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RESEARCH NOTE

Application of dynamic ray tracing in the 3-D inversion of seismic-reflection data

Vlastislav Červený[1] and Mirian A. de Castro[2]

[1] *Institute of Geophysics, Charles University, Ke Karlovu* 3, 121 16 *Praha* 2, *Czech Republic*

[2] *Instituto de Geociencias, Universidade Federal da Bahia, Rua Caetano Moura* 123, *Federação*, 40.210 *Salvador, Bahia, Brazil*

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SUMMARY

Possible applications of dynamic ray tracing to the 3-D Born and Kirchhoff inversion of seismic-reflection data are discussed. It is shown that the most important quantities in the inversion integrals, such as the Beylkin determinant and the amplitudes of the ray-theory Green functions, can be determined using the dynamic ray-tracing procedure. Dynamic ray tracing can be simply performed along known rays in a very general, laterally varying, layered background medium. The algorithms do not require the determination of the derivatives of traveltimes which play an important role in some other methods. They will find valuable applications particularly in complex 3-D structures.

Key words: dynamic ray tracing, inversion, reflection data, two-point eikonal.

1 INTRODUCTION

Recently, various inverse algorithms for seismic-reflection data have been proposed. Many of them are based on Born or on Kirchhoff approximations. See Beylkin (1985); Bleistein (1987); Beylkin & Burridge (1990); Hubral, Tygel & Schleicher (1991); Schleicher, Tygel & Hubral (1992); Weglein & Stolt (1992), etc. Many other references can be found in the above papers.

Here we shall consider the Cohen–Bleistein 3-D seismic-inversion expression, see Bleistein (1987), Arraya & Bleistein (1990),

$$\gamma(\mathbf{x}) = \frac{1}{8\pi^3}\iint d^2\boldsymbol{\xi} \times \frac{|h(\mathbf{x},\boldsymbol{\xi})|}{\mathscr{A}(\mathbf{x},\mathbf{x}_s)\mathscr{A}(\mathbf{x},\mathbf{x}_r)\,|\nabla T(\mathbf{x},\mathbf{x}_s)+\nabla T(\mathbf{x},\mathbf{x}_r)|^2} \times \int i\omega\, d\omega F(\omega)\exp\{-i\omega[T(\mathbf{x},\mathbf{x}_s)+T(\mathbf{x},\mathbf{x}_r)]\} \times D(\boldsymbol{\xi},\omega). \tag{1}$$

Here $\mathbf{x}_s$ and $\mathbf{x}_r$ denote the source and receiver positions along data surface S_0 (which may be curved). The surface is parameterized by two variables ξ_1, ξ_2, $\boldsymbol{\xi}\equiv(\xi_1,\xi_2)$. The distribution of sources and receivers along the surface is specified by equations $\mathbf{x}_s=\mathbf{x}_s(\xi_1,\xi_2)$, $\mathbf{x}_r=\mathbf{x}_r(\xi_1,\xi_2)$. For a more detailed explanation of these relations see Sections 4 and 5. The other quantities have the following meaning: $T(\mathbf{x},\mathbf{x}_s)$ and $T(\mathbf{x},\mathbf{x}_r)$ represent the traveltimes from the general point $\mathbf{x}$ to source $\mathbf{x}_s$ and to receiver $\mathbf{x}_r$, respectively, $\mathscr{A}(\mathbf{x},\mathbf{x}_s)$ represents the amplitude of the ray-theory Green function at point $\mathbf{x}$ due to a point source at $\mathbf{x}_s$, $\mathscr{A}(\mathbf{x},\mathbf{x}_r)$ is the same for the point source at $\mathbf{x}_r$, ω is the frequency, $F(\omega)$ is the spectrum of the source time function (tapered), $D(\boldsymbol{\xi},\omega)$ is the frequency domain representation of the acoustic reflection data (Fourier spectra of observed reflection seismograms) collected on surface S_0. Finally, $h(\mathbf{x},\boldsymbol{\xi})$ denotes the Beylkin determinant,

$$h(\mathbf{x},\boldsymbol{\xi}) = \begin{vmatrix} \nabla[T(\mathbf{x},\mathbf{x}_s)+T(\mathbf{x},\mathbf{x}_r)] \\ \dfrac{\partial}{\partial\xi_1}\nabla[T(\mathbf{x},\mathbf{x}_s)+T(\mathbf{x},\mathbf{x}_r)] \\ \dfrac{\partial}{\partial\xi_2}\nabla[T(\mathbf{x},\mathbf{x}_s)+T(\mathbf{x},\mathbf{x}_r)] \end{vmatrix}. \tag{2}$$

The result of integration, $\gamma(\mathbf{x})$, represents the reflection coefficient (scaled by the area under the filter).

