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Comparison of VSP and sonic-log data in nonvertical wells in a heterogeneous structure

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ABSTRACT

To compare the results of sonic-log measurements and of vertical seismic profiling (VSP), sonic-log velocities are used to estimate the corresponding traveltime in the geologic structure, which is then compared with the VSP traveltime. We show how to calculate the sonic-log traveltime in the geologic structure from the sonic-log velocities while taking into account the effects of the nonvertical propagation of seismic waves, resulting from the VSP-source offset and from heterogeneous velocity in the structure, together with the effects of the well trajectory deviating from strictly vertical. Errors caused by the commonly used assumption of vertical propagation may considerably exceed the difference of the measured VSP traveltimes from the sonic-log traveltimes.

INTRODUCTION

In borehole geophysics, sonic-log measurements and vertical seismic profiling (VSP) are two techniques which provide information about the seismic velocity in the structure surrounding a particular well. These techniques use significantly different frequencies of seismic signals. The results of sonic-log measurements may thus differ from the results of VSP by the velocity dispersion. To compare the results of the two measurements, the sonic-log velocities are used to estimate corresponding sonic-log traveltime τ in the geologic structure, which is then compared with VSP traveltime τ^{VSP}.

VSP provides traveltimes from the surface VSP source to the VSP receivers located in the well. These traveltimes are measured at frequencies from a few tens to a few hundreds hertz.

Sonic logging provides very detailed information about the slowness (inverse velocity) in the structure along the well. The slowness is averaged over the interval covered by sonic-log receivers, which is typically of the order of 1m. The slowness is measured typically at frequencies from 2 to 20 kHz.

Comparisons of VSP and sonic-log measurements have been addressed broadly in the literature since Gretener (1961). Many references can be found, e.g., in Stewart et al. (1984). If both measurements are done for a particular well, the so-called drift curve may be constructed. The drift curve displays the difference

$$\tau^{\mathrm{VSP}} - \tau \tag{1}$$

of measured VSP traveltimes τ^{VSP} from sonic-log traveltimes τ calculated by integrating the sonic-log slowness. The drift curve can be plotted along the part of the well where both the sonic-log slowness and VSP traveltimes have been measured. If the sonic-log slowness and VSP traveltimes are determined accurately and sonic-log traveltimes are calculated correctly, the drift curve should display the velocity dispersion between the sonic-log and VSP frequencies.

Gretener (1961) assumes a 1D layered structure. Starting with the shallowest data point common to both sonic-log and VSP measurements, the sonic-log slowness is integrated over the depth, yielding the traveltime increment for strictly vertical propagation. If the VSP source is situated at the well head and the well is straight and vertical, the drift curve represents the difference of the VSP traveltime increment from the integrated sonic-log slowness. For an offset between the VSP source and receivers, Gretener (1961) assumes a homogeneous background with straight rays and applies a simple cosine correction to the VSP traveltimes. He then constructs the drift curve as the difference between the corrected VSP traveltimes and the sonic-log traveltimes calculated for strictly vertical propagation. Because

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the velocity usually increases with depth, the cosine correction (straight-ray assumption) is usually inaccurate because of ray bending caused by both smooth large-scale heterogeneities and small-scale layering.

The aim of this paper is to show how to calculate sonic-log traveltime τ while taking into account the effects of the nonvertical propagation of seismic waves, resulting from the VSP-source offset and from the heterogeneous velocity in the structure, along with the effects of the well trajectory deviating from strictly vertical. Note that ray tracing in a reference velocity model, which is smooth or composed of a small number of layers or blocks with smooth velocity distribution, is used to calculate reference traveltimes τ^0 and reference slowness vectors.

Because we consider a heterogeneous velocity background, we calculate sonic-log traveltimes τ from the VSP source to receivers. These sonic-log traveltimes are already corrected for the offset between the VSP source and receivers, for the heterogeneous velocity in the structure and for the effects of the well trajectory deviating from strictly vertical. We then construct the drift curve as the difference, given by equation 1, between the measured VSP traveltimes and the correctly calculated sonic-log traveltimes. Moving the corrections from the VSP traveltimes to the sonic-log traveltimes between Gretener's (1961) and, in practice, our definition does not influence the drift curve.

