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On the retrieval of moment tensors from borehole data

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ABSTRACT

The complete moment tensors of seismic sources in homogeneous or vertically inhomogeneous isotropic structures cannot be retrieved using receivers deployed in one vertical borehole. The complete moment tensors can be retrieved from amplitudes of P-waves, provided that receivers are deployed in at least three boreholes. Using amplitudes of P- and S-waves, two boreholes are, in principle, sufficient. Similar rules also apply to transversely isotropic media with a vertical axis of symmetry.

In the case of limited observations, the inversion can be stabilized by imposing the zero-trace constraint on the moment tensors. However, this constraint is valid only if applied to observations of shear faulting on planar faults in isotropic media, which produces double-couple mechanisms. For shear faulting on non-planar faults, for tensile faulting, and for shear faulting in anisotropic media, the zero-trace constraint is no longer valid and can distort the retrieved moment tensor and bias the fault-plane solution.

Numerical modelling simulating the inversion of the double-couple mechanism from real data reveals that the errors in the double-couple and non-double-couple percentages of the moment tensors rapidly decrease with increase in the number of boreholes used. For noisy P- and S-wave amplitudes with noise of 15% of the top amplitude at each channel and for a velocity model biased by 10%, the errors in the double-couple percentage attain 25, 13 and 6% when inverting for the double-couple mechanism from one, two and three boreholes.

1 INTRODUCTION

Careful monitoring of induced microseismicity of reservoirs helps in understanding the fracture process and enables control of the rock fracturing to optimize reservoir production (Lumley 2001; Shoham 2001). The microseismic signals in reservoirs are commonly recorded by linear arrays of uniaxial or triaxial sensors deployed in wells. They are analysed by detection algorithms, and the events are traced to image fracturing as a function of time. Seismic waveforms are used to determine the source–time functions and moment tensors, which provide information about the size and orientation of fractures (Rutledge, Phillips and Schuessler 1998; Nolen-Hoeksema and Ruff 2001; Trifu and Shumila 2002; Rutledge and Phillips 2003). The moment tensors also indicate whether the fracture is opening or closing, which is the key to monitoring the connectivity of the fracture (Ross, Foulger and Julian 1999; Maxwell and Urbancic 2001; Foulger *et al.* 2004).

The complete moment tensor is described by six values and, in principle, can be determined from noise-free amplitudes of the P- and S-waves from the event, observed at three stations at least. The raypaths of waves must have different directions and must not lie in a plane. However, in practice, the moment tensor inversion is more involved: the data are noisy, the stations are deployed in wells in linear arrays thus forming special configurations, and P- and S-waves are not both always available in the recordings. In some cases, this might lead to questions about whether a particular experiment is properly designed and is capable of inverting for the complete moment tensor, and if not, how many moment tensor components can be retrieved. In this paper, I analyse the impact of various sensor configurations on the retrieval of the moment tensors, and

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the limitations of the moment tensor inversion due to inverting only selected types of waves. Using numerical modelling, I demonstrate both the robustness of the inversion when using different borehole configurations and the bias of the results if we apply the zero-trace moment tensor condition which is commonly used for stabilizing the inversion (see Dufumier and Rivera 1997; Nolen-Hoeksema and Ruff 2001).

2 RADIATION FUNCTIONS IN ISOTROPIC AND ANISOTROPIC MEDIA

The wavefield generated by a point seismic source is expressed as follows (Aki and Richards 2002, eq. 3.23):

$$u_i(\mathbf{x}, t) = m_{kl} * G_{ik,l}, \tag{1}$$

where $u_i(\mathbf{x}, t)$ is the displacement vector measured at point $\mathbf{x}$ and time t, $m_{kl}(t) = M_{kl} f(t)$ is the moment tensor function, M_{kl} is the seismic moment tensor, $f(t)$ is the source–time function, $G_{ik},_l$ is the spatial derivative of the Green's function, and the symbol $*$ denotes time convolution. The moment tensor function describes the properties of the source, and the Green's function describes the properties of the medium in which the source is situated. Assuming that the medium is homogeneous and isotropic and that the receivers are far from the source (at least several wavelengths), we can express equation (1) as (Aki and Richards 2002, eqs 4.24, 4.25):

$$u_i(\mathbf{x}, t) = \frac{R_i^P}{4\pi\rho\alpha^3}\frac{\dot{f}\left(t-\tau^P\right)}{r} + \frac{R_i^S}{4\pi\rho\beta^3}\frac{\dot{f}\left(t-\tau^S\right)}{r}, \tag{2}$$