According to Cohen, Hagin & Bleistein (1986), the general 3-D inversion formula 'does not represent a computationally feasible algorithm, primarily because a ray Jacobian determinant is not expressed in practical terms. In several important cases, however, this shortcoming can be overcome and expressions can be obtained that lead to feasible computing schemes'.

In this contribution, we shall show that the Beylkin determinant (2) and the amplitudes of Green functions $\mathscr{A}(\mathbf{x}, \mathbf{x}_s)$ and $\mathscr{A}(\mathbf{x}, \mathbf{x}_r)$ can be simply evaluated by dynamic ray tracing along known rays connecting point $\mathbf{x}$ with source $\mathbf{x}_s$ and receiver $\mathbf{x}_r$. The procedure may be applied to a fully general, 3-D laterally varying layered background. Dynamic ray tracing is a one-ray procedure, which does not require a numerical measurement of any elementary area, such as the elementary cross-sectional area of the ray tube, or the computation of the spatial derivatives of the traveltime field. All quantities are computed by direct numerical integration of the dynamic ray tracing system along a known ray.

2 TWO-POINT EIKONAL AND ITS MIXED DERIVATIVES

We shall consider a ray Ω connecting two points A, B in a medium described by the velocity $c(x_i)$. The slowness vector, tangent to the ray Ω, is denoted by $\mathbf{p}$.

Assume that we have performed ray tracing and dynamic ray tracing along ray Ω from A to B. Denote the relevant two-point traveltime (two-point eikonal) from A to B by $T(B, A)$. Now we consider a new point A' close to A and B' close to B. Point A' is displaced from A by $\mathbf{x}'$ and point B' from B by $\mathbf{x}''$. If displacements $\mathbf{x}'$ and $\mathbf{x}''$ are small, we can use dynamic ray tracing from A to B to determine the two-point traveltime $T(B', A')$, from A' to B'. The new ray from A' to B' need not be computed. The relevant equation is as follows,

$$T(B', A') = T(B, A) + x_i''p_i(B) - x_i'p_i(A) + \tfrac{1}{2}x_i''x_j''N_{ij}^+ - \tfrac{1}{2}x_i'x_j'N_{ij}^- - x_i'x_j''A_{ij}. \quad (3)$$

Here N_{ij}^+ are the elements of a 3×3 matrix of the second derivatives of the traveltime field at B due to a point source at A, and N_{ij}^- are the elements of a 3×3 matrix of the second derivatives of the traveltime field at A due to a point source at B. Both N_{ij}^+ and N_{ij}^- can be determined by dynamic ray tracing. Finally, the elements of the 3×3 matrix $\mathbf{A}$ are given by relation,

$$A_{ij} = e_{Ki}(A)e_{Lj}(B)[\mathbf{Q}_2^{-1}(B, A)]_{KL}. \quad (4)$$

Hereafter the capital letter indices $I, J, K, L, \ldots$ take the values 1, 2, and the lower-case indices $i, j, k, l, \ldots,$ the values 1, 2, 3. The Einstein summation is applied both with respect to small and capital indices.

The symbols e_{1i} and e_{2i} denote the Cartesian components of the unit vectors $\mathbf{e}_1$, $\mathbf{e}_2$, perpendicular to the ray Ω. The vectors $\mathbf{e}_1$, $\mathbf{e}_2$ are chosen in such a way to satisfy the following equations along the ray Ω:

$$\frac{d\mathbf{e}_1}{ds} = -c^2\left(\mathbf{e}_1 \cdot \frac{d\mathbf{p}}{ds}\right)\mathbf{p}, \qquad \mathbf{e}_2 = c\mathbf{p} \times \mathbf{e}_1. \quad (5)$$

