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Numerical modelling and inversion of travel times of seismic body waves in inhomogeneous anisotropic media

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Summary. Two approaches to travel-time computations in laterally inhomogeneous anisotropic media are presented. The first method is based on ray tracing in an anisotropic inhomogeneous medium, the second on the linearization procedure. The linearization procedure, which can be applied to inhomogeneous, slightly anisotropic media, does not require ray tracing in an anisotropic medium. Applications of linearized equations to the solutions of direct and inverse kinematic problems are discussed. A program package to perform the linearized computations for rather general 2-D laterally inhomogeneous layered structures is described and a numerical example is presented.

1 Introduction

The most important peculiarities of seismic body waves now used for investigating the anisotropic properties of the Earth's crust and the upper mantle are as follows:

(1) The azimuthal dependence of the travel-time field as observed along special observational systems on the Earth's surface. This azimuthal dependence has been used mainly to investigate the anisotropy of velocities of the quasi-compressional waves, but can also be used for quasi-shear waves.

(2) The time delay between two quasi-shear waves (shear waves splitting).

(3) The polarization effects and polarization anomalies of *S*-waves.

In most cases the anisotropic effects are relatively weak, hidden in, or at least combined with effects due to the lateral variability of the Earth's structure. It is natural to assume that for laterally inhomogeneous media the anisotropy may also vary in all directions. Not only the relative amount of anisotropy, but also the symmetry-system and orientation of the preferred axes change from place to place. Trying to investigate the anisotropic properties of the Earth's interior under the simplified assumption that the real medium is homogeneous may lead to serious errors. A more thorough approach must consider both these effects together, in laterally inhomogeneous anisotropic medium.

There are not very many methods available for numerical modelling of seismic body-wave fields in inhomogeneous anisotropic media. In fact, only high-frequency asymptotic methods (such as the ray method and its modifications) are general enough to solve the problem. In principle, ray tracing, travel-time, amplitude and polarization computations can be performed for arbitrarily laterally inhomogeneous anisotropic media where all the 21 elastic parameters change with all the three coordinates (Červený 1972; Petrashen 1980; Crampin 1981). The computations are simple and straightforward, but rather lengthy. They may be applied mainly to the solution of direct problems. It would be, however, hardly possible to use them to solve the inverse problem for general inhomogeneous anisotropic media as numerical modelling would be very time-consuming.

A simpler procedure for solving certain direct and inverse kinematic problems for inhomogeneous slightly anisotropic media is based on linearization. A linearization procedure was successfully used in the inversion of travel-time data for laterally heterogeneous isotropic structures (Firbas 1981) and may be used, in principle, for any slightly anisotropic medium. The linearization equations applicable to any seismic body wave propagating in a general, slightly anisotropic, laterally inhomogeneous medium were derived by Červený (1982) (see also Červený & Jech 1983). They provide us with a simple method of evaluating the change in the travel time between two specified points M^0 and M due to a slight change in the elastic parameters. The travel-time correction is expressed as an integral over the elastic parameter corrections (perturbations), multiplied by some weighting function. The integral is evaluated along the 'unperturbed' ray from M^0 to M, i.e. the ray computed in the unperturbed medium. Thus, to evaluate the travel-time correction caused by a slight perturbation of the elastic parameters, it is not necessary to evaluate the new ray in the perturbed medium. The relations between the travel-time correction and the elastic parameter corrections are linear. The corrections in elastic parameters may, of course, vary from place to place in the medium. In other words, they include both the effects of anisotropy and inhomogeneity. Thus, in the travel-time computation in an inhomogeneous anisotropic medium, the problem of discrepancy between anisotropic effects and the effects of inhomogeneity practically loses its meaning.

Let us emphasize one important point. As mentioned above, the travel-time corrections are obtained by an integration along the unperturbed ray. This, however, does not imply that the changes in the ray trajectory are in some sense second-order effects with respect to the travel-time corrections. The travel-time correction may remain small even when the ray trajectory from M^0 to M in the perturbed medium deviates considerably from the corresponding ray trajectory in the unperturbed medium. The method used to derive the linearized equations for travel-time corrections, presented in this paper, cannot be used to determine the perturbed ray, the change in the polarization of the displacement vector and the ray amplitudes. A more sophisticated approach must be used to solve the problem, starting with the elastodynamic equations. This approach will be described elsewhere.

2 Ray tracing in inhomogeneous anisotropic media

Methods of ray tracing and ray amplitude computation in inhomogeneous anisotropic media are well known. We shall therefore present only the most important conclusions and equations which we shall need in the following sections. For details see Babich (1961), Červený (1972), Červený, Molotkov & Pšenčík (1977), Petrashen (1980) and Crampin (1981).