where ρ is the density, α and β are the velocities of P- and S-waves, r is the distance from the source to the receiver, and τ^{P} and τ^{S} are the P- and S-wave traveltimes. The vectors $\mathbf{R}^{\mathrm{P}}$ and $\mathbf{R}^{\mathrm{S}}$ are the P- and S-wave radiation functions, given by

$$R_i^{\mathrm{P}} = n_i n_k n_l M_{kl}, \tag{3}$$

$$R_i^{\mathrm{S}} = -\left(n_i n_k - \delta_{ik}\right) n_l M_{kl}, \tag{4}$$

where the vector $\mathbf{n}$ denotes the unit vector directed from the source to the receiver (the ray vector), and δ_{kl} is the Kronecker delta. The Einstein summation convention over repeated subscripts is applied. Assuming an isotropic moment tensor,

$$M_{kl} = M_0 \delta_{kl}, \tag{5}$$

equations (3) and (4) yield

$$R_i^{\mathrm{P}} = M_0 n_i, \quad R_i^{\mathrm{S}} = 0. \tag{6}$$

Since the S-wave radiation function is zero in all directions, the source described by the isotropic moment tensor cannot be detected by measuring S-waves.

Assuming that the medium is homogeneous and anisotropic with no triplications and singularities, and the receivers are far from the source, we can express equation (1) as (Vavryčuk 2002, eq. 10):

$$\begin{aligned} u_i(\mathbf{x}, t) = {} & \frac{1}{4\pi\rho}\frac{R_i^{\mathrm{P}}}{v^{\mathrm{P}} c^{\mathrm{P}}\sqrt{K^{\mathrm{P}}}}\frac{\dot{f}\left(t-\tau^{\mathrm{P}}\right)}{r} \\ & + \frac{1}{4\pi\rho}\frac{R_i^{\mathrm{S1}}}{v^{\mathrm{S1}} c^{\mathrm{S1}}\sqrt{K^{\mathrm{S1}}}}\frac{\dot{f}\left(t-\tau^{\mathrm{S1}}\right)}{r} \\ & + \frac{1}{4\pi\rho}\frac{R_i^{\mathrm{S2}}}{v^{\mathrm{S2}} c^{\mathrm{S2}}\sqrt{K^{\mathrm{S2}}}}\frac{\dot{f}\left(t-\tau^{\mathrm{S2}}\right)}{r}, \end{aligned} \tag{7}$$

where v, c, K and τ are the group velocity, phase velocity, Gaussian curvature of the slowness surface and traveltime, respectively, and the superscripts denote the type of wave (P, S1 or S2). The radiation function $\mathbf{R}$ for each wave is expressed as

$$R_i = g_i g_k s_l M_{kl}, \tag{8}$$

where the vector $\mathbf{g}$ denotes the polarization vector of the wave, and $\mathbf{s}$ denotes the direction of the slowness vector. The vector $\mathbf{s}$ is generally different from the ray vector $\mathbf{n}$ calculated as the direction of the group-velocity vector, $\mathbf{n} = \mathbf{v}/v$.

3 MOMENT TENSOR INVERSION IN ISOTROPIC MEDIA

3.1 One-borehole experiment

Let us consider an experiment with a vertical string of three-component receivers placed in one vertical borehole. Let us fix the local coordinate system in such a way that the rays from the source to the receivers lie in the x_1–x_3 plane. The ray vector $\mathbf{n}$ is then expressed as $\mathbf{n} = (n_1, 0, n_3)^{\mathrm{T}}$ and the P- and S-wave radiation functions of the source in an isotropic medium are given by

$$\mathbf{R}^{\mathrm{P}} = \begin{bmatrix} n_1^3 M_{11} + 2n_1^2 n_3 M_{13} + n_1 n_3^2 M_{33} \\ 0 \\ n_1^2 n_3 M_{11} + 2n_1 n_3^2 M_{13} + n_3^3 M_{33} \end{bmatrix}, \tag{9}$$

$$\mathbf{R}^{\mathrm{S}} = \begin{bmatrix} n_3^2 n_1 \left(M_{11} - M_{33}\right) + \left(n_3^2 - n_1^2\right) n_3 M_{13} \\ n_1 M_{12} + n_3 M_{23} \\ n_1^2 n_3 \left(M_{33} - M_{11}\right) + \left(n_1^2 - n_3^2\right) n_1 M_{13} \end{bmatrix}. \tag{10}$$

Equations (9) and (10) indicate that the P-wave radiation function does not depend on the moment tensor components M_{12},

M_{22} and M_{23}, and the S-wave radiation function does not depend on the component M_{22}. Moreover, the S-wave radiation function is not sensitive to the trace of **M**, because it depends only on the difference M_{11}–M_{33}. This implies that data from a single borehole are insufficient to invert for the complete moment tensor. When inverting P-waves, we are able to obtain three components of the moment tensor: M_{11}, M_{33} and M_{13}. When inverting S-waves, we are able to obtain three components of the moment tensor: M_{12}, M_{13}, M_{23}, and the difference M_{11}–M_{33}. Inverting P- and S-waves simultaneously yields five components of the moment tensor. The unrecoverable component of **M** is M_{22}. This result has been derived for homogeneous isotropic media, but it obviously also applies to horizontally layered media or to inhomogeneous media with a vertical gradient. The impossibility of recovering the complete moment tensor from a single borehole has been recognized by Nolen-Hoeksema and Ruff (2001).

