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Kinematic inversion for qP- and qS-waves in inhomogeneous hexagonally symmetric structures

Jiří Jech and Ivan Pšenčík
Geophysical Institute, Czechoslovak Academy of Sciences, Boční II, 141 31 Praha 4, Czechoslovakia

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SUMMARY
We propose a kinematic inversion technique based on the first-order perturbation method for anisotropic media. In contrast to previous techniques, this one can be used to invert traveltimes of quasi-compressional as well as split quasi-shear waves, even if the initial unperturbed medium is isotropic. In principle, the technique is applicable to general 3-D laterally inhomogeneous anisotropic structures with an arbitrary degree of anisotropy. Here, however, for simplicity, media with hexagonal symmetry are considered. The effectiveness and limitations of the inversion technique are illustrated with synthetic VSP traveltime data. A simple linear variation of elastic parameters with depth is considered. The inversion scheme is tested both on data with pronounced shear wave splitting and data in which the shear wave splitting is negligible, the limiting case being data generated for a vertically inhomogeneous isotropic structure.

Key words: hexagonally symmetric media, quasi-P- and quasi-S-waves, traveltime inversion.

1 INTRODUCTION

Many traveltime inversion schemes for isotropic media are based on the simple formula for a first-order perturbation $\Delta\tau$, of the traveltime due to the slowness perturbation Δs,

$$\Delta\tau = \int_{\sigma_0}^{\sigma} \Delta s \, d\sigma'.$$

Here, the integration is performed between two points on the reference ray in the unperturbed medium, σ_0 and σ being the arclengths corresponding to the two points. The application of this formula to solving inverse problems was suggested e.g. by Romanov (1972). The formula is incorporated in the most widely applied inversion technique for isotropic media developed by Aki & Lee (1976) and their successors.

Červený (1982) and Hanyga (1982) proposed an extension of the above formula to inhomogeneous anisotropic media. Their formula for the traveltime perturbation $\Delta\tau$ due to the perturbation Δa_{ijkl} of density normalized elastic parameters a_{ijkl} along a ray path between two fixed points reads as follows:

$$\Delta\tau^{(m)} = -1/2 \int_{\tau_0}^{\tau} \Delta a_{ijkl} p_i^{(m)} p_l^{(m)} g_j^{(m)} g_k^{(m)} \, d\tau'. \qquad (1)$$

Here, $p_i = \partial\tau/\partial x_i$ and g_i are components of the slowness vector and of the unit particle motion vector in the unperturbed medium. The index m specifies the considered wave. Indices $m = 1$ and 2 correspond to qS_1- and qS_2-waves, and $m = 3$ corresponds to the qP-wave. The integration in (1) is performed along the unperturbed ray path.

Equation (1) can be used for any wave if the unperturbed medium is anisotropic and non-degenerate, i.e. if all the three waves propagating in the anisotropic medium propagate with different phase velocities. In the case of an isotropic unperturbed medium, equation (1) can only be used without any additional considerations for quasi-compressional waves. Therefore, the applications of formula (1) mostly concern the quasi-compressional waves. This was also true for inversion schemes; see, e.g., Hirahara & Ishikawa (1984) and Jech (1988).

Equation (1) cannot be used for quasi-shear waves in the case of an isotropic unperturbed medium, since an isotropic medium is degenerate (phase velocities of both shear waves coincide). In this case, the particle motion vectors of quasi-shear waves are not uniquely determined. The only constraint on the particle motion vectors is that they must be mutually perpendicular in the plane perpendicular to the vector $g_i^{(3)}$.

Jech & Pšenčík (1989) proposed a generalization of formula (1), which can be used for the determination of the traveltime perturbation of quasi-compressional as well as split quasi-shear waves even if the initial unperturbed medium is isotropic. The generalized formula reads

$$\Delta\tau^{(m)} = -1/4 \int_{\tau_0}^{\tau} \{(D_{11} + D_{22}) \pm [(D_{11} - D_{22})^2 + 4D_{12}^2]^{1/2}\} \, d\tau',$$

where

$$D_{MN} = \Delta a_{ijkl} p_i^{(m)} p_l^{(m)} e_j^{(M)} e_k^{(N)}, \qquad M, N = 1, 2.$$

The vectors $e_i^{(1)}, e_i^{(2)}$ are two mutually perpendicular unit vectors chosen arbitrarily in the plane perpendicular to the vector $g_i^{(3)}$. Recently, Nowack & Pšenčík (1991) showed that equation (1) could be used even in the degenerate case if the particle motion vectors $e_i^{(1)}, e_i^{(2)}$ were chosen so that

$$D_{12} = \Delta a_{ijkl} p_i^{(m)} p_l^{(m)} e_j^{(1)} e_k^{(2)} = 0, \tag{2}$$

where $m = 1$ or 2. Equation (2) is the sixth constraint (in addition to five conditions of orthonormality of vectors $e_i^{(1)}, e_i^{(2)}, g_i^{(3)}$) on the six components of the vectors $e_i^{(1)}, e_i^{(2)}$ so that they can be determined uniquely.

