SENSITIVITY GAUSSIAN PACKETS

Luděk Klimeš

Department of Geophysics, Faculty of Mathematics and Physics,
Charles University in Prague, Czech Republic; http://sw3d.cz

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We study how the perturbations of a generally heterogeneous isotropic or anisotropic structure manifest themselves in the wave field, and which perturbations can be detected within a limited aperture and a limited frequency band. We consider a smoothly varying heterogeneous generally anisotropic elastic background medium, and its arbitrarily varying generally anisotropic perturbations. We decompose the perturbations of elastic moduli and density into Gabor functions. The wave field scattered by the perturbations is then composed of waves scattered by the individual Gabor functions.

We assume a short–duration, broad–band, incident wave field with a smooth frequency spectrum. We approximate each wave scattered by one Gabor function by the *first–order Born approximation*, which describes the first–order sensitivity of the wave field to the infinitesimally small structural perturbations exactly. We make use of the *paraxial ray approximation* of the incident wave in the vicinity of the central point of the Gabor function, and of the *two–point paraxial ray approximation* of the Green tensor. The above–mentioned approximations enable us to calculate the waves scattered by the individual Gabor functions analytically [1].

The wave, scattered by one Gabor function, is composed of a few (i.e. 0 to 5 as a rule) Gaussian packets. Each of these "sensitivity" Gaussian packets has a specific frequency and propagates from the Gabor function in a specific direction, see Figures 1–3. Each sensitivity Gaussian packet is sensitive to just a single linear combination of the perturbations of elastic moduli and density, corresponding to the Gabor function. This information about the Gabor function is lost if the sensitivity Gaussian packet does not fall into the aperture covered by the receivers and into the legible frequency band. The situation improves with the increasing number of differently positioned sources. If we have many sources, the sensitivity Gaussian packets, propagating from a Gabor function, may be lost during the measurement corresponding to one source, but recorded during the measurement corresponding to another, differently positioned source. However, the problem is not only to record the Gaussian packets from a Gabor function, but to record them in as many different measurement configurations as to resolve the perturbations of all elastic moduli and density.

Figure 1: A single Gabor function.

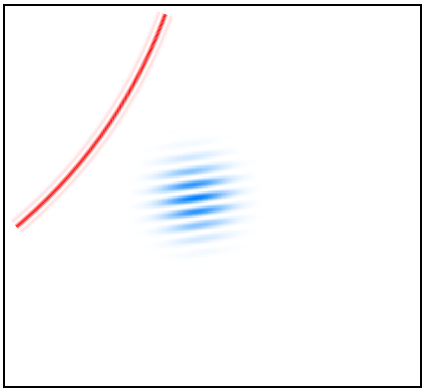

Figure 2: Broad–band wave incident at the Gabor function.

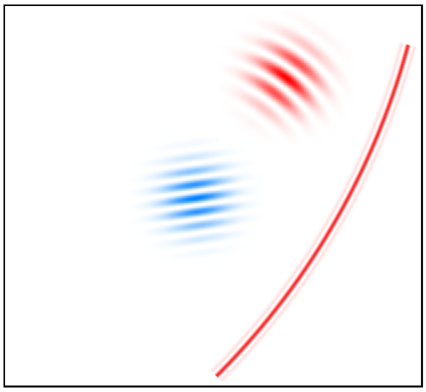

Figure 3: Gaussian packet scattered by the Gabor function.

In a numerical example, we consider the distribution of the P–wave velocity in the Marmousi structure. The velocity difference between the Marmousi structure and the smooth background medium is displayed in Figure 4. For the decomposition of the velocity difference, we generate the set of Gabor functions with their shapes optimized according to [2], see Figure 5. We then decompose the velocity difference from Figure 4 into the sum of Gabor functions. For each shot, we calculate the sensitivity Gaussian packets scattered by the individual Gabor functions. If a sensitivity Gaussian packet arrives at the receiver array within the registration time and frequency band, the recorded wave field contains

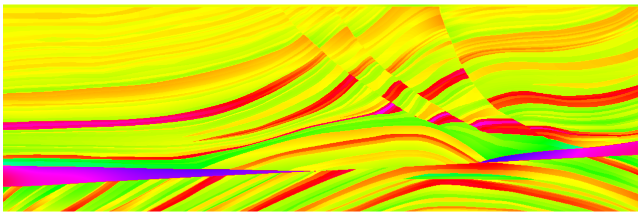

Figure 4: Velocity difference between the Marmousi structure and the velocity model.