3.2 Multi-borehole experiment

Let us consider an experiment with a vertical string of three-component receivers placed in several vertical boreholes. Let us define the local coordinate system in such a way that the rays from the source to the receivers in the first borehole lie in the x_1–x_3 plane. Obviously, data from additional boreholes improve the conditionality of the moment tensor equations. Inserting direction vectors **n** into equation (3) for receivers in all boreholes, we obtain a set of equations for the moment tensor. Depending on the number of boreholes used, we are able to determine a different number of moment tensor components (see Tables 1 and 2).

For determining the complete moment tensor we have to measure the amplitudes of P-waves from at least three boreholes, or the amplitudes of P- and S-waves from two boreholes. When utilizing the amplitudes of S-waves, the complete moment tensor cannot be retrieved from any number of boreholes, since S-waves are not sensitive to the trace of the moment tensor.

Table 1 Number of components of **M** recoverable in isotropy

Waves	1 well	2 wells	3 wells or more
P	3	5	6
S	4	5	5
P+S	5	6	6

Table 2 Components of **M** unrecoverable in isotropy. The moment tensor components are specified in the local coordinate system, in which the first borehole is along the x_3-axis and the rays lie in the x_1–x_3 plane

Waves	1 well	2 wells	3 wells or more
P	M_{12}, M_{22}, M_{23}	M_{12} /M_{22}	-
S	Tr(**M**), M_{22}	Tr(**M**)	Tr(**M**)
P+S	M_{22}	-	-

4 MOMENT TENSOR INVERSION IN ANISOTROPIC MEDIA

In this section, I first give a short discussion on the basic differences between moment tensor inversion in isotropy and in general anisotropy. I then focus on the inversion in transverse isotropy with a vertical axis of symmetry that is the most frequent anisotropy in exploration seismic.

4.1 One-borehole experiment in triclinic anisotropy

Let us consider a one-borehole experiment situated in a medium with triclinic anisotropy. Let us define the local coordinate system so as to observe the rays from the source to the receivers in the x_1–x_3 plane. The ray vector **n** is then expressed as $\mathbf{n} = (n_1, 0, n_3)^\mathrm{T}$ and the radiation functions of a seismic source are expressed by equation (8). Since the slowness and polarization vectors in anisotropic media have directions $\mathbf{g} = (g_1, g_2, g_3)^\mathrm{T}$, $\mathbf{s} = (s_1, s_2, s_3)^\mathrm{T}$, which need not be parallel or perpendicular to the ray vector, the radiation functions depend on all components of the moment tensors. Consequently, one-borehole data can be used, in principle, for inverting for the complete moment tensor. However, from a practical viewpoint, this task is infeasible for several reasons. Firstly, we do not usually have an accurate and complete knowledge of triclinic anisotropy. Secondly, in the majority of geophysically interesting cases, anisotropy displays some symmetry. This means that the radiation functions need not depend on all moment tensor components, as in isotropic media, and therefore the complete moment tensor cannot be retrieved. Thirdly, anisotropy is usually weak and the medium is inhomogeneous, which causes the inversion for the complete moment tensor to become unstable.

4.2 One- and multi-borehole experiments in transverse isotropy

Let us now consider a transversely isotropic medium with a vertical axis of symmetry. In this case, the seismic source generates P-, SV- and SH-waves with the following polarization vectors and directions of the slowness vectors:

$$\mathbf{g}^{\mathrm{P}}=\begin{bmatrix} g_1^{\mathrm{P}} \\ 0 \\ g_3^{\mathrm{P}} \end{bmatrix},\quad \mathbf{g}^{\mathrm{SV}}=\begin{bmatrix} g_1^{\mathrm{SV}} \\ 0 \\ g_3^{\mathrm{SV}} \end{bmatrix},\quad \mathbf{g}^{\mathrm{SH}}=\begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix},$$

$$\mathbf{s}^{\mathrm{P}}=\begin{bmatrix} s_1^{\mathrm{P}} \\ 0 \\ s_3^{\mathrm{P}} \end{bmatrix},\quad \mathbf{s}^{\mathrm{SV}}=\begin{bmatrix} s_1^{\mathrm{SV}} \\ 0 \\ s_3^{\mathrm{SV}} \end{bmatrix},\quad \mathbf{s}^{\mathrm{SH}}=\begin{bmatrix} s_1^{\mathrm{SH}} \\ 0 \\ s_3^{\mathrm{SH}} \end{bmatrix}. \tag{11}$$