We shall consider an inhomogeneous anisotropic medium described by 21 elastic parameters c_{ijkl} and density ρ. The elastic parameters c_{ijkl} and density ρ and their first and second spatial derivatives are assumed to be continuous functions of rectangular Cartesian coordinates x_i. For simplicity, we do not consider the structure interfaces here, but all the results may be modified even for laterally varying layered structures as in the standard ray theory for isotropic media.

Instead of elastic parameters c_{ijkl} we shall also use the parameters

$$a_{ijkl} = c_{ijkl}/\rho, \tag{1}$$

and we shall also refer to them as elastic parameters, for simplicity. The dimension of a_{ijkl} is (distance/time)^2, so that it corresponds to the square of velocity.

Let us describe any wavefront of a high-frequency body wave propagating in an anisotropic inhomogeneous medium by the equation

$$\tau(x_i) = t, \tag{2}$$

where t is the time. We also denote by p the slowness vector with components

$$p_i = \partial\tau/\partial x_i. \tag{3}$$

The slowness vector p has the direction of the unit normal N to the wavefront. We can then write $p_i = N_i/V$, where V denotes the phase (normal) velocity.

A basic role in the ray theory of eleastic waves in an inhomogeneous anisotropic medium is played by the symmetric 3×3 matrix Γ with elements given by the relation,

$$\Gamma_{jk} = p_i p_l a_{ijkl}, \qquad (i, j, k, l = 1, 2, 3). \tag{4}$$

Note that the Einstein summation convention is used throughout this paper. The matrix Γ is often called the Christoffel matrix. We now denote by $U_i (i = 1, 2, 3)$ the components of the displacement vector of the high-frequency body wave under consideration, in the zeroth-order ray approximation. It is not difficult to show that U_k satisfy the following equations,

$$(\Gamma_{jk} - \delta_{jk}) U_k = 0, \tag{5}$$

where δ_{jk} is the Kronecker symbol, $\delta_{jk} = 1$ for $j = k$, $\delta_{jk} = 0$ for $j \neq k$.

As we can see from equation (5), the determination of the kinematic properties of a high-frequency elastic wave propagating in an inhomogeneous anisotropic medium is a typical eigenvalue problem. Let us denote the three eigenvalues of Γ by $G_m (m = 1, 2, 3)$ and the corresponding eigenvectors by $g^{(m)} (m = 1, 2, 3)$. The eigenvalues G_m are the three solutions of the characteristic equation

$$\text{Det}\,(\Gamma_{jk} - \delta_{jk} G_m) = 0, \tag{6}$$

and the eigenvectors can be determined from the equations

$$(\Gamma_{jk} - \delta_{jk} G_m) g_k^{(m)} = 0, \qquad j = 1, 2, 3 \tag{7}$$

(no summation over m). We assume that $G_1 \neq G_2 \neq G_3$.

Equation (5) is satisfied if any of the three eigenvalues G_m is equal to 1,

$$G_m(p_1, p_2, p_3, x_1, x_2, x_3) = 1. \tag{8}$$

This follows immediately from the comparison of equations (5) and (6). Since $p_i = \partial\tau/\partial x_i$, equation (8) is a non-linear partial differential equation for $\tau(x_i)$. As in isotropic media, we shall refer to equation (8) as the eikonal equation.

Thus, in an inhomogeneous anisotropic media (when a_{ijkl} and their first and second spatial derivatives are continuous functions of coordinates), three independent wavefronts may propagate. One of them (say, $G_1 = 1$) corresponds to the so-called quasi-compressional wave qP, the other two to the two different quasi-shear waves $qS1$ and $qS2$ (say, $G_2 = 1$ and

$G_3 = 1$). Note that in inhomogeneous isotropic media $G_1 = \alpha^2 p_i p_i$, $G_2 = G_3 = \beta^2 p_i p_i$, where α and β are the velocities of P- and S-waves, respectively.

It also follows from (5) and (7) that the displacement vectors corresponding to these three waves have the direction of the eigenvectors $\mathbf{g}^{(m)}$; $m = 1, 2, 3$.

As the matrix Γ is positive-definite (see Babich 1961), the eigenvalues $G_m (m = 1, 2, 3)$ are real-valued and positive. The functions G_m are homogeneous functions of the second order in p_i. Inserting $p_i = N_i/V$, equation (8) immediately yields the relation for the phase velocity

$$V = [G_m(N_i, x_i)]^{1/2}. \tag{9}$$

Note that the eignvalues G_m can be expressed simply in terms of the components of the eigenvectors $\mathbf{g}^{(m)}$. Equation (7) immediately yields,

$$G_m = \Gamma_{jk} g_k^{(m)} g_j^{(m)} = a_{ijkl} p_i p_l g_k^{(m)} g_j^{(m)} \tag{10}$$

(no summation over m).