2 ANISOTROPIC MEDIUM WITH HEXAGONAL SYMMETRY

Let us consider an anisotropic medium with hexagonal symmetry described by five independent elastic parameters $A_{11}, A_{33}, A_{55}, A_{66}$ and A_{13}. A condensed notation for elastic parameters is used such that $A_{11} = a_{1111}$, $A_{33} = a_{3333}$, $A_{55} = a_{1313}$, $A_{66} = a_{1212}$ and $A_{13} = a_{1133}$. The parameters are specified in a local Cartesian coordinate system x_i', the axis of symmetry being parallel to the local x_3' axis. Let us further consider a general Cartesian coordinate system x_i with axes x_1 and x_2 situated in a horizontal plane and axis x_3 vertical, positive downwards. The axis of hexagonal symmetry of the considered anisotropic medium (i.e., x_3' axis) can be arbitrarily orientated in the general Cartesian coordinate system. The orientation of the x_3' axis can be specified by angles α_0 and β_0. Angle α_0 $(0 \le \alpha_0 \le \pi)$ is the angle between positive axes x_3 and x_3' of the general and local Cartesian coordinate systems, measured positively from the general x_3 axis. Angle β_0 $(0 \le \beta_0 \le 2\pi)$ is the angle between the projection of the positive x_3' axis into the horizontal plane and the positive axis x_1. The angle β_0 is measured positively anticlockwise from x_1. For this choice of angles α_0 and β_0 the x_3' axis is specified by the following unit vector:

$$(\sin\alpha_0 \cos\beta_0, \sin\alpha_0 \sin\beta_0, \cos\alpha_0).$$

In an inhomogeneous medium, the values of angles α_0 and β_0 generally can vary from point to point.

In media with hexagonal symmetry the propagation properties only depend on the cosine, denoted by Θ, of the angle between the phase normal and the axis of symmetry. For a wave with the phase normal

$$(\sin\alpha \cos\beta, \sin\alpha \sin\beta, \cos\alpha),$$

the quantity Θ is given by

$$\Theta = \sin\alpha \sin\alpha_0 \cos(\beta - \beta_0) + \cos\alpha \cos\alpha_0. \tag{3}$$

The angles α and β are defined in the same way as α_0 and β_0. For the above specifications, if condition (2) is satisfied, equation (1) reduces to

$$\tau^{(1)} = -1/2 \int_{\tau_0}^{\tau} D_1\, d\tau', \qquad \tau^{(2)} = -1/2 \int_{\tau_0}^{\tau} D_2\, d\tau',$$
$$\tau^{(3)} = -1/2 \int_{\tau_0}^{\tau} D_3\, d\tau, \tag{4}$$

for qS_1-, qS_2- and qP-waves, respectively. In (4),

$$\begin{aligned} D_1 &= v_S^{-2}[\Theta^4(2C_5 - C_1 - C_2) + \Theta^2(C_1 + C_2 - 2C_5) + C_3], \\ D_2 &= v_S^{-2}[\Theta^2(C_3 - C_4) + C_4], \\ D_3 &= v_P^{-2}[\Theta^4(C_1 + C_2 - 2C_5) + 2\Theta^2(C_5 - C_1) + C_1], \\ C_1 &= \Delta A_{11}, \quad C_2 = \Delta A_{33}, \quad C_3 = \Delta A_{55}, \quad C_4 = \Delta A_{66}, \\ C_5 &= \Delta A_{13} + 2\Delta A_{55}, \end{aligned} \tag{5}$$

Symbols v_P and v_S in (5) denote P- and S-wave velocities in the initial unperturbed isotropic medium.

The goal of the inversion is the determination of angles α_0 and β_0 specifying the orientation of the axis of symmetry and coefficients C_i related to the perturbations of the elastic parameters, see (5), which minimize the function

$$\Omega = \sum_{i=1}^{N} (\tau_i^{\text{obs}} - \tau_i^{\text{cal}} - \Delta\tau_i)^2. \tag{6}$$

Here τ_i^{obs} is the ith of N observed traveltimes, τ_i^{cal} is the ith of N calculated traveltimes for an initial unperturbed isotropic medium and $\Delta\tau_i$ is the ith traveltime perturbation of the corresponding considered wave, given by equation (4). Traveltime perturbations depend linearly on perturbations of elastic parameters, see (4) and (5), and non-linearly on the angles α_0 and β_0 specifying the orientation of the axis of symmetry; see (4) and (3). Therefore, the Levenberg–Marquardt least squares algorithm (Bevington 1969), is used to find the parameters minimizing Ω in equation (6). The algorithm is iterative, and the result of each iterative step is the new estimate of the vector $\mathbf{x}$ formed from the sought parameters C_i and the angles α_0 and β_0 (altogether M unknowns):

$$\mathbf{x} = (\mathbf{B}^{\mathrm{T}}\mathbf{B} + \lambda\mathbf{I})^{-1}\mathbf{B}^{\mathrm{T}}\mathbf{y}. \tag{7}$$

In (7), $\mathbf{y}$ is a vector of dimension N containing differences $\tau_i^{\text{obs}} - \tau_i^{\text{cal}}$, $\mathbf{B}$ is $N \times M$ matrix of the partial derivatives of traveltimes with respect to the sought parameters, $\mathbf{I}$ denotes the $M \times M$ identity matrix and λ is a damping factor. The value of the damping factor varies during the iterative process. It is greater in the beginning, when we are far from the minimum of the sum of squares. For such λ the method resembles the gradient method. The parameter λ decreases step by step while the minimum of sum of squares is approached. For small values of λ the method resembles the linearization method, which converges well in the neighbourhood of the minimum. In the beginning of iterations, the values of λ are usually $\lambda = 0.001$–0.01. If the sum of squares of residuals in the next iterative step is less than in the previous step, λ is divided by 2–10; if not, λ is multiplied by 2–10. The iterative procedure is terminated when a stop criterion is satisfied, e.g. if in two successive iterations elastic parameters and angles of orientation of the axis of symmetry agree, component by component, to a prescribed number of digits, or if the sum of squares of residuals changes from iteration to iteration by less than a prescribed value. A typical number of iterations in the Levenberg–Marquardt least-squares algorithm in the following numerical examples was 8–10.