The radiation functions are expressed as follows:

$$\mathbf{R}^{\mathrm{W}}=$$
$$\begin{bmatrix} \left(g_1^{\mathrm{W}}\right)^2 s_1^{\mathrm{W}} M_{11}+g_1^{\mathrm{W}}\left(g_1^{\mathrm{W}} s_3^{\mathrm{W}}+g_3^{\mathrm{W}} s_1^{\mathrm{W}}\right) M_{13}+g_1^{\mathrm{W}} g_3^{\mathrm{W}} s_3^{\mathrm{W}} M_{33} \\ 0 \\ g_1^{\mathrm{W}} g_3^{\mathrm{W}} s_1^{\mathrm{W}} M_{11}+g_3^{\mathrm{W}}\left(g_1^{\mathrm{W}} s_3^{\mathrm{W}}+g_3^{\mathrm{W}} s_1^{\mathrm{W}}\right) M_{13}+\left(g_3^{\mathrm{W}}\right)^2 s_3^{\mathrm{W}} M_{33} \end{bmatrix}, \tag{12}$$

$$\mathbf{R}^{\mathrm{SH}}=\begin{bmatrix} 0 \\ s_1^{\mathrm{SH}} M_{12}+s_3^{\mathrm{SH}} M_{23} \\ 0 \end{bmatrix}, \tag{13}$$

where W stands for P or SV.

Equations (12) and (13) indicate that the P- and SV-wave radiation functions do not depend on the components M_{12}, M_{22} and M_{23}, and the SH-wave radiation function does not depend on the components M_{11}, M_{22}, M_{33} and M_{13}. Hence, when inverting the P- or SV-waves gathered at one borehole, we are able to obtain three components of the moment tensor, and when inverting the SH-waves, we are able to obtain two components of the moment tensor (see Table 3). Inverting the P-, SV- and SH-waves simultaneously yields five components of the moment tensor. As in the case of isotropic media, we cannot recover the M_{22} component using the data gathered at one vertical borehole (see Table 4). This result is also valid for other symmetries of anisotropy, such as orthorhombic anisotropy, provided the seismic source and the borehole lie in the symmetry plane.

Table 3 Number of components of **M** recoverable in transverse isotropy with a the vertical axis of symmetry

Waves	1 well	2 wells	3 wells or more
P	3	5	6
SV	3	5	6
SH	2	3	3
SV+SH	5	6	6
P+SV+SH	5	6	6

Table 4 Components of **M** unrecoverable in transverse isotropy with a vertical axis of symmetry. The moment tensor components are specified in the local coordinate system, in which the first borehole is along the x_3-axis and rays lie in the x_1–x_3 plane

Waves	1 well	2 wells	3 wells or more
P	M_{12}, M_{22}, M_{23}	M_{12} /M_{22}	-
SV	M_{12}, M_{22}, M_{23}	M_{12} /M_{22}	-
SH	M_{11}, M_{22}, M_{33}, M_{13}	M_{11}, M_{22}, M_{33}	M_{11}, M_{22}, M_{33}
SV+SH	M_{22}	-	-
P+SV+SH	M_{22}	-	-

In the case of the two-borehole experiment, we can determine five components of the moment tensor when inverting P- or SV-waves, and we can determine three components of the moment tensor when inverting SH-waves. If we combine SV- and SH-waves, or P-, SV- and SH-waves, then the complete moment tensor can be determined.

5 NUMERICAL MODELING

In this section, I focus on estimating errors produced by the moment tensor inversion of amplitudes of P- and S-waves observed at one, two or three boreholes located in an isotropic medium. To simulate conditions of real experiments, I assume in the inversion: (1) an inaccurate knowledge of the medium, (2) a mislocated source and (3) noisy amplitudes of waves. Moreover, the inversion is performed under the assumption that the moment tensors have a zero trace. It is necessary to apply this condition to the one-borehole experiment, where only five components of the moment tensors can be inverted (see Table 1), in order to stabilize the inversion (see Nolen-Hoeksema and Ruff 2001). Application of this condition in multi-borehole experiments is not necessary, but is still frequently adopted in practice. Here, I show how the results can be distorted if the source is not shear and the true moment tensor has a non-zero trace.