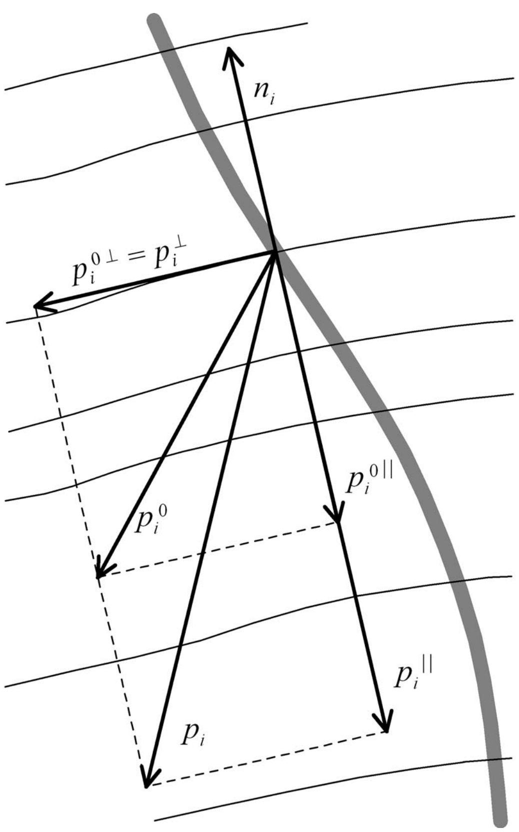

Figure 1. A simple sketch of a well (bold gray line) in a layered geologic structure (thin lines). Arrows show unit vector $n_i(\mathbf{x})$ perpendicular to layers, reference slowness vector $p_i^0(\mathbf{x})$ calculated in the reference velocity model, approximate slowness vector $p_i(\mathbf{x})$ in the geologic structure calculated using the sonic-log slowness, and the projections of the slowness vectors onto the plane perpendicular to n_i, and onto the direction of n_i.

The difference of the measured VSP traveltimes from the sonic-log traveltimes calculated using the sonic-log velocities is typically 1.7 ms/1000 ft (Gretener, 1961). In the numerical example in this paper, the error of the above-mentioned relative traveltime difference, caused by the assumption of vertical propagation, reaches 2.3 ms/1000 ft. Thus we should not neglect the error caused by not taking into account the deviation of the VSP slowness vector from the vertical. We estimate the errors caused by the commonly used assumption of vertical propagation in Appendix A.

We denote vectors simultaneously by bold letters such as $\mathbf{x}$ and by their components x_i, where $i = 1,2,3$, and use the Einstein summation over the pairs of identical indices, e.g., $|\mathbf{x}|^2 = x_i x_i$.

SONIC-LOG TRAVELTIME FOR A NONVERTICAL WELL IN A HETEROGENEOUS STRUCTURE

We calculate the reference traveltime and the reference slowness vector in a reference velocity model, which is sufficiently smooth for application of ray methods. Using the reference slowness vector, we estimate the slowness vector in the locally 1D strongly heterogeneous structure inferred from sonic logs, and integrate it along the well to obtain the sonic-log traveltime from the VSP source.

Assumptions about the geologic structure

To deal with true well trajectories deviating from the strictly vertical direction, and to consider the horizontal offset between the VSP source and receivers, we need to make some assumptions about the velocity distribution in the geologic structure. We thus assume that a sufficiently smooth reference velocity model is available, and that the strong variations of the slowness in the geologic structure are locally 1D in the direction given by unit vector $n_i = n_i(\mathbf{x})$ normal to the local layering (see Figure 1). The strong 1D slowness variations are thus locally functions of *level* $\xi = n_i x_i$. Note that in many cases the smooth reference velocity model is 1D with the slowness varying only vertically, and vector n_i is constant and vertical, but here we consider general smoothly varying $n_i = n_i(\mathbf{x})$ for the sake of generality.

Reference velocity model

The reference velocity model may be smooth or, if there are pronounced velocity interfaces indicated, composed of a small number of layers or blocks with smooth velocity distribution (Červený et al., 1988). The reference velocity model should be obtained by simultaneous inversion of all available data, e.g., of sonic logs and VSP, crosswell, or refraction measurements. Because the reference velocity model should be sufficiently smooth for application of ray methods, the inversion should be restricted by minimizing simultaneously the Sobolev norm composed of the second velocity derivatives in the model (Bulant, 2002).