The perturbations of elastic parameters and the angles of orientation of the axis of symmetry determined in this way could be used to construct a refined model, which could serve as an unperturbed model for the next iteration of the inversion procedure. In this paper, however, we limit ourselves to the first iteration of the inversion scheme, which is easy to perform and, as shown in the following, can give sufficiently good results.

3 NUMERICAL EXAMPLES

The above-described inversion procedure based on the first-order perturbation theory for anisotropic media was used to invert synthetic VSP traveltime data in several numerical experiments. A procedure for tracing rays through 3-D inhomogeneous anisotropic structures (Gajewski & Pšenčík 1990) was used to generate the 'observed' traveltimes. The elastic parameters proposed by Shearer & Chapman (1989) for the surface of their effectively anisotropic earth's crust model were used for generating synthetic traveltimes.

The models considered in this paper are described by the five elastic parameters $A_{11} = 19.11$, $A_{33} = 11.91$, $A_{55} = 5.10$, $A_{66} = 6.38$, $A_{13} = 4.40\ \mathrm{km^2\,s^{-2}}$, specified at the surface. A constant vertical gradient of elastic parameters is introduced in such a way that the anisotropy of the medium vanishes at a depth of 5 km and the medium there becomes isotropic with $v_P = 5.5\ \mathrm{km\,s^{-1}}$ and $v_S = 3.28\ \mathrm{km\,s^{-1}}$. Two different orientations of the axis of symmetry, constant throughout the model, were successively considered. Synthetic traveltimes of direct qP-, qS_1- and qS_2-waves were computed for the following VSP configuration: three sources distributed along a horizontal surface profile at distances of 0.25, 0.5 and 0.75 km from the borehole and 25 receivers distributed along a vertical profile in the depth range of 0.1–0.7 km with a step of 25 m. Computed traveltimes served as observations in the solution of the inverse problem. As an illustration, Fig. 1 shows some of the rays of qP-waves in the model under consideration.

In the first experiment, the axis of symmetry was situated in the vertical plane containing the surface profile of sources, i.e., the angle β_0 was specified $\beta_0 = 0^0$. The angle α_0 (denoted by EFI in Fig. 2), made by the axis of symmetry and the vertical, was 60°. As an initial model for the inversion, a homogeneous isotropic model with $v_P = 3.7\ \mathrm{km\,s^{-1}}$ and $v_S = 2.32\ \mathrm{km\,s^{-1}}$ was considered. These values were derived from approximately averaged traveltimes from all the three sources to the uppermost and lowest receivers. The sought parameters were five values of elastic parameters at the surface, five values of constant vertical gradients of these elastic parameters and the angle α_0 (denoted by PFI in Fig. 2), specifying the orientation of the axis of symmetry in the vertical plane containing the profile of sources.

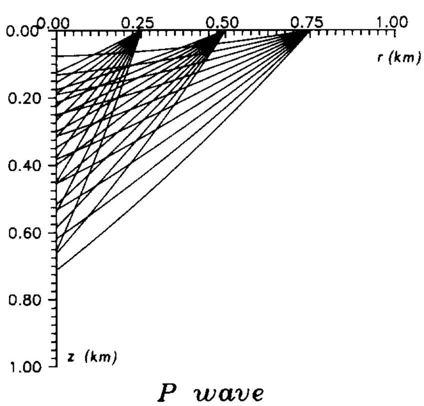


Figure 1. Configuration of the VSP experiment with the ray diagram of the qP-wave in the anisotropic model of hexagonal symmetry. The axis of symmetry is situated in the vertical plane containing the profile of sources and makes 60° with the vertical. Elastic parameters vary linearly with depth; their values are specified in the text.