5.1 Geometry of the experiment

Let us assume a seismic source located at depth of 1.5 km. The three vertical observation boreholes are positioned at a distance of 0.5 km from the source with azimuths of 90° (borehole 1), 60° (borehole 2) and 30° (borehole 3). Each borehole is contains 11 three-component receivers deployed at depths from 0.5 to 1.5 km in steps of 100 m. The medium is

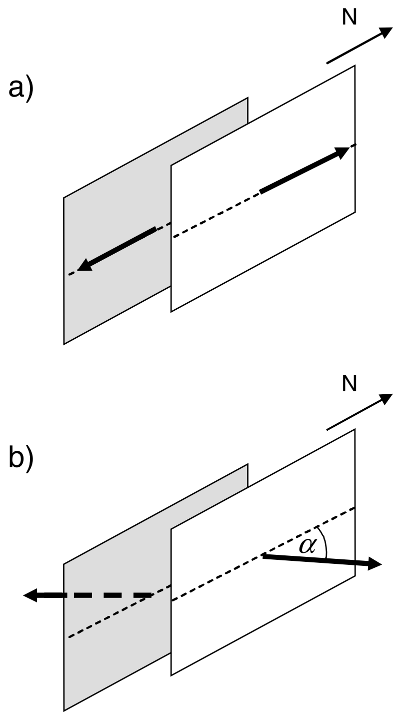

Figure 1 Geometry of faulting: (a) shear faulting, (b) tensile faulting. The fault is vertical in the N–S direction, the slip lies in the horizontal plane.

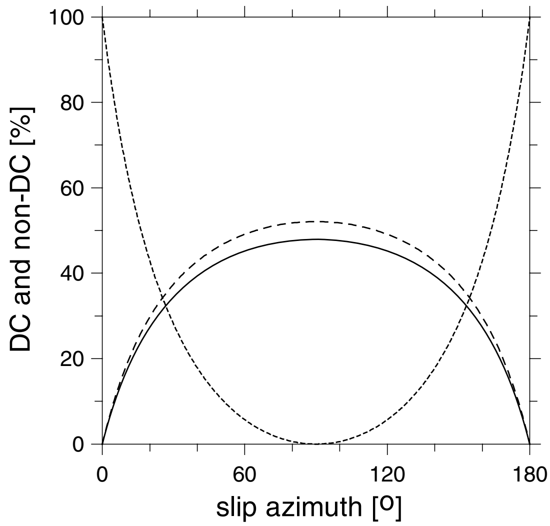

Figure 2 The double-couple and non-double-couple components of the moment tensor. The percentages of the isotropic (solid line), CLVD (dashed line), and double-couple (DC) (dotted line) components are shown as a function of the slip azimuth. The value of the azimuth coincides with the deviation of the slip from the fault plane. The percentages of the double-couple and non-double-couple components were calculated using formulae (8a–c) of Vavryčuk (2001).

homogeneous and isotropic with P- and S-wave velocities, $v^{\mathrm{P}} = 4.0$ km/s and $v^{\mathrm{S}} = 2.5$ km/s, respectively.

The seismic source is defined by the orientation of the fault plane and by the slip vector (see Vavryčuk 2001, eq. 1). The fault plane is vertical and is characterized by a strike of 0°. The slip lies in the horizontal plane and its azimuth varies from 0° to 180°. Hence, the sources studied cover all possible combinations, from the pure shear source (see Fig. 1a, the slip azimuths are 0° and 180°) to the pure tensile source (the slip azimuth is 90°). Physically, the tensile sources correspond to opening of faults or fractures during the hydrofracture experiments (see Fig. 1b)

Since the sources under study are generally non-shear, the moment tensors contain the double-couple part as well as the non-double-couple part (see Vavryčuk 2001). The non-double-couple part consists of the compensated linear vector dipole (CLVD) component and the isotropic component. The percentages of the isotropic and CLVD components depend on the orientation of the slip with respect to the fault normal (see Fig. 2). If the slip lies in the fault plane (slip azimuth 0° or 180°), the source is pure shear and contains no non-double-couple components. For the other directions, the source is partly tensile and the non-double-couple components are non-zero. For a pure tensile source (slip azimuth 90°), the double-couple percentage is zero, and the isotropic and CLVD components are positive and attain their maximum values.