Reference traveltime and slowness vector in a smooth velocity model

We denote the reference traveltime calculated from the VSP source in the smooth reference velocity model $\tau^0(\mathbf{x})$, and the corresponding slowness vector $p_i^0(\mathbf{x})$. They may be calculated, e.g., by wavefront tracing from the VSP source (Vinje et al., 1993), controlled initial-value ray tracing from the VSP source (Bulant, 1999) followed by interpolation within ray cells (Bulant and Klimeš, 1999), or two-point ray tracing from the VSP receivers (Bulant, 1996) supplemented by interpolation described in Appendix B.

To calculate the sonic-log traveltime, we need to know the reference slowness vector at points $\mathbf{x}$ along the part of the well, where slowness $u(\mathbf{x})$ has been determined by a sonic log. Moreover, if we also know the reference traveltime along that part of the well, we can not only construct the drift curve defined by equation 1, but also compare sonic-log traveltime $\tau(\mathbf{x})$ corresponding to the sonic-log slowness with reference traveltime $\tau^0(\mathbf{x})$ corresponding to the smooth reference model.

Approximate slowness vector in the geologic structure

In accordance with our assumption of strong slowness variations in the geologic structure appearing only in the direction given by unit vector n_i, we assume the projection of the slowness vector p_k in the geologic structure onto the plane perpendicular to n_i to be about equal to the same projection of the reference slowness vector p_k^0,

$$\begin{aligned} p_i^{\perp}(\mathbf{x}) &= [\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k(\mathbf{x}) \\ &= [\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k^0(\mathbf{x}) \end{aligned} \tag{2}$$

(see Figure 1). Here Kronecker delta δ_{ij} represents the components of the identity matrix. The slowness vector in the geologic structure may thus be expressed as the sum of its projections,

$$\begin{aligned} p_i(\mathbf{x}) &= p_i^{\perp}(\mathbf{x}) + p_i^{\parallel}(\mathbf{x}) \\ &= [\delta_{ik} - n_i(\mathbf{x})n_k(\mathbf{x})]p_k(\mathbf{x}) + n_i(\mathbf{x})n_k(\mathbf{x})p_k(\mathbf{x}) \end{aligned} \tag{3}$$

(see Figure 1). Here, projection $p_i^{\perp}(\mathbf{x})$ perpendicular to $n_i(\mathbf{x})$ is given by equation 2. The parallel component

$$p^{\parallel} = n_k(\mathbf{x})p_k(\mathbf{x}) \tag{4}$$

of the slowness vector $p_k(\mathbf{x})$ can be calculated from the measured sonic-log slowness $u(\mathbf{x})$ using eikonal equation

$$p_i(\mathbf{x})p_i(\mathbf{x}) = [u(\mathbf{x})]^2. \tag{5}$$

Inserting equations 2 and 4 into equation 3, we obtain

$$p_i(\mathbf{x}) = p_i^0(\mathbf{x}) - n_i(\mathbf{x})n_k(\mathbf{x})p_k^0(\mathbf{x}) + n_i(\mathbf{x})p^{\parallel}, \tag{6}$$

and inserting equation 6 into equation 5, we obtain

$$\begin{aligned} p^{\parallel} = {} & \mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})] \\ & \times \sqrt{[u(\mathbf{x})]^2 - p_k^0(\mathbf{x})p_k^0(\mathbf{x}) + [n_k(\mathbf{x})p_k^0(\mathbf{x})]^2}. \end{aligned} \tag{7}$$

The slowness vector in the geologic structure is thus approximated by equation 7 and

$$p_i(\mathbf{x}) = p_i^0(\mathbf{x}) + n_i(\mathbf{x})[p^{\parallel} - n_k p_k^0(\mathbf{x})], \tag{8}$$

which is equivalent to equation 6.