The eikonal equation (8) can be solved by means of characteristics. The characteristics correspond to rays and the relevant equations to ray tracing systems. The ray tracing systems can be written in several forms, each of them may be suitable in certain cases.

The first form is as follows:

$$\frac{dx_i}{d\tau} = \frac{1}{2} \frac{\partial G_m}{\partial p_i}, \qquad \frac{dp_i}{d\tau} = -\frac{1}{2} \frac{\partial G_m}{\partial x_i}. \tag{11}$$

The parameter along the ray τ corresponds to the time of propagation along the ray. This form of the ray tracing system is not quite suitable in the case of a general anisotropic medium. The analytical expressions for G_m are usually rather complicated; they are solutions of the cubic equation (6). To seek the analytical expressions for the partial derivatives $\partial G_m/\partial p_i$ and $\partial G_m/\partial x_i$ from (6) would be a really cumbersome procedure. Equations (11) are, however, suitable in simpler cases, when the characteristic equation (6) can be factorized.

The second form is obtained from (11), when we use the theorem on implicit functions to determine the partial derivatives $\partial G_m/\partial p_i$ and $\partial G_m/\partial x_i$ directly from (6). In this way we obtain

$$\frac{dx_i}{d\tau} = a_{ijkl} p_l D_{jk}/D, \qquad \frac{dp_i}{d\tau} = -\frac{1}{2} \frac{\partial a_{ljks}}{\partial x_i} p_l p_s D_{jk}/D, \tag{12}$$

where D_{jk} and D are given by relations

$$\begin{aligned} D_{11} &= (\Gamma_{22}-1)(\Gamma_{33}-1) - \Gamma_{23}^2, & D_{12} = D_{21} &= \Gamma_{13}\Gamma_{23} - \Gamma_{12}(\Gamma_{33}-1), \\ D_{22} &= (\Gamma_{11}-1)(\Gamma_{33}-1) - \Gamma_{13}^2, & D_{13} = D_{31} &= \Gamma_{12}\Gamma_{23} - \Gamma_{13}(\Gamma_{22}-1), \\ D_{33} &= (\Gamma_{11}-1)(\Gamma_{22}-1) - \Gamma_{12}^2, & D_{23} = D_{32} &= \Gamma_{12}\Gamma_{13} - \Gamma_{23}(\Gamma_{11}-1), \\ D &= D_{11} + D_{22} + D_{33}. \end{aligned} \tag{13}$$

In the ray tracing system (12), it is not necessary to evaluate the eigenvalues G_m of the matrix Γ when computing rays. All the expressions in the system (12) are explicit. The ray tracing system (12) was first given by Červený (1972). It cannot be used when $D = 0$, that is when the relevant eigenvalue coincides with any of the two remaining eigenvalues.

The third form is obtained from (10) and (11),

$$\frac{dx_i}{d\tau} = a_{ijkl} p_l g_k^{(m)} g_j^{(m)}, \qquad \frac{dp_i}{d\tau} = -\frac{1}{2} \frac{\partial a_{ljks}}{\partial x_i} p_l p_s g_k^{(m)} g_j^{(m)}. \tag{14}$$

Form (14) is very close to (12), in which the components of the eigenvectors are expressed analytically (for $D \neq 0$).

Thus, we can see that ray tracing in general anisotropic inhomogeneous media is in principle simple and straightforward. It is, however, substantially more time-consuming than ray tracing in isotropic inhomogeneous media due to the complicated summations.

As a byproduct of ray tracing, we can evaluate the travel time τ and a number of other quantities at any point of the ray:

(a) The components of the slowness vector $p_i (i = 1, 2, 3)$, the components of the unit normal to the wavefront $N_i (i = 1, 2, 3)$ and the phase (normal) velocity $V = (p_i p_i)^{-1/2} = [G_m (N_i, x_i)]^{1/2}$.

(b) The components of the group velocity vector $v_i = dx_i/d\tau$, the group (ray) velocity $v = [(dx_i/d\tau)\ (dx_i/d\tau)]^{1/2}$ and the components of the unit vector n_i tangent to the ray, $v_i = v n_i$.

(c) The components of the eigenvector $\mathbf{g}^{(m)}$, which determine the direction of the displacement vector of the corresponding wave. Thus, the eigenvectors $\mathbf{g}^{(m)}$ determine the polarization of the wave (in the zero approximation of the ray theory).