5.1 Inversion scheme

In the next numerical tests, the inversion using far-field amplitudes of P- and S-waves is used. The amplitudes A_i observed at one three-component sensor are expressed as follows:

$$A_i^{\mathrm{W}} = M_{kl} G_{ik,l}^{\mathrm{W}}, \tag{14}$$

where W denotes P or S, and $G_{ik,l}^{\mathrm{P}}$ and $G_{ik,l}^{\mathrm{S}}$ are defined as

$$G_{ik,l}^{\mathrm{P}} = \frac{n_i n_k n_l}{4\pi\rho\alpha^3 r} \quad \text{and} \quad G_{ik,l}^{\mathrm{S}} = -\frac{(n_i n_k - \delta_{ik})\, n_l}{4\pi\rho\beta^3 r}. \tag{15}$$

By observing amplitudes at many sensors, equation (14) can be used to form a system of linear equations for unknown components M_{kl}. The optimum moment tensor is then solved by minimizing the sum of differences between the theoretical and observed amplitudes in the least-squares sense.

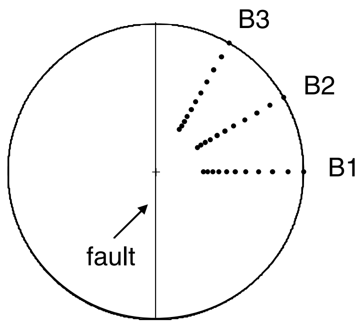

Figure 3 The configuration of the experiment in the upper-hemisphere equal-area projection. B1, B2 and B3 denote the boreholes; dots mark the positions of the sensors. The plus sign marks the position of the source. The straight line shows the projection of the fault.

5.3 Inversion from noise-free amplitudes

First, I assume that the observed amplitudes are noise-free. I compare results of the moment tensor inversion performed under the assumptions of an exact and an approximate knowledge of the medium and of an accurately located and a mislocated source. The approximate model of the medium differs from the exact model by 10%, and is characterized by P- and S-wave velocities of 4.4 and 2.75 km/s, respectively. The approximate directions of rays from the source to the receivers are shifted due to the mislocation of the source by 10° in azimuth.

For the above-described geometry of faulting and the borehole configuration (see Fig. 3), I calculated the true moment tensor of the source using equation (1) of Vavryčuk (2001) and theoretical amplitudes of the P- and S-waves radiated from the source and recorded by borehole sensors using equations (14) and (15). I then inverted the amplitudes for the complete moment tensor using exact and approximate Green's functions. I performed three numerical experiments by inverting the data from boreholes: (a) B1, (b) B1 and B2, and (c) B1, B2 and B3. The retrieved moment tensors were constrained to have zero trace, which is equivalent to zero isotropic components.

Figure 4 shows the errors in the percentages of compensated linear vector dipole (CLVD) and double-couple components of the source, and the errors in the strike of the fault calculated when inverting with the exact medium model and with the exact source location. The figures indicate that all inversions were successful for pure shear sources: for the slip azimuths of 0° and 180°, the inversions yield correct values of the CLVD and double-couple components, and the retrieved orientation of the fault is also correct. However, the results are less accurate for the other directions. For the one-borehole experiment, the errors in the CLVD and double-couple components reach values of up to 80% and 60%, respectively. For multi-borehole experiments, the errors in the CLVD components reach values of up to 40%. The errors in the double-couple components decrease with an increasing number of boreholes, being about 15% for the three-borehole experiment. Also the retrieved strike of the fault is incorrect. The errors can reach values of up to 45° and their dependence on the slip azimuth is roughly the same for all experiments.

The maximum errors in the strike of the fault and in the percentages of the double-couple and CLVD components are produced when the pure tensile source is inverted (slip azimuth 90°). This is caused by the assumption of the zero-trace moment tensors, which is completely inconsistent with the pure tensile source. Of course, if we do not apply the zero-trace constraint then we obtain the correct strike of the fault as well as the correct percentages of the double-couple, CLVD and isotropic components. However, this is possible only in multi-borehole experiments, because they are robust enough to invert for the complete moment tensor without applying any *a priori* constraints.

Figure 5 shows the errors in the percentages of the CLVD and double-couple components of the source, and the errors of the strike of the fault, when inverting with an approximate medium model and with a mislocated source. In this case, the inversion yields inaccurate values of the double-couple and CLVD components and the strike, even for pure shear sources (slip azimuths 0° and 180°). For the one-borehole experiment, the error in the double-couple and CLVD components is about 20%. For the two- and three-borehole experiments the error decreases to 12% and 5%, respectively. The error of the strike is roughly 10° for all experiments and is produced by the mislocation of the source. For tensile sources, the errors of the double-couple and CLVD components and the strike increase to values similar to those produced when inverting for the moment tensor with an exact modelled medium and with the exact source location. This indicates that an incorrect assumption imposed on the source (the zero isotropic component) distorts the results more seriously than the application of the inaccurate medium model or the mislocated source.