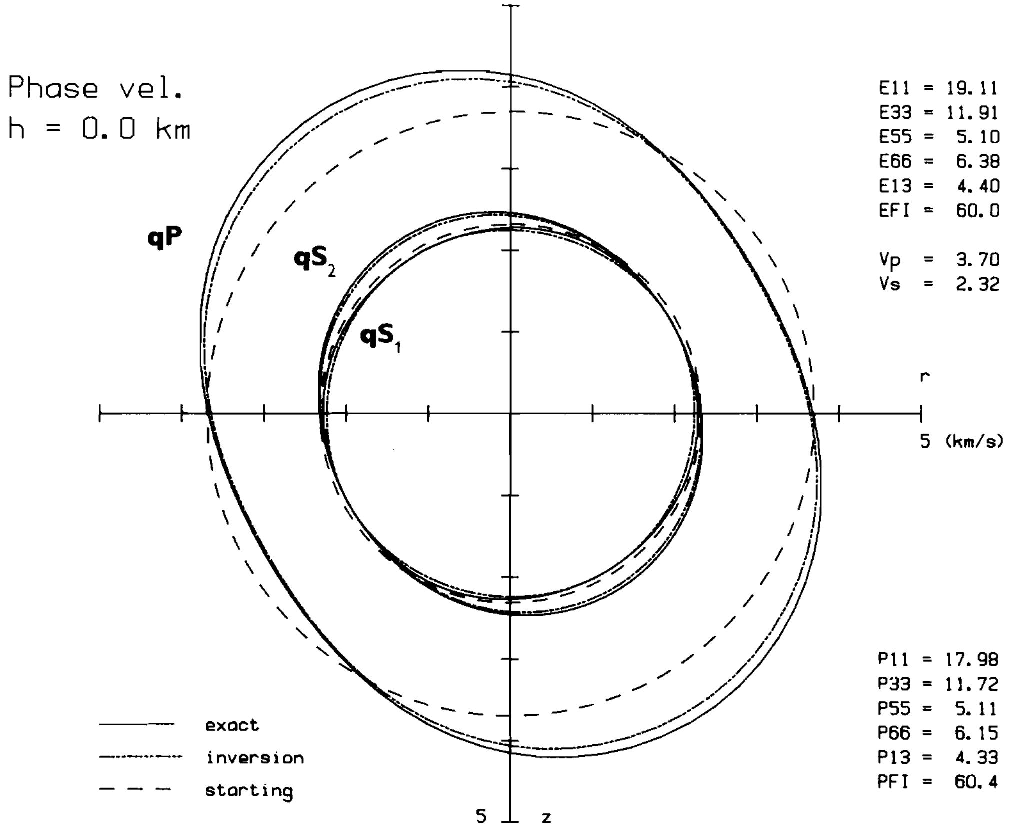
Figure 2 shows the cross-sections of the qP, qS_1 and qS_2 phase velocity surfaces with the vertical plane containing the profile of sources. The sections are taken at the surface of the model (top) and at a depth of 0.7 km (bottom), which is the depth of the deepest receiver. The exact phase velocities which were used for the computation of 'observed' arrival times are denoted by solid lines. The velocities of the initial isotropic model are denoted by sparse dashed lines and the inverted phase velocities are denoted by dense dashed lines. On the right-hand sides of graphs, values of exact (denoted by E11, E33, . . . , EFI) and inverted (denoted by P11, P33, . . . , PFI) elastic parameters and angles α_0 of the axis of symmetry are shown. Although the initial unperturbed model is homogeneous and isotropic, the inverted results are surprisingly close to the exact ones. (There seems to be no reason why the inverted model should not have different anisotropy and inhomogeneity from the exact one.) Even the angle of orientation of the axis of symmetry is determined very well. Greater differences for all the waves can be observed around the direction from the upper left to the lower right. The reason for this are larger differences in phase velocities between the starting and exact models in the discussed direction. The sum of squares of residuals

$$\sum_{i=1}^{N} (\tau_i^{\mathrm{obs}} - \tau_i^{\mathrm{cal}})^2$$

for the inverted model is 8 per cent of that for the unperturbed medium.

Because the phase velocity sheets of the quasi-shear waves are very close to each other in the direction in which waves travel from the sources towards receivers, the corresponding traveltimes differ by no more than several milliseconds. It would be very difficult to distinguish the arrival times of the two quasi-shear waves in real seismograms. Therefore, traveltimes of the faster qS-wave were taken as the traveltimes of both quasi-shear waves. Nevertheless, the inversion resulted in two distinct qS phase velocity sheets. This is a consequence of the joint qP and qS traveltime inversion.

Figure 3 shows the same results as above, only in another graphic form. It shows the distribution of exact (solid lines) and inverted (dashed lines) horizontal (||) and vertical ($^{\perp}$) phase velocities with depth for qP-, qS_1- and qS_2-waves. Apparently great deviations of exact and inverted velocity–depth profiles are caused by a very detailed velocity scale. In fact, at the surface, the maximum deviations are about 2 and 1.5 per cent for vertical qP and qS_2 phase velocities. The corresponding deviations of the exact vertical phase velocities and initial unperturbed velocities are at the same time about 11 and 7 per cent.