It is well known that the unit vectors **n**, **N** and $\mathbf{g}^{(m)}$ are generally different in an inhomogeneous anisotropic medium.

We shall not discuss the problems of the initial conditions for ray tracing systems, the determination of the geometrical spreading and ray amplitudes here. These problems, even though very important, are not directly related to the purpose of this paper. For a detailed discussion of these problems see the above references.

3 Linearized approach to travel-time computations in anisotropic media

The ray tracing and travel-time computation described in Section 2 are simple and straightforward in principle and can be used to solve direct kinematic problems in arbitrary laterally inhomogeneous 3-D anisotropic media. It would, however, be rather time consuming and cumbersome to use the methods to solve inverse problems by numerical modelling.

A simpler procedure for the solution of both direct and inverse kinematic problems in an inhomogeneous anisotropic medium is based on linearization. A linearization procedure was successfully applied in the inversion of travel-time data for laterally heterogeneous isotropic structures and may, in principle, also be used for anisotropic inhomogeneous media.

We shall first consider an inhomogeneous anisotropic medium described by the elastic parameters a^0_{ijkl} (dependent on coordinates x_i) and call it the H^0 medium. Let us consider two points $M^0(x^0_i)$ and $M(x_i)$ (see Fig. 1a) and one of the three waves which can propagate in the H^0 medium (either the quasi-compressional or one of the two quasi-shear waves). We assume that the ray L^0 of this wave and corresponding travel time τ^0 from M^0 to M are known.

Let us change the medium H^0 slightly and denote the new perturbed medium by H. Also denote the elastic parameters in the H medium by a_{ijkl},

$$a_{ijkl} = a^0_{ijkl} + a^1_{ijkl}. \qquad (15)$$

Let a^1_{ijkl} be small corrections (or perturbations) in the elastic parameters. We can again evaluate the ray L from M^0 to M and the corresponding travel time τ from M^0 to M (see Fig. 1b). We can write

$$\tau(M^0, M) = \tau^0(M^0, M) + \tau^1(M^0, M). \qquad (16)$$

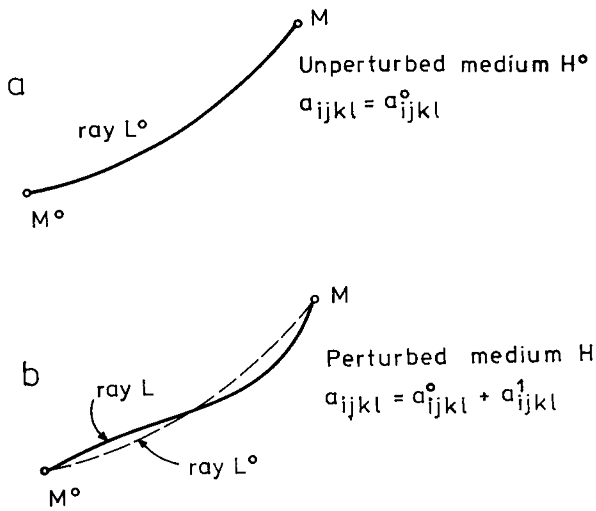

Figure 1. Rays L^0 and L in the unperturbed and perturbed media.

The basic linearization equation, derived by Červený (1982), gives a linear relation between $\tau^1(M^0, M)$ and a^1_{ijkl}. It reads

$$\tau^1(M^0, M) = -\frac{1}{2}\int_{L^0}\left(\frac{\partial G_m}{\partial a_{ijkl}}\right)_0 a^1_{ijkl}\, d\tau^0. \tag{17}$$

The integral in (17) is taken along the unperturbed ray L^0. Similarly, $d\tau^0$ corresponds to an infinitesimal travel-time increment along the punperturbed ray L^0. The index 'o' in the expression for $\partial G_m/\partial a_{ijkl}$ denotes that the components of the slowness vector p_i and the elastic parameters a_{ijkl} correspond to the H^0 medium and to the unperturbed ray L^0.

Thus, the basic linearization equation (17) can be used to determine the travel-time correction $\tau^1(M^0, M)$ by a quadrature along L^0 from M^0 to M of the small perturbation in the elastic parameters, multiplied by a simple weighting function.