5.4 Inversion from noisy amplitudes

Numerical experiments similar to those described in Section 5.3 have also been performed for noisy data. The noisy

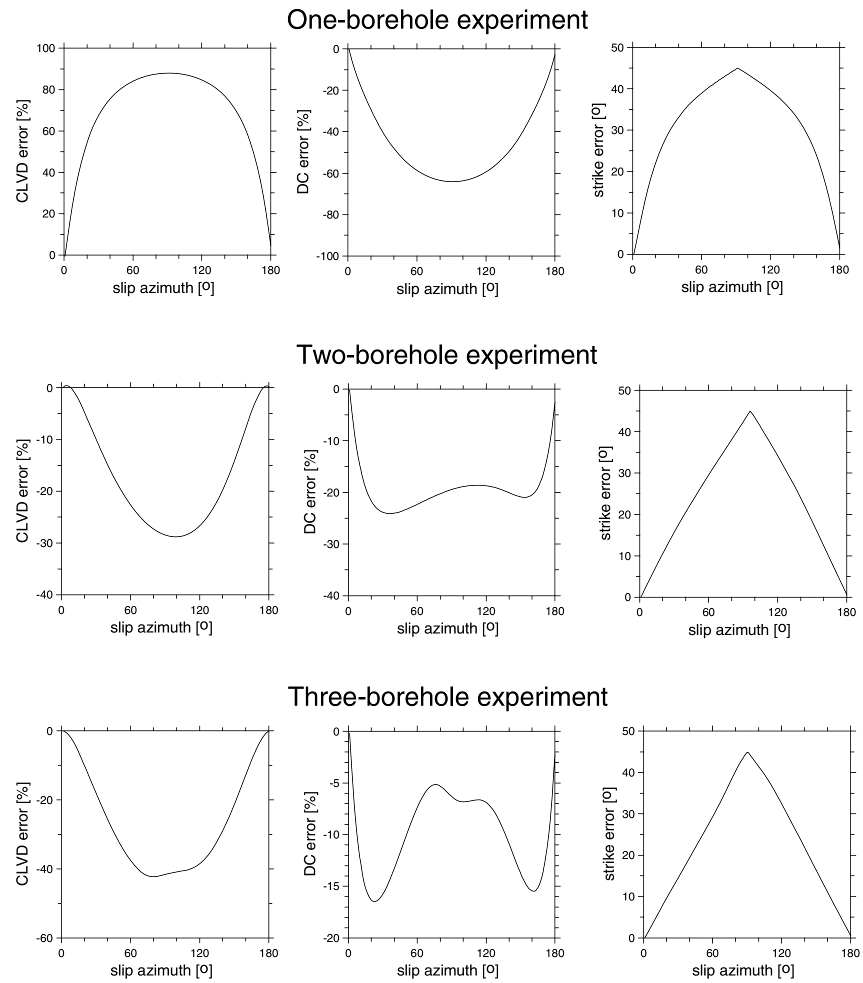

Figure 4 The errors in the CLVD and double-couple components and the strike as a function of the slip direction for the one-borehole, two-borehole and three-borehole experiments. The errors are calculated as the difference between the exact and calculated values. The inversion is performed using the exact medium model and the accurate location of the source.

P- and S-wave amplitudes were generated from the true P- and S-wave amplitudes by adding noise with a uniform distribution and with the maximum amplitude reaching ±15% of the top amplitude at each channel. In order to obtain statistically relevant results, we performed 100 moment tensor inversions, each with a different noise realization. In the inversions, we consider the approximate medium model and the mislocated source as in Section 5.3. We assumed two types of source: a pure double-couple source (with slip azimuth of 0°) and a tensile source (with slip azimuth of 60°). The moment tensors were constrained to have the zero trace.

Figure 6 shows the histograms of the CLVD and double-couple components and strike errors for the shear event. The most frequent error in the percentages of the CLVD and double-couple components is about 24% for the one-borehole experiment. For the two- and three-borehole experiments, the errors decrease to 13% and 6%, respectively. Hence, as expected, the one-borehole experiment is most vulnerable to errors in amplitudes and to mismodelling of the medium. The error in the strike is about 10° for all experiments and is due to the mislocation of the source.

Figure 7 shows the histograms of the CLVD, double-couple and strike errors for the tensile event with a slip azimuth of 60°. In this case, the inversion totally fails for the one-borehole experiment. The two- and three-borehole experiments also fail in estimating the strike of the fault and the CLVD percentage. The estimation of the double-couple percentage is, however, better: the three-borehole experiment yields the most frequent error in the double-couple percentage of 6% only. This value is in agreement with the results obtained by the inversion from noise-free data. This indicates that random noise is not the primary factor causing deterioration in the accuracy of the results. The primary origin of errors of the moment tensor inversion in the multi-borehole experiments is the zero-trace constraint.