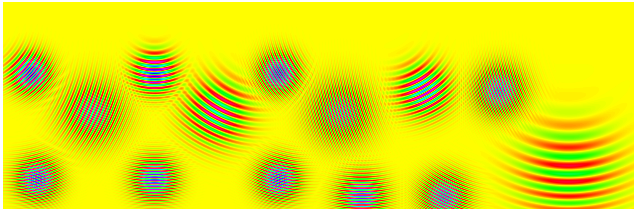

Figure 5: Example showing 14 ones of 67014 optimized Gabor functions used to decompose the velocity difference.

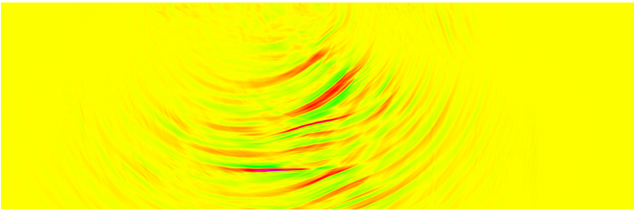

Figure 6: Sum of the Gabor functions influencing the seismograms recorded for shot 70.

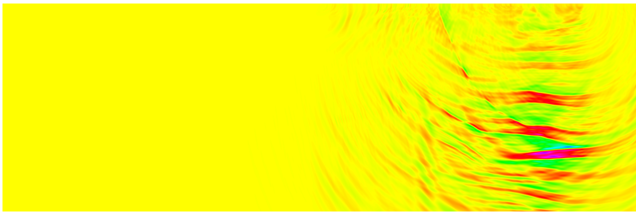

Figure 7: Sum of the Gabor functions influencing the seismograms recorded for shot 220.

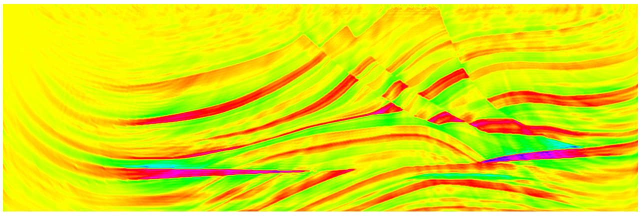

Figure 8: Sum of the Gabor functions influencing the seismograms collected from all shots.

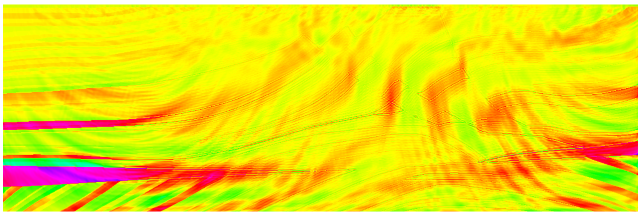

Figure 9: Part of the velocity difference from Figure 4 influencing no recorded seismogram.

information on the corresponding Gabor function. The sums of the Gabor functions influencing the seismograms recorded for shots 70 and 220 are displayed in Figures 6 and 7. The velocity difference from Figure 4 can be decomposed into the part to which the recorded seismograms are not sensitive and into the part to which the recorded seismograms are sensitive. The sum of the Gabor functions influencing the seismograms collected from all shots is displayed in Figure 8. This is the part of the velocity difference to which the recorded seismograms are sensitive. The remaining part of the velocity difference, influencing no recorded seismogram within the first–order Born approximation, is displayed in Figure 9. This part of the velocity difference cannot be recovered from the Marmousi seismograms.

The sensitivity Gaussian packets can enable migrations to be replaced by true linearized inversions of seismic reflection data. For the algorithm of the linearized inversion of the complete set of seismograms recorded for all shots, refer to [3].

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References

[1] L. Klimeš. Sensitivity of seismic waves to the structure. *Stud. geophys. geod.*, **56**, No. 2, in press, 2012.

[2] L. Klimeš. Optimization of the structural Gabor functions in a homogeneous velocity model for a zero–offset surface seismic reflection survey. In: *Seismic Waves in Complex 3–D Structures, Report 18*, pp. 115–127, Dep. Geophys., Charles Univ., Prague, 2008, online at “http://sw3d.cz”.

[3] L. Klimeš. Stochastic wavefield inversion using the sensitivity Gaussian packets. In: *Seismic Waves in Complex 3–D Structures, Report 18*, pp. 71–85, Dep. Geophys., Charles Univ., Prague, 2008, online at “http://sw3d.cz”.