The basic linearization equation (17) can be rewritten in several other forms, like the ray tracing systems. The second form is as follows,

$$\tau^1(M^0, M) = -\frac{1}{2}\int_{L^0} (p_i p_l D_{jk}/D)_0\, a^1_{ijkl}\, d\tau^0, \tag{18}$$

where D_{jk} and D are given by (13). It is assumed that $D \neq 0$. Finally, the third form reads

$$\tau^1(M^0, M) = -\frac{1}{2}\int_{L^0} (p_i p_l g_k^{(m)} g_j^{(m)})_0\, a^1_{ijkl}\, d\tau^0, \tag{19}$$

(no summation over m), where $g_j^{(m)}$ are the components of the eigenvector $\mathbf{g}^{(m)}$, corresponding to the wave under study.

The linearized equations simplify considerably if the characteristic equation (6) can be factorized.

In the most important case of the isotropic H^0 medium, the ray L^0 may correspond either to a P-wave or to an S-wave.

Let us first assume that the ray L^0 corresponds to a P-wave and denote the travel time of the P-wave from M^0 to M by $\tau_P^0(M^0, M)$. Then, for the travel time of a quasi-compressional wave $\tau_P(M^0, M)$ in the perturbed H medium we obtain the expression:

$$\tau_P(M^0, M) = \tau_P^0(M^0, M) + \tau_P^1(M^0, M); \tag{20}$$

where

$$\tau_P^1(M^0, M) = -\frac{1}{2} \int_{L^0} \alpha^{-2} (N_i N_j N_k N_l)_0 \, a_{ijkl}^1 \, d\tau^0. \tag{21}$$

Here $\alpha = \alpha(x_i)$ denotes the P-wave velocity in the H^0 medium.

The situation is more complicated for S-waves. We denote the two quasi-shear waves which propagate in the anisotropic H medium by $S1$ and $S2$. We assume that the ray L^0 in the H^0 medium corresponds to an S-wave and denote the travel time of the S-wave from M^0 to M by $\tau_S^0(M^0, M)$. We can then write for the H medium

$$\tau_{S1}(M^0, M) = \tau_S^0(M^0, M) + \tau_{S1}^1(M^0, M),$$

$$\tau_{S2}(M^0, M) = \tau_S^0(M^0, M) + \tau_{S2}^1(M^0, M). \tag{22}$$

In isotropic media, the eigenvectors $\mathbf{g}^{(2)}$ and $\mathbf{g}^{(3)}$ corresponding to shear waves are not uniquely determined, we only know that they are mutually perpendicular and perpendicular to L^0. In this way, the orientation of the displacement vectors of $S1$ and $S2$ may be chosen arbitrarily. However, as soon as their orientation is specified at any point of the ray, it is also uniquely determined along the whole ray. This is not the case in the anisotropic medium, where the orientations of $S1$ and $S2$ are strictly determined; waves $S1$ and $S2$ generally propagate along different rays with different velocities. This fact causes the well-known shear-wave splitting. However, it is simple to recognize that the expression $g_k^{(2)} g_j^{(2)} + g_k^{(3)} g_j^{(3)}$ is uniquely determined at any point of the ray even in isotropic media. Without difficulties we obtain

$$g_k^{(2)} g_j^{(2)} + g_k^{(3)} g_j^{(3)} = \delta_{kj} - N_k N_j, \tag{23}$$

when the H^0 medium is isotropic. Thus, (22), (23) and (19) yield

$$\tau_{S1}^1(M^0, M) + \tau_{S2}^1(M^0, M) = -\frac{1}{2} \int_{L^0} \beta^{-2} [N_i N_l(\delta_{kj} - N_k N_j)]_0 \, a_{ijkl}^1 \, d\tau^0. \tag{24}$$

Equation (24) was also derived by a different approach in Červený (1982). This reference also gives the linearized expression for τ_{S1}^1, τ_{S2}^1 and for the time delay $\tau_{S2} - \tau_{S1}$ for various special cases of practical importance (e.g. when the unperturbed ray L^0 is a plane curve).

The accuracy of the linearized equations was investigated by Červený & Jech (1983). A vertically inhomogeneous model of the Earth's crust was considered, the H^0 medium was isotropic and the H medium anisotropic. The average coefficient of anisotropy was close to 10 per cent. The linearized equations yield the travel times of quasi-compressional waves with an accuracy higher than 0.05 s within the range of epicentral distances of ~150 km. These results, together with the results of other (unpublished) computations, suggest that the accuracy of linearized equations is sufficient to apply them to practical intereptations of seismic measurements. To test the accuracy of the linearization approach more thoroughly, a one-purpose computer program was written. The results of computation (Firbas 1982) support the above conclusions and will be published elsewhere.