Here s denotes the arc length along Ω; the quantities $\mathbf{p}$ and $d\mathbf{p}/ds$ are known from ray tracing. If the unit vector $\mathbf{e}_1$ is chosen to be orthogonal to the ray Ω at an initial point of the ray Ω, the triplet of unit vectors $\mathbf{e}_1, \mathbf{e}_2, \mathbf{e}_3 \equiv c\mathbf{p}$ satisfying (5) is orthogonal and right handed along the whole ray Ω. Finally, $\mathbf{Q}_2(B, A)$ is a 2×2 minor of the 4×4 ray propagator matrix $\mathbf{\Pi}(B, A)$,

$$\mathbf{\Pi}(B, A) = \begin{pmatrix} \mathbf{Q}_1(B, A) & \mathbf{Q}_2(B, A) \\ \mathbf{P}_1(B, A) & \mathbf{P}_2(B, A) \end{pmatrix}. \quad (6)$$

The 2×2 minors $\mathbf{Q}_1$, $\mathbf{Q}_2$, $\mathbf{P}_1$, $\mathbf{P}_2$ can be determined by the dynamic ray tracing along the ray Ω.

For a more detailed explanation see Červený, Klimeš & Pšenčík (1984); Červený (1985, 1987a,b). A detailed derivation of (3) can be found in Červený (1987b). All these references also present equations for N_{ij}^+ and N_{ij}^-.

Equation (3) for a two-point eikonal $T(B', A')$ offers many useful applications. One of them is the determination of the mixed partial derivative of the traveltime,

$$\frac{\partial^2 T}{\partial x_i'\, \partial x_j''} = -A_{ij}. \quad (7)$$

This is the basic equation for the determination of the Beylkin determinant.

Matrix $\mathbf{Q}_2(B, A)$ which plays an important role in the evaluation of Beylkin determinant can be also applied to evaluate the complex-valued amplitudes $\mathscr{A}(B, A)$ of the Green function. In a smooth, constant-density $(\rho = 1)$ medium, the amplitude of the time-harmonic Green function corresponding to a point source situated at A is given by the relation

$$\mathscr{A}(B, A) = \frac{1}{4\pi}\left[\frac{c(B)c(A)}{|\det \mathbf{Q}_2(B, A)|}\right]^{1/2} e^{i\delta T(B,A)}. \quad (8)$$

Here $\delta T(B, A)$ is the phase shift due to caustics.

3 COMPUTATION OF THE BEYLKIN DETERMINANT IN A 3-D MEDIUM

Using eqs (4) and (7), we can easily determine the Beylkin determinant (2). Assume an arbitrary data surface S_0, parameterized by parameters ξ_1, ξ_2. The source-receiver configuration of the seismic experiment may be specified by equations

$$\mathbf{x}_s = \mathbf{x}_s(\xi_1, \xi_2), \qquad \mathbf{x}_r = \mathbf{x}_r(\xi_1, \xi_2). \quad (9)$$

We shall use the following notation,

$$\frac{\partial x_{si}}{\partial \xi_K} = q_{iK}^{(s)}, \qquad \frac{\partial x_{ri}}{\partial \xi_K} = q_{iK}^{(r)}. \quad (10)$$

Hence,

$$\frac{\partial^2 T(\mathbf{x}, \mathbf{x}_s)}{\partial x_i\, \partial \xi_K} = \frac{\partial^2 T(\mathbf{x}, \mathbf{x}_s)}{\partial x_i\, \partial x_{sj}} q_{jK}^{(s)} = -A_{ij}^{(s)} q_{jK}^{(s)} = -e_{Mi}^{(s)}(\mathbf{x})e_{Nj}^{(s)}(\mathbf{x}_s)q_{jK}^{(s)}[\mathbf{Q}_2^{-1}(\mathbf{x}_s, \mathbf{x})]_{MN}.$$

Similar expressions can also be obtained for the mixed derivatives of $T(\mathbf{x}, \mathbf{x}_r)$. We shall use the notation

$$\begin{aligned} B_{iK}^{(s)} &= -\frac{\partial^2 T(\mathbf{x}, \mathbf{x}_s)}{\partial x_i\, \partial \xi_K} = e_{Mi}^{(s)}(\mathbf{x})e_{Nj}^{(s)}(\mathbf{x}_s)q_{jK}^{(s)}[\mathbf{Q}_2^{-1}(\mathbf{x}_s, \mathbf{x})]_{MN}, \\ B_{iK}^{(r)} &= -\frac{\partial^2 T(\mathbf{x}, \mathbf{x}_r)}{\partial x_i\, \partial \xi_K} = e_{Mi}^{(r)}(\mathbf{x})e_{Nj}^{(r)}(\mathbf{x}_r)q_{jK}^{(r)}[\mathbf{Q}_2^{-1}(\mathbf{x}_r, \mathbf{x})]_{MN}. \end{aligned} \quad (11)$$

