

# COUPLING RAY SERIES

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There are two different standard zero-order high-frequency asymptotic ray theories for S waves in elastic media: the *isotropic ray theory* based on the assumption of equal velocities of both S waves, and the *anisotropic ray theory* assuming both S waves strictly decoupled. Note that here the term “different” means that the anisotropic ray theory is not a generalization of the isotropic ray theory and that both theories yield really different S waves in equal “weakly anisotropic” velocity models.

In the isotropic ray theory, the S-wave polarization vectors do not rotate about the ray, whereas in the anisotropic ray theory they coincide with the eigenvectors of the Christoffel matrix which may rotate rapidly about the ray.

In “weakly anisotropic” media, at moderate frequencies, the S-wave polarization tends to remain unrotated round the ray, but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with frequency. This behaviour of the S-wave polarization is described by the *coupling ray theory* proposed by Coates and Chapman (1990), or by its rough quasi-isotropic approximation proposed by Kravtsov (1969). The frequency-dependent coupling ray theory is the generalization of both the zero-order isotropic and anisotropic ray theories and provides continuous transition between them. The coupling ray theory is applicable to S waves at all degrees of anisotropy, from isotropic to considerably anisotropic velocity models.

Both the isotropic ray theory and the anisotropic ray theory can be derived as the zero-order terms of the standard ray series applied to the elastodynamic equation.

We propose the *coupling ray series* (Klimeš, 2007), which yields the coupling ray theory in a similar way as the standard ray series yields the standard ray-theory solution of the elastodynamic equation. We insert the coupling ray series into the elastodynamic equation and obtain the coupling ray theory as the zero-order high-frequency asymptotic approximation.

The standard ray series yields a good approximation of the additional amplitude components but fails in the description of S-wave coupling. The coupling ray series describes S-wave coupling well, but its approximation of the additional amplitude components is not satisfactory. We thus also propose a more general ray series, which represents a generalization of both the standard ray series and the coupling ray series.

We demonstrate the behaviour of S-wave polarization and S-wave coupling and splitting on numerical examples.

## References

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