All the linearized equations presented above are applicable to media without interfaces. In solving the direct problem for complex media, it is necessary to verify that the linearized solutions can also be used across interfaces. This aspect has been studied and the proof will be published by Firbas elsewhere. The conclusion following from these investigations is that the linearized equations can be applied even to reflected/refracted waves (including multiple reflections) in going from isotropic to slightly anisotropic models when the shape of the interfaces is fixed.

4 Application of linearized equations to the solution of direct kinematic problems

The question may arise why the approximate (linearized) solutions should be derived and used, if the exact methods of computing the kinematic characteristics are known (see Section 2). We can give some good reasons which may form a basis for utilizing the linearized solutions.

The most important reason consists in the computer efficiency of the linearized solutions. They need at least ten times less computer time than exact solutions.

The linearized equations automatically include two-point ray tracing. This again considerably increase the efficiency of the linearization approach, mainly for 3-D rays. In exact calculations, two-point ray tracing complicates the computations considerably. The applications of various approximations to two-point ray tracing introduce errors which may be comparable to or even higher than the error caused by linearization.

The linearization approach would be very suitable for modelling time residuals due to long-term changes of anisotropy along fixed systems of receivers and sources. These long-term changes of anisotropy may be caused, for example, by long-term changes in the stress field in mines, in areas of high seismic activity, etc.

In some cases, we only wish to study the influence of anisotropy on the travel-time field qualitatively; the knowledge of exact travel-time data is not required. Then, of course, the linearization approach is the most suitable.

The linearized equations were used to write a rather extensive computer program to evaluated the travel-time fields of seismic body waves in a 2-D laterally inhomogeneous anisotropic layered structure (see Firbas 1982). The unperturbed medium H is considered to be isotropic. It may contain block structures, isolated bodies, vanishing layers, fractures, etc. The velocity distribution within individual layers may change both horizontally and vertically. To perform the ray tracing and travel-time computations in the H^0 medium, the 2-D software package SEIS81, written by Červený & Pšenčík (1981), was included in the program, with some slight modifications. The new anisotropic version of this package enables travel times to be computed and plotted for arbitrary, slightly anisotropic 2-D media. The individual layers may be either isotropic or anisotropic. The anisotropic properties may vary in the individual layers both horizontally and vertically. The input data for a^1_{ijkl} may be determined in a grid and a linear interpolation between grid points is used.

An example of model calculations, using the package described above, is presented. The model is described by four layers over a half-space. The first layer (the crust) and the third layer (the low-velocity layer in the upper mantle) are isotropic. The second and the fourth layers are assumed to be slightly anisotropic; they differ from the isotropic case in the value of the elastic parameter a_{3333}. The P-wave velocity α within the individual layers in the unperturbed isotropic model H^0 changes only in the vertical direction. It is given by the relation $\alpha(x) = \alpha_0 + bz$ km s^{-1}, with $\alpha_0 = 6.0$ km s^{-1}, $b = 0.048$ s^{-1} (first layer), $\alpha_0 = 7.15$ km s^{-1}, $b = 0.035$ s^{-1} (second layer), $\alpha_0 = 8.1$ km s^{-1}, $b = 0.0$ s^{-1} (third layer), $\alpha_0 = 7.4$ km s^{-1}, $b = 0.025$ s^{-1} (fourth layer). The interfaces are curved and their shape is shown in Fig. 2. This figure also shows the ray diagram of P-waves reflected from the bottom of the fourth layer and refracted inside that layer, in the unperturbed isotropic medium H^0. The perturbed anisotropic H model is described by the relation $a^1_{3333} = A + B$ $(z-15)$ km s^{-2}, with $A = B = 0$ for the first and third layer, $A = 10$, $B = 0.0258$ for the second layer, and $A = 11.328$, $B = 0.014$ for the fourth layer. The travel times for the isotropic case (crosses) and the anisotropic model (circles) are shown in Fig. 3.