The expression for the Beylkin determinant $h(\mathbf{x}, \boldsymbol{\xi})$ then reads

$$h(\mathbf{x}, \boldsymbol{\xi}) = \begin{vmatrix} p_i^{(s)}(\mathbf{x}) + p_i^{(r)}(\mathbf{x}) \\ B_{i1}^{(s)} + B_{i1}^{(r)} \\ B_{i2}^{(s)} + B_{i2}^{(r)} \end{vmatrix}, \tag{12}$$

where $B_{iK}^{(r)}$ and $B_{iK}^{(s)}$ ($i = 1, 2, 3, K = 1, 2$) are given by (11). All the quantities in (12) and (11) can be computed by ray tracing and dynamic ray tracing.

Expression (12) with (11) for the Beylkin determinant can be simplified further, if we take into account that basis vectors $\mathbf{e}_1$, $\mathbf{e}_2$, $\mathbf{e}_3$ are orthogonal and right handed, so that $\mathbf{e}_3 = \mathbf{e}_1 \times \mathbf{e}_2$. We shall also choose the same unit vectors $\mathbf{e}_2^{(r)}(\mathbf{x})$ and $\mathbf{e}_2^{(s)}(\mathbf{x})$: $\mathbf{e}_2^{(s)}(\mathbf{x}) = \mathbf{e}_2^{(r)}(\mathbf{x}) = \mathbf{e}_2(\mathbf{x})$, perpendicular to the plane specified by $\mathbf{e}_3^{(r)}(\mathbf{x})$ and $\mathbf{e}_3^{(s)}(\mathbf{x})$. Eq. (12) then becomes

$$h(\mathbf{x}, \boldsymbol{\xi}) = \varepsilon_{ijk}[p_i^{(s)} + p_i^{(r)}][B_{j1}^{(s)} + B_{j1}^{(r)}][B_{k2}^{(s)} + B_{k2}^{(r)}].$$

where ε_{ijk} is a Levi–Civitta symbol. After some manipulation, we obtain

$$h(\mathbf{x}, \boldsymbol{\xi}) = \frac{1}{c(\mathbf{x})}(1 + \cos\theta)\det\mathbf{D}, \tag{13}$$

where θ is the angle between the directions of rays to the source ($\mathbf{x}_s$) and to the receiver ($\mathbf{x}_r$) at point $\mathbf{x}$,

$$\cos\theta = e_{3i}^{(r)}(\mathbf{x})e_{3i}^{(s)}(\mathbf{x}). \tag{14}$$

The 2×2 matrix $\mathbf{D}$ has elements,

$$D_{IJ} = P_{IJ}^{(r)} + P_{IJ}^{(s)}, \tag{15}$$

with

$$P_{IJ}^{(r)} = e_{Lk}^{(r)}(\mathbf{x}_r)q_{kI}^{(r)}[\mathbf{Q}_2^{-1}(\mathbf{x}_r, \mathbf{x})]_{JL},$$
$$P_{IJ}^{(s)} = e_{Lk}^{(s)}(\mathbf{x}_s)q_{kI}^{(s)}[\mathbf{Q}_2^{-1}(\mathbf{x}_s, \mathbf{x})]_{JL}. \tag{16}$$

Equation (13) with (14)–(16) represent the final expression for the Beylkin determinant in an arbitrary source-receiver configuration.

4 SOURCE-RECEIVER MEASUREMENT CONFIGURATIONS

Equation (13) for the Beylkin determinant is very general. We now wish to specify it for specific source-receiver measurement configurations along a particular data surface, S_0. In the whole section, we shall denote general Cartesian coordinates by x_1, x_2, x_3.

We assume that the sources and receivers are distributed along a data surface S_0. In Cartesian coordinates, data surface S_0 is given by the relation

$$x_3 = f(x_1, x_2). \tag{17}$$

We consider seismic experiments in which the sources and receivers depend on each other. We first specify the positions of the receivers, and then determine the positions of the sources, depending in general on the positions of the receivers.