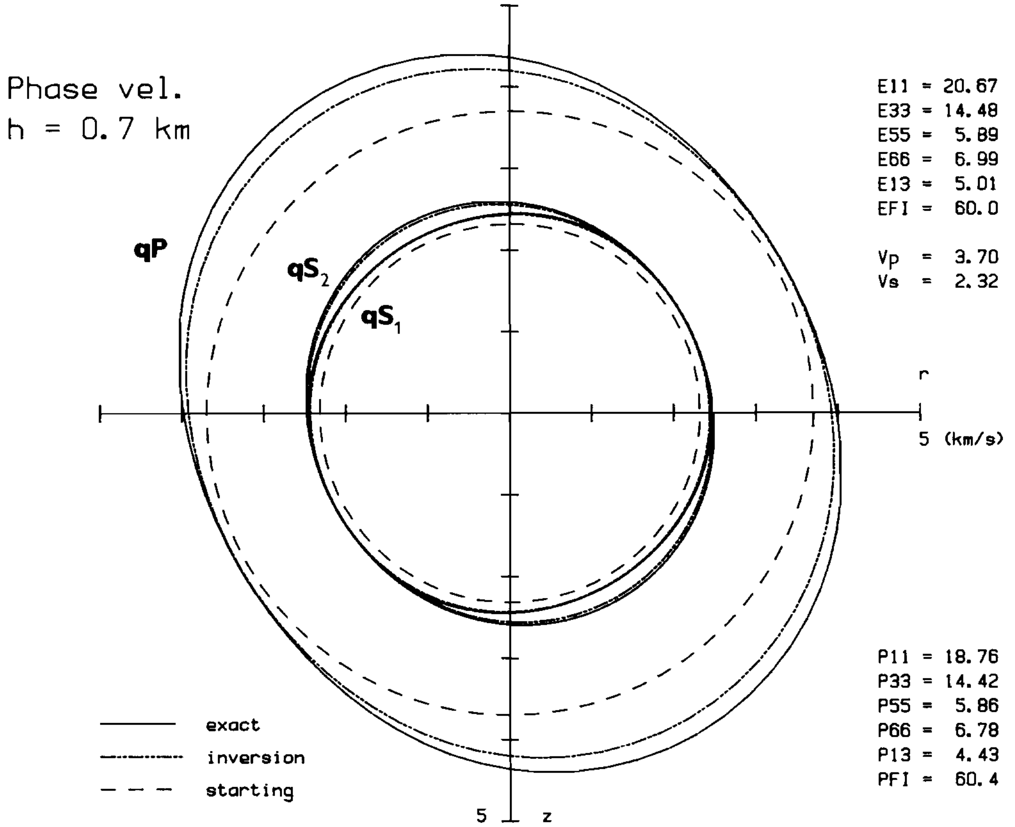


Figure 2. Inversion of traveltime data computed for the anisotropic model shown by solid line. Cross-sections of the qP, qS_1 and qS_2 phase velocity surfaces with a vertical plane containing the profile of sources at the surface (top) and at the depth of 0.7 km (bottom). Exact (E11, E33, E55, E66, E13 and EFI) and inverted (P11, P33, . . . , PFI) values of elastic parameters and the angle α_0 of the axis of hexagonal symmetry are shown on the right-hand side. Symbols v_P and v_S denote P- and S-wave velocities of the isotropic homogeneous starting model.

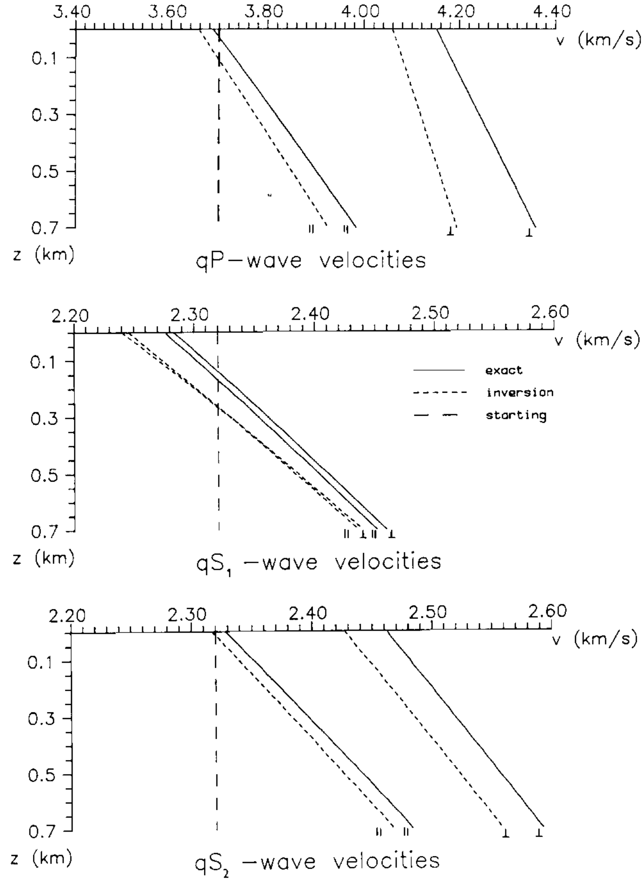


Figure 3. Inversion of traveltime data computed for the anisotropic model shown in Fig. 2. The distribution of horizontal (||) and vertical ($^{\perp}$) exact and inverted qP and qS phase velocities with depth and of P- and S-wave velocities in the isotropic starting model.

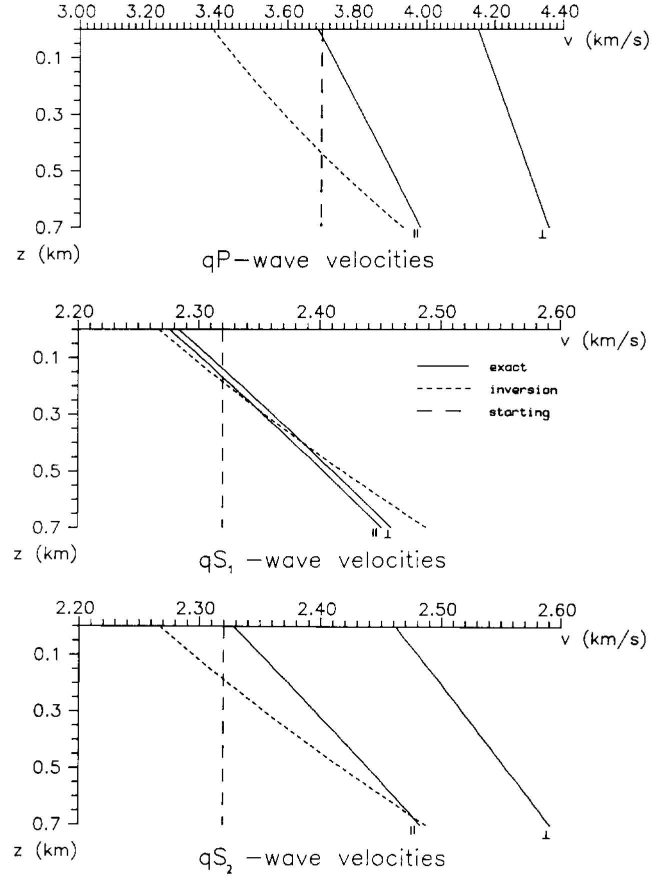


Figure 4. Inversion of traveltime data computed for the *anisotropic* model from Fig. 2 under the assumption of *isotropy* of the sought model. The distribution of horizontal (||) and vertical ($^{\perp}$) qP and qS phase velocities and of P-and S- wave velocities with depth in the anisotropic and inverted isotropic models and of P- and S-wave velocities in the isotropic starting model.