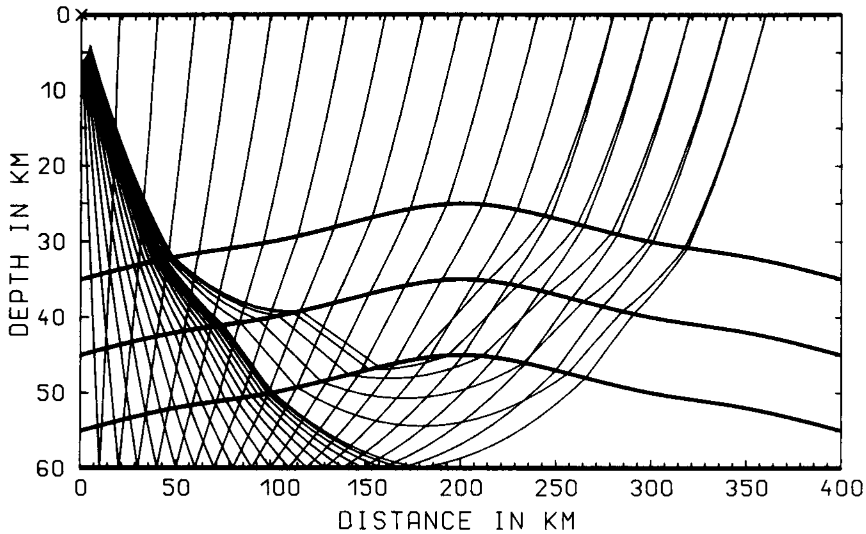

Figure 2. Ray diagram of *P*-waves reflected from the bottom of the fourth layer and refracted inside the layer in the unperturbed isotropic layered structure, used for evaluating the travel times shown in Fig. 3. For a more detailed description of the model see text.

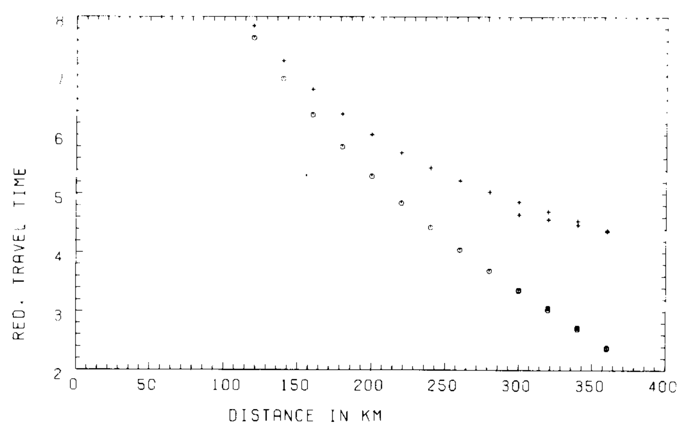

Figure 3. The travel times of waves shown in Fig. 2 for the isotropic case (crosses) and the anisotropic case (circles). The anisotropic model is obtained by a change in the elastic parameter a_{3333} (for details see text). The travel times for the anisotropic model are calculated by the linearization approach from the travel times for the isotropic model.

Fig. 3 does not need any explanation. It should only be stressed that the presence of anisotropy may not only change the shape of the travel-time curves, but may cause some phases (reflected, refracted) to become nearly coincident, even though they are separated in the unperturbed isotropic H^0 model (see Fig. 3). The same applies to the opposite case.

Other examples of similar computations can be found in Firbas (1982).

5 Applications of linearized equations to the solution of inverse kinematic problems

In the solution of the inverse kinematic problem for anisotropic inhomogeneous media, nothing can be said about the causes of anisotropy. We consider the anisotropy to correspond to some effective, macroscropic scale anisotropy. As the anisotropic effects are frequency dependent, the anisotropic properties of the medium, obtained by the solution of the inverse kinematic problem, correspond approximately to the prevailing frequency of the observed wavefield used in the interpretation.

Perhaps the most important observational fact concerning the properties of the Earth's crust and the uppermost mantle is that the media for which we wish to solve the inverse problem are only slightly anisotropic, with an anisotropy smaller than say 10 per cent. (The exceptions may correspond primarily to the sedimentary layers; they are not considered here.) In this case, a unique possibility appears to take advantage of the linearization approach. The accuracy of the method seems to be quite satisfactory, far below the errors of measurements.

Before solving the inverse problem, it is necessary to find a suitable representation for the anisotropic properties of the medium. At present, using all 21 independent elastic parameters seems to be hopeless. (It is, of course, simple to consider all 21 elastic parameters in the solution of the direct problem, but not in the inverse problem.) In taking advantage of the reduced number of parameters, it is necessary to keep in mind the purpose of the computations: whether or not the obtained parameters should be further interpreted in terms of the material composition of the medium.

The simplest approach can be based on the determination of maximum and minimum velocities at each point, together with some specified azimuthal dependence. An alternative possibility is to use ellipsoidal anisotropy (with two elastic constants controlling the quasi-compressional waves and two other controlling the propagation of the quasi-shear waves), or some other simple type of anisotropy, say a transversely isotropic medium with a vertical or inclined axis of symmetry.

With a reduced number of parameters, we can try to solve the inverse kinematic problem, assuming that the system of travel-time curves along the Earth's surface is sufficiently dense. There are two approaches to the solution.