Sonic-log traveltime

At a selected point $\mathbf{x} = \mathbf{x}^{\mathrm{select}}$, usually at the shallowest point for which the sonic-log slowness is measured, we put

$$\tau(\mathbf{x}^{\mathrm{select}}) = \tau^0(\mathbf{x}^{\mathrm{select}}). \tag{9}$$

Then we use the trapezoidal quadrature along the well,

$$\tau(\mathbf{x}^{\mathrm{new}}) = \tau(\mathbf{x}) + \frac{1}{2}[p_i(\mathbf{x}^{\mathrm{new}}) + p_i(\mathbf{x})][x_i^{\mathrm{new}} - x_i], \tag{10}$$

and obtain the sonic-log traveltime $\tau(\mathbf{x})$ along the part of the well where slowness $u(\mathbf{x})$ has been determined by a sonic log. The slowness vectors in equation 10 are determined by equations 7 and 8.

Note that in other formulations, the calculation of the sonic-log traveltime is initiated usually at the shallowest point $\mathbf{x}^{\mathrm{Rselect}}$ for which both the VSP and sonic-log measurements are available by putting $\tau(\mathbf{x}^{\mathrm{Rselect}}) = \tau^{\mathrm{VSP}}(\mathbf{x}^{\mathrm{Rselect}})$. We believe that our equation 9 for initiation of calculation of τ is more appropriate, because the resulting drift curve does not reflect the systematic shift caused by a possible error of the VSP traveltime at the initial point $\mathbf{x}^{\mathrm{Rselect}}$.

NUMERICAL EXAMPLE

The authors were motivated to carry out this study when they faced the task of comparing the results of sonic logging and of VSP in a well which deviated considerably from the vertical (see Figure 2). The depth scale of Figure 2 is in meters with zero depth slightly below the well head. The well head was at a depth of -20.27 m and the bottom of the well at a depth of 2021.76 m. Sonic logging was performed between the depths of 1312.24 and 2019.39 m at steps of 0.1528 m (0.5 ft) along the well. There were 64 VSP receivers at depths ranging from 101.77 to 1970.37 m at steps of 30.24 m (100 ft) along the well.

The well is located in Texas, U.S.A., where the geologic structure is dominated by approximately horizontal sedimentary layers. Thus the reference velocity model chosen was smooth 1D with vertical velocity variations only (see Figure 3). The model was constructed by inverting simultaneously sonic-log velocities and VSP traveltimes, and minimizing the Sobolev norm composed of the second velocity derivatives in the model (Bulant, 2002).

The difference

$$\tau(\mathbf{x}) - \tau^0(\mathbf{x}) \tag{11}$$

of the sonic-log traveltime from the reference traveltime in the reference velocity model was calculated according to equation 10, and is plotted as the solid line in Figure 4. We also have calculated the approximate sonic-log traveltime under the assumption of vertical propagation using equation A-2, and plotted its difference from thereference traveltime by the dotted line in Figure 4. We can see that the difference of the correct sonic-log traveltime from the sonic-log traveltime calculated under the assumption of vertical propagation is 5.3 ms over the 695 m of the depth interval of the measurements. The error of the sonic-log traveltime, caused by the assumption of vertical propagation, is thus 2.3 ms/1000 ft, which is comparable with typical difference 1.7 ms/1000 ft (Gretener, 1961) of the measured VSP traveltimes from the sonic-log traveltimes. The main thing is that this error may be even larger in other VSP configurations.