In the next numerical experiment, the above synthetic traveltimes computed for the anisotropic structures were inverted under the assumption of isotropy and a constant vertical gradient of the square of the slowness of the sought model. The assumption of isotropy of the model could be deduced from the indistinguishable arrival times of the two quasi-shear waves. Traveltimes of the faster qS-wave were considered. The results of inversion are shown in Fig. 4. The resulting 'isotropic' velocity–depth distribution does not represent an average of horizontal and vertical phase velocities (note that horizontal and vertical phase velocities do not correspond to maximum and minimum velocities, see Fig. 2) of the exact medium. Such a result would be obtained for a transversely isotropic medium with a vertical axis of symmetry, see, e.g., Červený & Pšenčík (1972). Thus, due to the inclination of the axis of symmetry in the experiment presented here, the 'isotropic' inversion yields completely distorted results.

In another numerical experiment, synthetic traveltimes were computed for an isotropic structure with a constant vertical gradient of P- and S-wave velocity. The configuration of the experiment was the same as in the previous cases. Assuming that S-wave arrival times represent traveltimes of coinciding qS_1- and qS_2-waves, the synthetic data were inverted using the 'anisotropic' inversion scheme. The results for the depths of 0 and 0.7 km are shown in Fig. 5. The inverted model differs only very slightly from the exact isotropic model. This is somewhat surprising since there seemed to be no reason why a part of the model inhomogeneity should not transform into anisotropy during the inversion procedure.

Figure 6 shows vertical sections of phase velocity surfaces at depths of 0 and 0.7 km for the same experiment as in Fig. 2 but for different anisotropy. In this case the axis of symmetry was specified by angles $\alpha_0 = 30°$ and $\beta_0 = 90°$, i.e., the axis of symmetry was situated in the vertical plane perpendicular to the profile of sources on the surface, and inclined 30° from the vertical. For this configuration, there were observable differences between traveltimes of both quasi-shear waves. As in Fig. 2, the agreement of inverted results with exact data is very good. In this case the sum of squares of time residuals

$$\sum_{i=1}^{N} (\tau_i^{\mathrm{obs}} - \tau_i^{\mathrm{cal}})^2$$

for the inverted model is only 0.6 per cent of that for the unperturbed model.