The positions of the receivers along data surface S_0 are determined by parametric equations,

$$x_{r_1} = x_{r_1}(\xi_1, \xi_2), \qquad x_{r_2} = x_{r_2}(\xi_1, \xi_2),$$
$$x_{r_3} = f(x_{r_1}(\xi_1, \xi_2), x_{r_2}(\xi_1, \xi_2)). \tag{18}$$

In (18), the first two parameteric equations determine the x_1- and x_2-Cartesian coordinates of the receiver specified by parameters ξ_1, ξ_2. The third coordinate x_{r_3} is chosen to situate the receiver on surface S_0, see (17).

The position of the sources depends on the positions of the receivers. We shall adopt very general relations,

$$x_{s_1} = a_{11}x_{r_1}(\xi_1, \xi_2) + a_{12}x_{r_2}(\xi_1, \xi_2) + b_1$$
$$x_{s_2} = a_{21}x_{r_1}(\xi_1, \xi_2) + a_{22}x_{r_2}(\xi_1, \xi_2) + b_2. \tag{19}$$

Here a_{IJ} and b_I ($I, J = 1, 2$) are constants. By proper selection of constants a_{IJ} and b_I, we can simulate very general source-receiver configurations used in seismic experiments. As the sources are also distributed along data surface S_0 given by (17),

$$x_{s_3} = f(a_{11}x_{r_1}(\xi_1, \xi_2) + a_{12}x_{r_2}(\xi_1, \xi_2) + b_1, a_{21}x_{r_1}(\xi_1, \xi_2) + a_{22}x_{r_2}(\xi_1, \xi_2) + b_2). \tag{20}$$

Using relations (17)–(20), we can compute $q_{iJ}^{(r)}$ and $q_{iJ}^{(s)}$, $i = 1, 2, 3, J = 1, 2$, see (10). We obtain four basic quantities $q_{IJ}^{(r)}$, $I, J = 1, 2$,

$$q_{IJ}^{(r)} = \frac{\partial x_{r_I}}{\partial \xi_J}. \tag{21}$$

All other expressions $q_{iJ}^{(r)}$ and $q_{ij}^{(s)}$ can then be expressed in terms of the four basic quantities (21),

$$q_{IJ}^{(s)} = a_{IK}q_{KJ}^{(r)},$$
$$q_{3J}^{(r)} = f_I^{(r)}q_{IJ}^{(r)}, \qquad q_{3J}^{(s)} = f_I^{(s)}a_{IK}q_{KJ}^{(r)}. \tag{22}$$

Here we have used the notation,

$$f_I^{(r)} = \left[\frac{\partial f(x_1, x_2)}{\partial x_I}\right]_{x_1 = x_{r_1}, x_2 = x_{r_2}},$$
$$f_I^{(s)} = \left[\frac{\partial f(x_1, x_2)}{\partial x_I}\right]_{x_1 = x_{s_1}, x_2 = x_{s_2}}. \tag{23}$$

Using the above relations, we can express $P_{IJ}^{(r)}$ and $P_{IJ}^{(s)}$ in (16) as follows,

$$P_{IJ}^{(r)} = Q_{JL}^{(r)-1}[e_{LK}^{(r)} + e_{L3}^{(r)}f_K^{(r)}]q_{KI}^{(r)},$$
$$P_{IJ}^{(s)} = Q_{JL}^{(s)-1}[e_{LN}^{(s)} + e_{L3}^{(s)}f_N^{(s)}]a_{NK}q_{KI}^{(r)}. \tag{24}$$

Hence,

$$P_{IJ}^{(r)} + P_{IJ}^{(s)} = q_{KI}^{(r)}\{Q_{JL}^{(r)-1}[e_{LK}^{(r)} + e_{L3}^{(r)}f_K^{(r)}] + Q_{JL}^{(s)-1}[e_{LN}^{(s)} + e_{L3}^{(s)}f_N^{(s)}]a_{NK}\}. \tag{25}$$