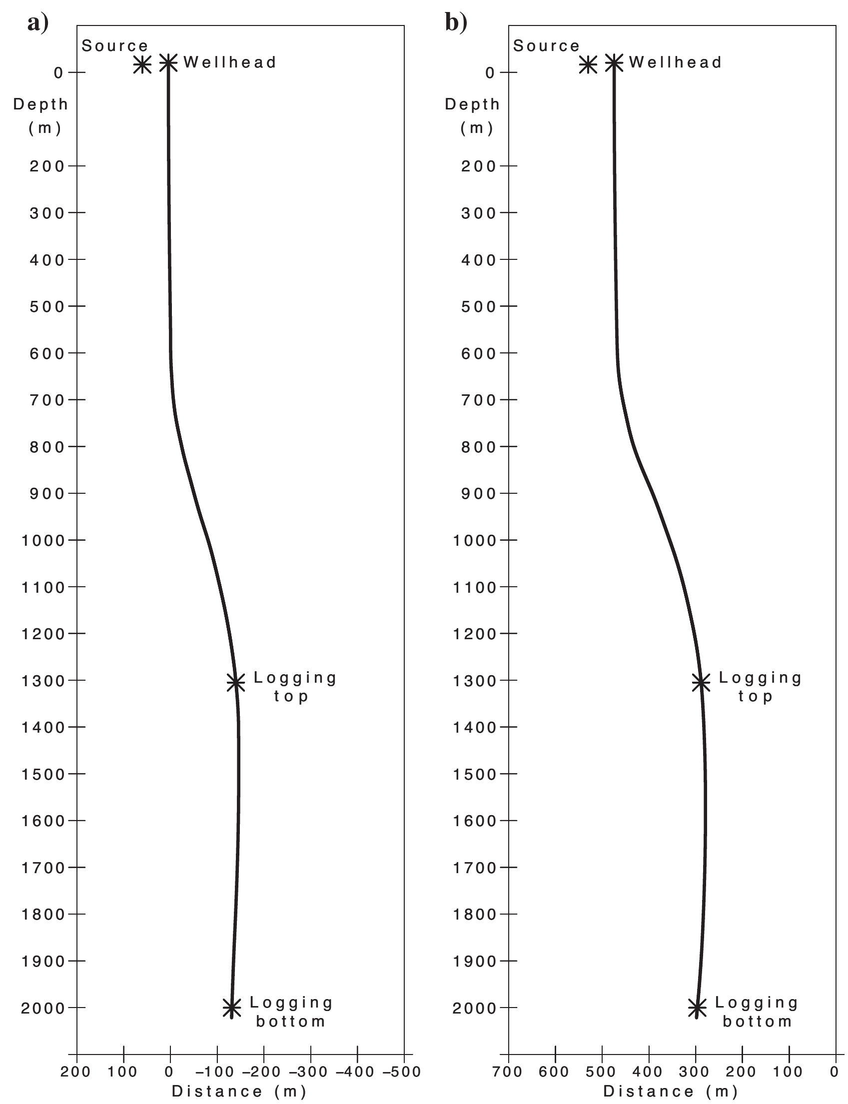

Figure 2. The trajectory of the well in which the measurements were carried out. (a) Projection onto the x_1x_3 plane and (b) projection onto the x_2x_3 plane. The top of the sonic-log depth interval is denoted by *Logging top*, the bottom of the sonic-log depth interval is denoted by *Logging bottom*, and the VSP source is denoted by *Source*.

