media by incidence of a P wave or a coupled S wave. In order to compute first-order synthetic seismograms of these waves in laterally varying layered media, we must supplement the procedure by transformation relations for the FORT and FODRT across the interface. The transformation of FORT consists in transformation of the first-order slowness vector across the interface, which is described in Sec.3.1. The transformation of FODRT is formally the same as in the exact case, see, e.g., Farra & Le Bégat (1995) or Červený (2001). It is only necessary to substitute the exact quantities by their first-order counterparts in the corresponding equations.

The main goal of this contribution was to show how to compute amplitudes of reflected and transmitted waves in layered, weakly anisotropic media. We ignored an important aspect of presented equations, specifically of eqs (16), which is the computation of R/T coefficients. Let us only briefly mention that the problem of determination of R/T coefficients may be approached from several directions. The most natural is to seek R/T coefficients of P and coupled S waves. As the first step in their derivation, it is necessary to normalize the vectors $\mathbf{f}^{[i]}$ of all waves to unit vectors, and to substitute individual terms in eq. (16) by terms with normalized vectors. The normalized vectors $\mathbf{f}^{[I]}$ are arbitrarily oriented in the plane perpendicular to the normalized vector $\mathbf{f}^{[3]}$. We can always rotate the normalized vectors $\mathbf{f}^{[I]}$ so that one of them is horizontal. We can then compute R/T coefficients in a way similar to their computation in isotropic media. The S wave vectorial amplitude is formally split into the SH and SV components, and we can thus calculate P-P, P-SV, SV-P, SV-SV and SH-SH R/T coefficients. Another possible specification of R/T coefficients is such that they represent coefficients of separate S-wave modes (S1, S2) specified by the first-order S-wave polarization vectors. We are going to concentrate on the subject of R/T coefficients in a forthcoming publication.

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References

Červený, V., 2001. *Seismic Ray Theory*, Cambridge Univ. Press, Cambridge.

Chapman, C. H., 2004. *Fundamentals of seismic wave propagation.* Cambridge: Cambridge Univ. Press.

Daley, P.F., & Hron, F., 1977. Reflection and transmission coefficients for transversely isotropic media. *Bull. Seismol. Soc. Am.*, **67**, 661–675.

Dehghan, K., Farra, V. & Nicolétis, L., 2007. Approximate ray tracing for qP-waves in inhomogeneous layered media with weak structural anisotropy. *Geophysics*, **72**, SM47–SM60.

Farra, V. & Le Bégat, S. 1995. Sensitivity of qP-wave traveltimes and polarization vectors to heterogeneity, anisotropy and interfaces. *Geophys.J.Int.*, **121**, 371384.

Farra, V. & Pšenčík, I., 2008. First-order ray computations of coupled S waves in inhomogeneous weakly anisotropic media. *Geophys.J.Int.*, **173**, 979-989.

Farra, V. & Pšenčík, I., 2009 Coupled S waves in inhomogeneous weakly anisotropic media using first-order ray tracing, in Seismic waves in complex 3-D structures, Report 19, pp. 63–91, Charles University, Faculty of Mathematics and Physics, Dept. of Geophysics, Praha. (Available online at "http://sw3d.mff.cuni.cz".)

Fedorov, F.I., 1968. *Theory of elastic waves in crystals*, Plenum, New York.

Gajewski, D. & Pšenčík, I., 1987. Computation of high-frequency seismic wavefileds in 3-D laterally inhomogeneous anisotropic media. *Geophys.J.R.astr.Soc.*, **91**, 383–411.

Jech, J. & Pšenčík, I., 1989. First-order perturbation method for anisotropic media. *Geophys.J.Int.*, **99**, 369–376.

Jílek, P., 2002. Converted PS-wave reflection coefficients in weakly anisotropic media. *Pure Appl. Geophys.*, **159**, 1527–1562.

Klimeš, L., 2003. Weak-contrast reflection-transmission coefficients in a generally anisotropic background. *Geophysics*, **68**, 2063-2072.

Musgrave, M.P.J., 1970. *Crystal Acoustics*, Holden-Day, San Francisco.

Pšenčík, I. & Farra, V., 2005. First-order ray tracing for qP waves in inhomogeneous weakly anisotropic media. *Geophysics*, **70**, D65-D75.

Pšenčík, I. & Farra, V., 2007. First-order P-wave ray synthetic seismograms in inhomogeneous weakly anisotropic media. *Geophys.J.Int.*, **170**, 1243–1252.

Rueger, A., 1997. P-wave reflection coefficients for transversely isotropic models with vertical and horizontal axis of symmetry. *Geophysics*, **62**, 713-722.

Rueger, A., 2002. *Reflection Coefficients and Azimuthal AVO Analysis in Anisotropic Media*, SEG, Tulsa.

Ursin, B., & Haugen,G.U., 1996. Weak-contrast approximation of the elastic scattering matrix in anisotropic media. *Pure Appl. Geophys.*, **148**, 685-714.

Vavryčuk V., 1999. Weak-contrast R/T coefficients in weakly anisotropic elastic media: P-wave incidence. *Geophys. J. Int.*, **138**, 553–562.

Vavryčuk, V., & Pšenčík, I., 1998. PP wave reflection coefficients in weakly anisotropic media, *Geophysics*, **63**, 2129-2141.

Zillmer, M., Gajewski,D., & Kashtan,B. M.. 1998, Anisotropic reflection coefficients for a weak-contrast interface. *Geophys. J. Int.*, **132**, 159166.