As we can see from (25) and (15), the expression for $\det\mathbf{D}$ in (13) can be factorized,

$$\det\mathbf{D} = \det\mathbf{D}^{(1)}\det\mathbf{D}^{(2)}, \tag{26}$$

where $\mathbf{D}^{(1)}$ and $\mathbf{D}^{(2)}$ are 2×2 matrices with elements

$$D_{IJ}^{(1)} = q_{IJ}^{(r)}, \qquad D_{IJ}^{(2)} = \bar{P}_{IJ}^{(r)} + \bar{P}_{IJ}^{(s)}, \tag{27}$$

and

$$\bar{P}_{IJ}^{(r)} = Q_{IL}^{(r)-1}[e_{LJ}^{(r)} + e_{L3}^{(r)}f_J^{(r)}],$$
$$\bar{P}_{IJ}^{(s)} = Q_{IL}^{(s)-1}[e_{LN}^{(s)} + e_{L3}^{(s)}f_N^{(s)}]a_{NJ}. \tag{28}$$

For plane data surface S_0 situated at $x_3 = 0$, see (17) with $f(x_1, x_2) = 0$, $f_J^{(r)} = f_N^{(s)} = 0$. Then

$$\bar{P}_{IJ}^{(r)} = Q_{IL}^{(r)-1}e_{LJ}^{(r)}, \qquad \bar{P}_{IJ}^{(s)} = Q_{IL}^{(s)-1}e_{LN}^{(s)}a_{NJ}. \tag{29}$$

The final equation for the Beylkin determinant now reads

$$h(\mathbf{x}, \boldsymbol{\xi}) = \frac{1}{c(\mathbf{x})}(1 + \cos\theta) \det \mathbf{D}^{(1)} \det \mathbf{D}^{(2)}, \tag{30}$$

see (26)–(28) for $\mathbf{D}^{(1)}$ and $\mathbf{D}^{(2)}$.

5 CONCLUSIONS

The derived eq. (30) for the Beylkin determinant $h(\mathbf{x}, \boldsymbol{\xi})$ can be used in an arbitrary 3-D, laterally varying, layered and blocked, isotropic structure. The limitations of eq. (30), of course, are given by the validity conditions of the ray method. This is, however, not a decisive limitation, as the Cohen–Bleistein 3-D seismic inversion integral (1) is also based on the ray principles. The equation can be also used for an arbitrarily curved, but smooth, data surface S_0.

In eq. (30), the Beylkin determinant $h(\mathbf{x}, \boldsymbol{\xi})$ is expressed in terms of the 2×2 matrix $\mathbf{Q}_2$ which can be simply evaluated by dynamic ray tracing along known rays. The method does not require the evaluation of the first and second derivatives of traveltimes, and/or numerical measurements of any elementary area connected with the ray field.

Equation (30) can be used for very general source-receiver configurations. We shall present here three important simple examples.

(a) In the zero offset configuration, we put

$$a_{11} = a_{22} = 1, \qquad a_{12} = a_{21} = b_1 = b_2 = 0.$$

(b) In the common source configuration, we have

$$a_{11} = a_{22} = a_{12} = a_{21} = 0, \qquad b_1 = x_{s1}, \qquad b_2 = x_{s2}.$$

(c) Finally, in the common offset configuration, we choose the relevant constants as follows:

$$a_{11} = a_{22} = 1, \qquad a_{12} = a_{21} = 0, \qquad b_1 = \Delta x_1, \qquad b_2 = \Delta x_2.$$

Here Δx_1 and Δx_2 denote the constant horizontal distances between the source and receiver in the $x_1 x_2$ plane, along the direction of the x_1 axis and the x_2 axis, respectively.

In certain special situations, (30) can be considerably simplified. For a homogeneous medium, we have $\mathbf{Q}_2(\mathbf{x}_s, \mathbf{x}) = c\,|\mathbf{x} - \mathbf{x}_s|\,\mathbf{I}$, $\mathbf{Q}_2(\mathbf{x}_r, \mathbf{x}) = c\,|\mathbf{x} - \mathbf{x}_r|\,\mathbf{I}$, where $\mathbf{I}$ is the 2×2 identity matrix. The unit vectors $\mathbf{e}_1, \mathbf{e}_2$ remain constant along the ray in this case and may be simply expressed in terms of x_i, x_{si} and x_{ri}. For a plane datum surface S_0, situated at $x_3 = 0$, we have

$$q_{3I}^{(r)} = q_{3I}^{(s)} = f_J^{(r)} = f_J^{(s)} = 0.$$

Finally, the equations can be also simplified for a 2.5-D case, when they lead to well known equations by Bleistein, Cohen & Hagin (1987).

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