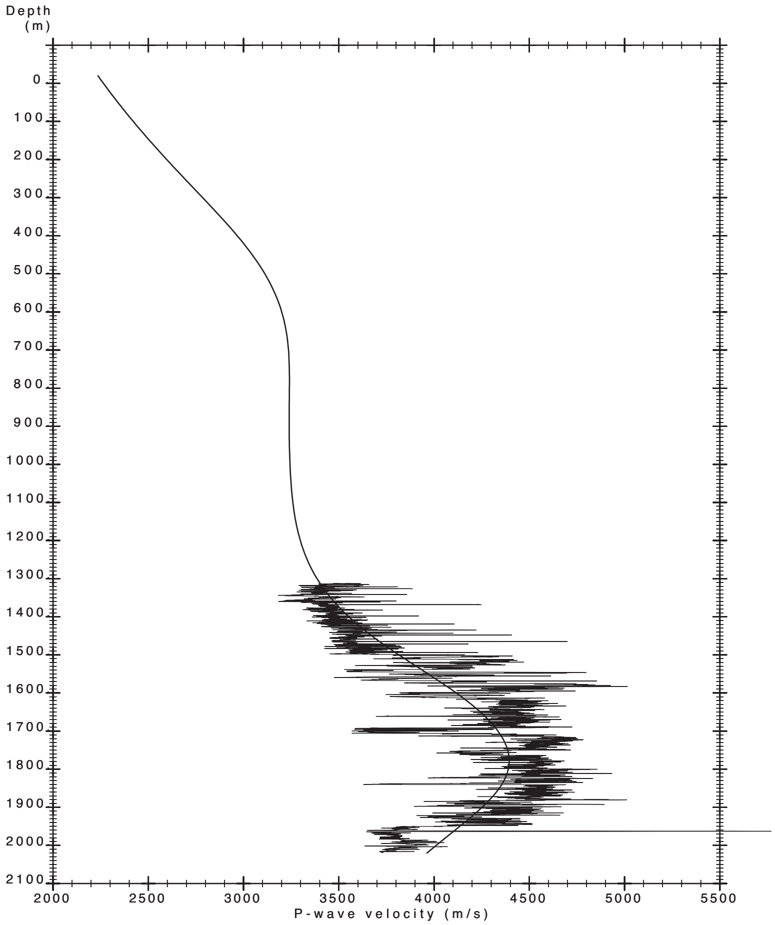

Figure 3. P-wave velocity measured by sonic logging (thin line) in the well, and P-wave velocity (bold line) in the smooth reference velocity model.

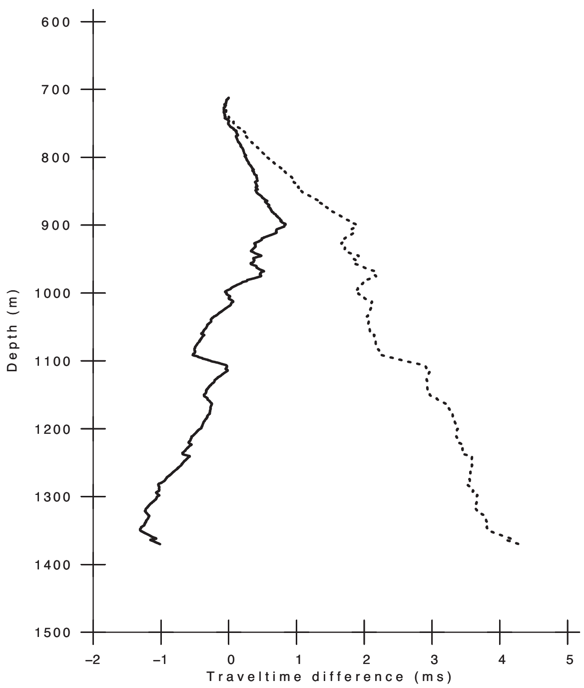

Figure 4. The difference of the sonic-log traveltime τ from the reference traveltime τ^0 (solid line), and the same difference calculated under the assumption of vertical propagation (dotted line).

CONCLUSIONS

We have proposed a method for calculating the sonic-log traveltime in a geologic structure from the values of slowness measured by sonic logging, while taking into account the effects of the nonvertical propagation of seismic waves, resulting from the VSP-source offset and from the heterogeneous velocity in the structure, along with the effects of the well trajectory deviating from strictly vertical. In Appendix A, we also estimated analytically the errors caused by the commonly used assumption of vertical propagation. In the numerical example, the sonic-log traveltime error caused by the assumption of vertical propagation exceeds the typical difference of the measured VSP traveltimes from the sonic-log traveltimes, which indicates clearly that the effects of the nonvertical propagation of seismic waves and of the well trajectory deviating from strictly vertical should not be neglected when calculating the sonic-log traveltime.

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APPENDIX A

ERROR OF SONIC-LOG TRAVELTIME CAUSED BY ASSUMPTION OF QUASI-VERTICAL PROPAGATION

Under quasi-vertical propagation we understand the assumption that

$$p_i(\mathbf{x}) = n_i(\mathbf{x})u(\mathbf{x})\mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})], \qquad \text{(A-1)}$$

i.e., the assumption that all the rays are always parallel with vector n_i. As already mentioned in the section about assumptions about the geologic structure, vector n_i will in most cases be vertical, and the assumption expressed by equation A-1 will then be the assumption of vertical propagation.