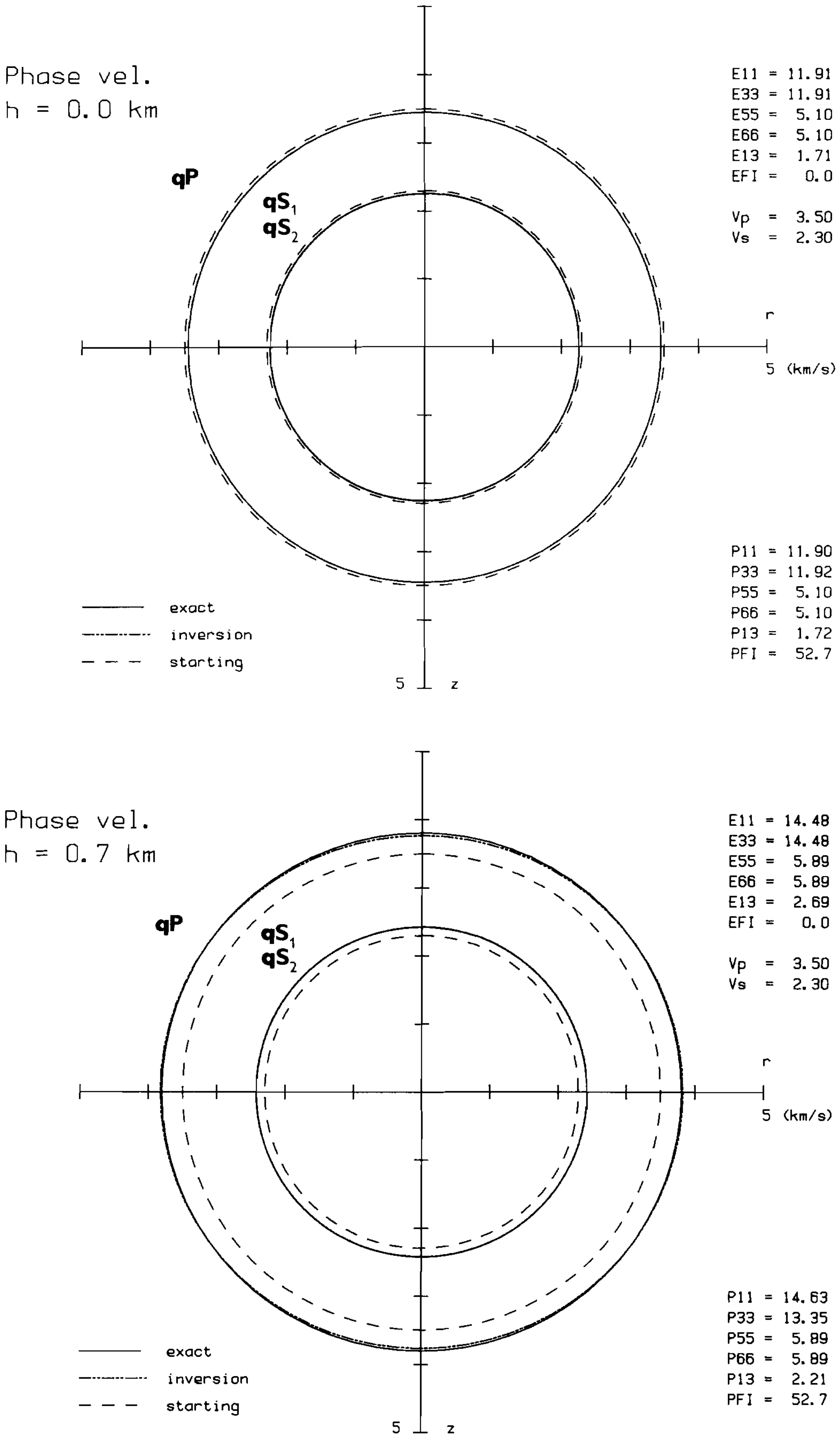
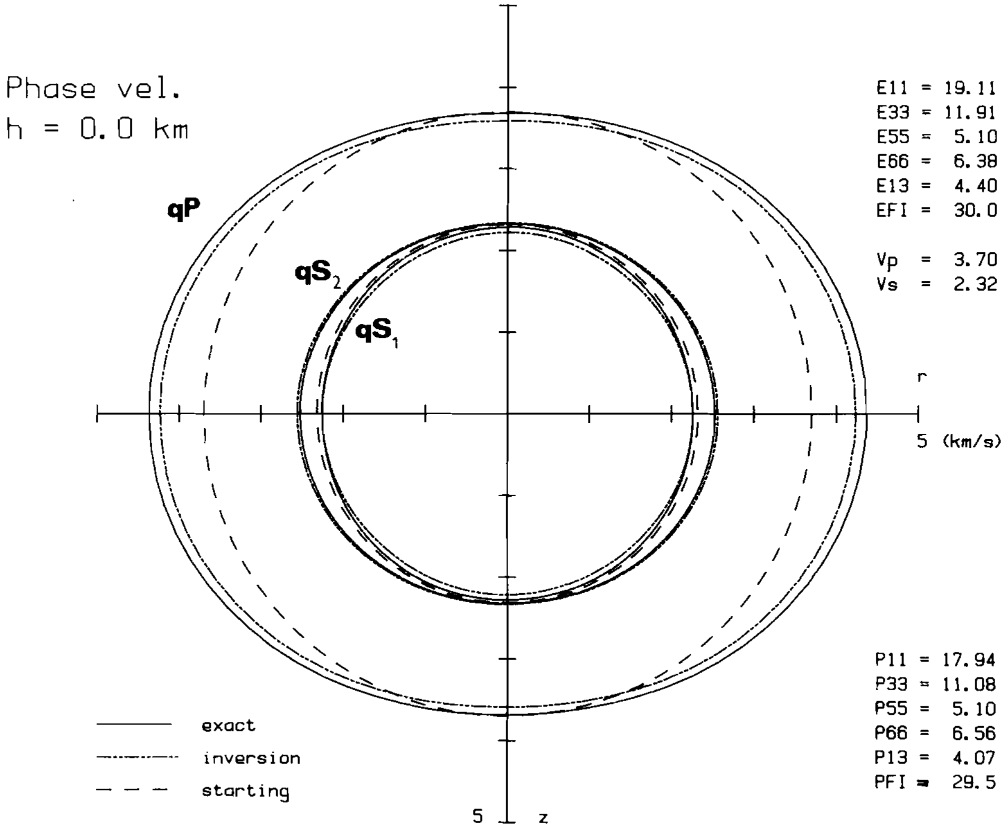


Figure 5. Inversion of traveltime data computed for the *isotropic* model shown by solid lines under the assumption of *anisotropy* of the sought model. Symbols and configuration of the figure are the same as in Fig. 2.