The first approach is based on numerical modelling in successive solutions of the direct problem to get the best fit with the observed data. In principle, the exact ray tracing and travel-time computations described in Section 2 can also be used for this purpose, but the linearization approach will give similar result with much less computation involved.

The second approach is used in the direct solution of the inverse kinematic problem, using the linearized equations presented in this paper. Such a linearization approach was used with success in solving the inverse kinematic problem for 2- and 3-D laterally inhomogeneous isotropic media in the past (Romanov 1972; Spencer & Gubbins 1980; Firbes 1981; Novotný 1981). For laterally heterogeneous slightly anisotropic media, the linearized equations described in this paper can be used in the same way. For simpler types of media such as for the homogeneous isotropic H^0 medium, a similar approach based on Backus' (1965) equations has already been applied with success to various oceanic and continental regions (Backus 1965; Bamford 1976, 1977; Crampin & Bamford 1977; Vetter & Minster 1981, etc.). The linearized approach described here can, however, also take into account the lateral heterogeneity of the medium. It also allows various wavefields (both reflected and refracted) with quite different curved rays to be considered. In fact, it would be very useful to combine the reflection and refraction measurements in the linearization approach. We need as many rays crossing the investigated region in different directions as possible.

Similarly, the resolving power will increase considerably if quasi-shear-wave data are used, so that the time delay between the two quasi-shear waves can be estimated. The whole procedure can, of course, be applied iteratively, as the linearized equations are applicable even when the initial model is anisotropic. Some general linear inversion schemes can be used for this purpose. Exact (non-linearized) ray tracing in inhomogeneous anisotropic media would, however, be needed at each step of the iterative linearization procedure.

References

Babich, V. M., 1961. Ray method for the computation of the intensity of wave fronts in the elastic inhomogeneous medium, in *Problems in the Dynamic Theory of the Propagation of Seismic Waves,* ed. Petrashen, G. I., **5,** 36–44, Leningrad University Press (in Russian).

Backus, G. E., 1965. Possible forms of seismic anisotropy of the uppermost mantle under oceans, *J. geophys. Res.,* **70,** 3429–3439.

Bamford, D., 1976. Mozaic time-term analysis, *Geophys. J. R. astr. Soc.,* **44,** 433.

Bamford, D., 1977. P_n velocity anisotropy in a continental upper mantle, *Geophys. J. R. astr. Soc.,* **49,** 29–48.

Červený, V., 1972. Seismic rays and ray intensities in inhomogeneous anisotropic media, *Geophys. J. R. astr. Soc.,* **29,** 1–13.

Červený, V., 1982. Direct and inverse kinematic problems for inhomogeneous anisotropic media, *Contr. Geophys. Inst. Slov. Acad. Sci.,* **13,** 127–133.

Červený, V. & Jech, J., 1983. Linearized solutions of kinematic problems of seismic body waves in inhomogeneous slightly anisotropic media, *J. Geophys.,* **51,** submitted.

Červený, V. & Pšenčík, I., 1981. *2-D seismic ray package,* Research Report, Institute of Geophysics, Charles University, Prague.

Červený, V., Molotkov, I. A. & Pšenčík, I., 1977. *Ray Method in Seimology,* Universita Karlova, Praha.

Crampin, S., 1981. A review of wave motions in anisotropic and cracked media, *Wave Motion,* **3,** 343–391.

Crampin, S. & Bamford, D., 1977. Inversion of *P*-wave velocity anisotropy, *Geophys. J. R. astr. Soc.,* **49,** 123–132.

Firbas, P., 1981. Inversion of travel-time data for laterally heterogeneous velocity structure – linearization approach, *Geophys. J. R. astr. Soc.,* **66,** 189–198.

Firbas, P., 1982. Travel times, amplitudes and synthetic seismograms for slightly anisotropic media, *Open File Rep. No. 108,* Geophysikalisches Institute University of Karlsruhe.

Novotný, M., 1981. Two methods for solving the linearized two-dimensional inverse kinematic problem, *J. Geophys.,* **50,** 7–15.

Petrashen, G. I., 1980. *Wave Propagation in Anisotropic Elastic Media,* Nauka, Leningrad (in Russian).

Romanov, V. G., 1972. *Some Inverse Problems for Hyperbolic Equations,* Nauka, Novosibirsk (in Russian).

Spencer, C. & Gubbins, D., 1980. Travel-time inversion for simultaneous earthquake and velocity structure determination in laterally varying media, *Geophys. J. R. astr. Soc.,* **63,** 95–116.

Vetter, K. U. & Minster, J. B., 1981. P_n velocity anisotropy in Southern California, *Bull. seism. Soc. Am.,* **71,** 1511–1530.