By inserting equation A-1 into equation 10 for the trapezoidal quadrature along the well, we obtain the approximation of the sonic-log traveltime for quasi-vertical propagation in the form of

$$\tilde{\tau}(\mathbf{x}^{\mathrm{new}}) = \tilde{\tau}(\mathbf{x}) + \frac{1}{2}[u(\mathbf{x}^{\mathrm{new}}) + u(\mathbf{x})]n_i(\mathbf{x}) \times[x_i^{\mathrm{new}} - x_i]\,\mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})]. \quad \text{(A-2)}$$

We denote the tangent vector to the well by $t_i(\mathbf{x})$, and obtain the increment

$$\Delta\tau(\mathbf{x}) = p_i(\mathbf{x})t_i(\mathbf{x}) \quad \text{(A-3)}$$

of the sonic-log traveltime in the direction along the well. Replacing $p_i(\mathbf{x})$ in equation A-3 by the assumption expressed in equation A-1, we obtain the increment

$$\Delta\tilde{\tau}(\mathbf{x}) = n_i(\mathbf{x})t_i(\mathbf{x})u(\mathbf{x})\,\mathrm{sgn}[n_i(\mathbf{x})p_i^0(\mathbf{x})] \quad \text{(A-4)}$$

of the approximate sonic-log traveltime in the direction along the well under the assumption of quasi-vertical propagation. The local relative sonic-log traveltime error is then

$$\rho\tau = \frac{\Delta\tilde{\tau} - \Delta\tau}{\Delta\tau}. \quad \text{(A-5)}$$

We may calculate and inspect this relative error along the part of the well where the slowness $u(\mathbf{x})$ has been determined by a sonic log.

Note that the calculation of the sonic-log traveltime described in this paper excludes this error caused by the assumption of quasi-vertical propagation, which is the reason we consider this method of calculating the sonic-log traveltime to be superior to Gretener's (1961) method.

We now approximate the relative error given by equation A-5. We decompose the slowness vector into the vector parallel with n_i and into vector $p_i^{\perp}$ perpendicular to n_i,

$$p_i = n_i p^{\parallel} + p_i^{\perp}, \quad \text{(A-6)}$$

and insert it into equation A-3,

$$\Delta\tau = n_i t_i p^{\parallel} + p_i^{\perp} t_i. \quad \text{(A-7)}$$

We insert equation A-6 into equation 5, express $u(\mathbf{x})$ in terms of $p^{\parallel}$ and $p_i^{\perp}(\mathbf{x})$, and insert it into equation A-4,

$$\Delta\tilde{\tau} = n_i t_i p^{\parallel}\sqrt{1 + p_i^{\perp}p_i^{\perp}(p^{\parallel})^{-2}}. \quad \text{(A-8)}$$

We see that $\rho\tau = 0$ if $p_i^{\perp} = 0$. Note that vector $p_i^{\perp} = p_i^{0\perp}$, unlike $p^{\parallel}$, varies smoothly. If we insert equations A-7 and A-8 into equation A-5 and neglect $p_i^{\perp}t_i$ in the denominator, we may decompose the relative error given by equation A-5 into two terms,

$$\rho\tau = \left[\sqrt{1 + \frac{p_i^{\perp}}{p^{\parallel}}\frac{p_i^{\perp}}{p^{\parallel}}} - 1\right] - \frac{p_i^{\perp}}{p^{\parallel}}\frac{t_i}{n_k t_k}. \quad \text{(A-9)}$$

For small $p_i^{\perp}/p^{\parallel}$, the first term may further be approximated,

$$\rho\tau = \frac{1}{2}\frac{p_i^{\perp}}{p^{\parallel}}\frac{p_i^{\perp}}{p^{\parallel}} - \frac{p_i^{\perp}}{p^{\parallel}}\frac{t_i}{n_k t_k}. \quad \text{(A-10)}$$

Finally, introducing the decomposition

$$t_i = n_i t^{\parallel} + t_i^{\perp} \quad \text{(A-11)}$$

and inserting it into equation A-10 we arrive at

$$\rho\tau = \frac{1}{2}\frac{p_i^{\perp}}{p^{\parallel}}\frac{p_i^{\perp}}{p^{\parallel}} - \frac{p_i^{\perp}}{p^{\parallel}}\frac{t_i^{\perp}}{t^{\parallel}}. \quad \text{(A-12)}$$

The first error term is independent of the well inclination and depends on the quasi-horizontal offset of the VSP source from the studied part of the well only. The second error term depends on the quasi-horizontal offset of the VSP source and on the well inclination and may be much larger than the first term for wells deviated from the quasi-vertical. Equation A-12 provides a better understanding of the sonic-log traveltime error caused by neglecting the nonverticality of the well and by not taking into account the nonverticality of the VSP slowness vector in a heterogeneous structure.