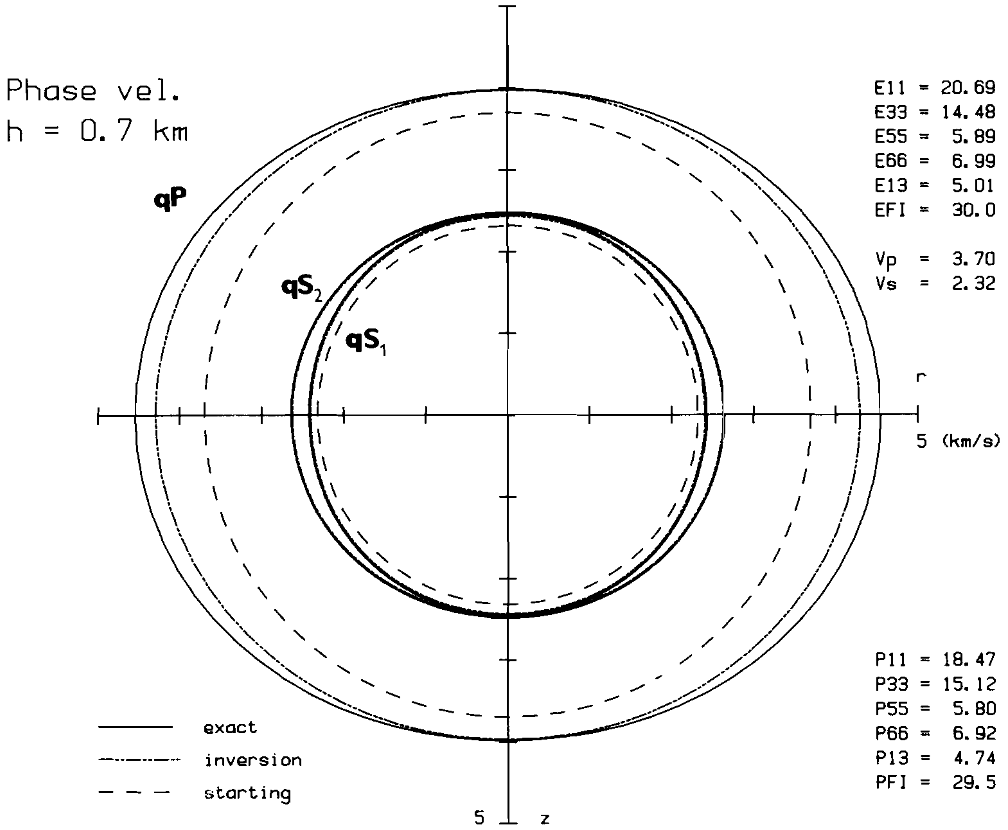


Figure 6. Inversion of traveltime data computed for the anisotropic model shown by solid lines. The same as in Fig. 2 but for an anisotropic model with the axis of hexagonal symmetry situated in the vertical plane perpendicular to the profile of sources. The axis is inclined 30° from the vertical.

4 DISCUSSION

There are several possibilities for solving inverse problems for inhomogeneous anisotropic media.

(a) The traveltime data can be first inverted by 'isotropic' inversion which will yield an inhomogeneous isotropic structure. This structure can be used as an initial unperturbed medium to solve the inverse problem with the use of an 'anisotropic' inversion scheme. As was shown in Fig. 4, this may lead to quite distorted results, which will not improve after an 'anisotropic' inversion. In the example shown in Fig. 4, after the 'isotropic' inversion it was even impossible to apply 'anisotropic inversion' since there was practically no misfit of 'observed' data and data corresponding to the inverted isotropic vertically inhomogeneous model.

(b) Červený & Simões-Filho (1991) suggest a completely different order of operations for fractorized anisotropic media. They propose to invert first the time delays between *qS*-waves into elastic parameters of the factorized anisotropic medium. In the second step structural inversion can be performed. In an experiment resembling this procedure (results of this experiment are not presented in this paper), the synthetic traveltime data were first inverted into a homogeneous anisotropic structure. This structure was then used as an initial unperturbed medium for the 'anisotropic' inversion. A structure with slightly different anisotropy and a considerably weaker vertical gradient than in the exact model was obtained.

(c) The experiments described in this paper support the idea of 'simultaneous' inversion, i.e. inversion in which anisotropy as well as structural inhomogeneity are sought simultaneously. In such cases the inverted results were, at least for the simple examples presented here, very close to the exact models no matter whether these models were isotropic or anisotropic. In order to make this conclusion general, further testing on more complicated models is, of course, necessary.

In many other experiments different from those described above, the procedure was tested on traveltime data generated by only two or even one source. Reduction of the number of sources leads to greater dependence of quality of the inversion on the starting model. A similar phenomenon, i.e. greater dependence of quality of the inversion on the starting model, was observed when the axis of symmetry was oriented arbitrarily in space, i.e., both angles α_0 and β_0 specifying the axis were sought. The procedure was also tested on *qP*-wave traveltime data only. The results confirmed what can be seen from equations (5). Only the orientation of the axis of symmetry and values of the quantities A_{11}, A_{33} and $A_{13}+2A_{55}$ were determined.

The proposed inversion scheme was applied to the real data obtained in The Geysers experiment in California (Daley *et al.* 1989). Although the measured data were of very high quality, the application of the inversion procedure was complicated by the existence of a traveltime data set from only one source. Consequently, several quite different models were obtained, depending on the choice of the initial unperturbed model, which fitted observed traveltimes with good accuracy. This ambiguity could possibly be reduced by forward modelling of synthetic seismic wavefields and their comparison with measured seismograms.

5 CONCLUSIONS

An inversion procedure based on the first-order perturbation theory for anisotropic media is proposed. Arrival times of all three waves propagating in anisotropic media can be used for a simultaneous determination of anisotropy and structural inhomogeneity of the medium. The procedure works well even if shear wave splitting is unobservable. Simple tests described in this paper seem to indicate that 'simultaneous' inversion leads to the best results and that anisotropy and heterogeneity should not be sought independently; see the discussion in the previous section. Although the presented procedure is based on the first iteration of the inverse scheme only (an iterative inversion procedure is under preparation), it can be used for inverting models with rather strong anisotropy. The performance of the inversion scheme will, of course, improve when an iterative procedure is used. Since the procedure is based on kinematics only, it is less affected by problems connected with the existence of singularities of the wave propagation in anisotropic media.

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