APPENDIX B

INTERPOLATION OF THE REFERENCE TRAVELTIME AND SLOWNESS VECTOR

A standard procedure in ray-theory-based methods is that, during ray tracing, all quantities are calculated along the rays, and these quantities are then interpolated between the rays. Assume that we have calculated reference traveltime $\tau^0(\mathbf{x})$ and the thecorresponding reference slowness vector $p_i^0(\mathbf{x})$ at the VSP receivers $\mathbf{x}^{\mathrm{R}}$ situated in the well or in its vicinity (to allow for position measurement uncertainties),

$$\tau^{\mathrm{R}} = \tau^0(\mathbf{x}^{\mathrm{R}}), \quad p_i^{\mathrm{R}} = p_i^0(\mathbf{x}^{\mathrm{R}}). \quad \text{(B-1)}$$

We first interpolate the reference traveltime and the corresponding slowness vector along the straight lines between receivers to *level* $\xi = n_i x_i$. Along the straight lines, we shall apply the linear interpolation of coordinates x_i^{A} and slowness-vector components p_i^{A}, and the cubic interpolation of reference traveltime τ^{A}. We then extrapolate the reference traveltime from points $\mathbf{x}^{\mathrm{A}}$ of the straight line to points $\mathbf{x}$ of the curved well by the linear Taylor expansion. The slowness vector is extrapolated as a constant vector.

Let $x_i^{\mathrm{B}}, \tau^{\mathrm{B}}, p_i^{\mathrm{B}}$ and $x_i^{\mathrm{C}}, \tau^{\mathrm{C}}, p_i^{\mathrm{C}}$ be two consecutive multiplets of values $x_i^{\mathrm{R}}, \tau^{\mathrm{R}}, p_i^{\mathrm{R}}$. We denote

$$w^{\mathrm{B}} = \frac{n_i(\mathbf{x})[x_i - x_i^{\mathrm{C}}]}{n_j(\mathbf{x})[x_j^{\mathrm{B}} - x_j^{\mathrm{C}}]} \quad \text{(B-2)}$$

and

$$w^{\mathrm{C}} = \frac{n_i(\mathbf{x})[x_i - x_i^{\mathrm{B}}]}{n_j(\mathbf{x})[x_j^{\mathrm{C}} - x_j^{\mathrm{B}}]} = 1 - w^{\mathrm{B}}. \quad \text{(B-3)}$$

The linear interpolation of coordinates then reads (Bulant and Klimeš, 1999, equation 2)

$$x_i^{\mathrm{A}} = w^{\mathrm{B}}x_i^{\mathrm{B}} + w^{\mathrm{C}}x_i^{\mathrm{C}}, \quad \text{(B-4)}$$

and the linear interpolation of the slowness vector reads

$$p_k^{\mathrm{A}} = w^{\mathrm{B}}p_k^{\mathrm{B}} + w^{\mathrm{C}}p_k^{\mathrm{C}}. \quad \text{(B-5)}$$

The cubic interpolation of reference traveltime reads (Bulant and Klimeš, 1999, equation 25)

$$\tau^{A} = a(w^{B})\tau^{B} + a(w^{C})\tau^{C} + (w^{B})^{2}w^{C}p_{i}^{B}[x_{i}^{C} - x_{i}^{B}] - w^{B}(w^{C})^{2}p_{i}^{C}[x_{i}^{C} - x_{i}^{B}], \quad \text{(B-6)}$$

where

$$a(w) = w^{2}(3 - 2w) \quad \text{(B-7)}$$

(Bulant and Klimeš, 1999, equation 21).

Then the slowness vector is extrapolated as a constant vector,

$$p_{i}^{0}(\mathbf{x}) = p_{i}^{A}, \quad \text{(B-8)}$$

and the reference traveltime is extrapolated by the linear Taylor expansion,

$$\tau^{0}(\mathbf{x}) = \tau^{A} + p_{i}^{A}[x_{i} - x_{i}^{A}]. \quad \text{(B-9)}$$

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