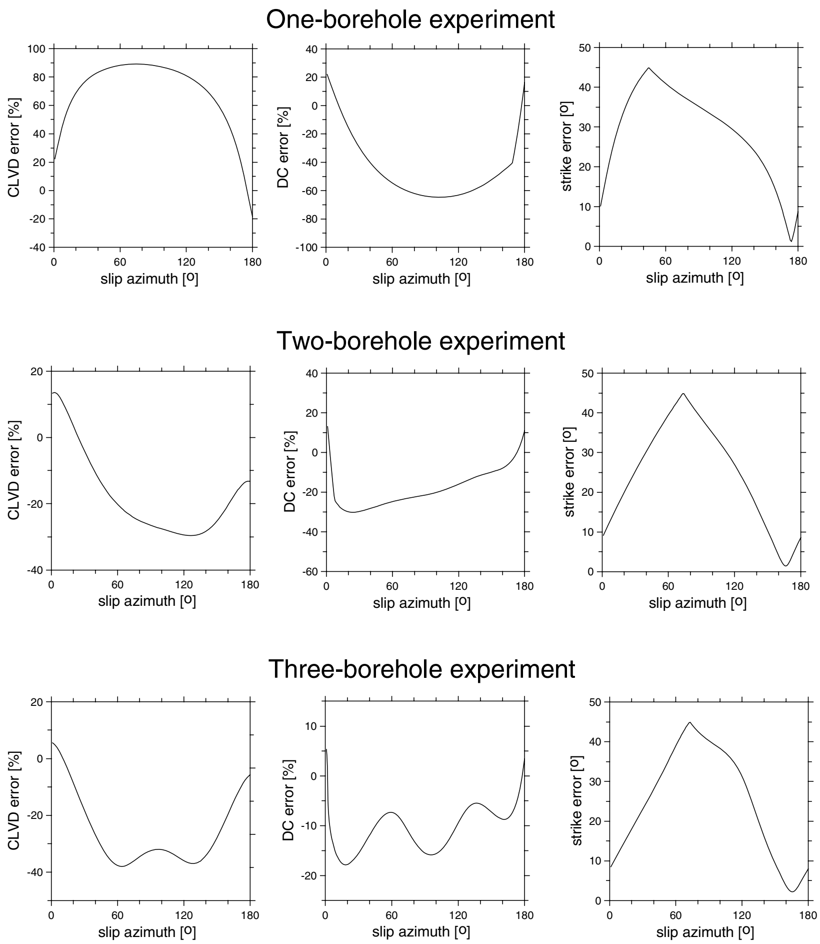

Figure 5 The errors in the CLVD and double-couple components and the strike as a function of the slip direction for the one-borehole, two-borehole, and three-borehole experiments. The errors are calculated as the difference between the exact and calculated values. The inversion is performed using the approximate medium model and the mislocated source.

6 CONCLUSIONS

It is not possible to retrieve the complete moment tensor of seismic sources in a homogeneous isotropic structure from data gathered by receivers deployed in one vertical (or straight inclined) borehole. Three components of the moment tensor can be retrieved using amplitudes of P-waves, four components using amplitudes of S-waves, and five components using amplitudes of both P- and S-waves. The complete moment tensor can be retrieved from amplitudes of P-waves, provided that receivers are deployed in three parallel boreholes or in at least two non-parallel boreholes. The boreholes must be in different azimuths from the source. Using the amplitudes of P- and S-waves, two (either parallel or non-parallel) boreholes are, in principle, sufficient to retrieve the complete moment tensor. Using amplitudes of the S-waves only, we can never determine the trace of the moment tensor, regardless of the number of boreholes used.

The above conclusions also apply to layered isotropic structures or to isotropic structures with a velocity gradient, provided the normal to the layers or the gradient direction is along the direction of the borehole. If the medium is transversely isotropic with the symmetry axis parallel to the borehole, the moment tensors can be retrieved under conditions similar to those that are valid for isotropic media. The only difference is that the SV- and SH-waves in transversely isotropic media contain slightly more information than the S-waves in isotropic media. For example, we can invert for the complete moment tensor using the amplitudes of the SV- and SH-waves in transversely isotropic media, gathered at least at two boreholes (see Table 3), while we can never do this using the amplitudes of the S-waves in isotropic media (see Table 1).

If the geometry of the experiment prevents retrieval of the complete moment tensor, the inversion can be stabilized by imposing the condition of zero trace on the moment tensor. This works for inverting from the P- and S-wave amplitudes

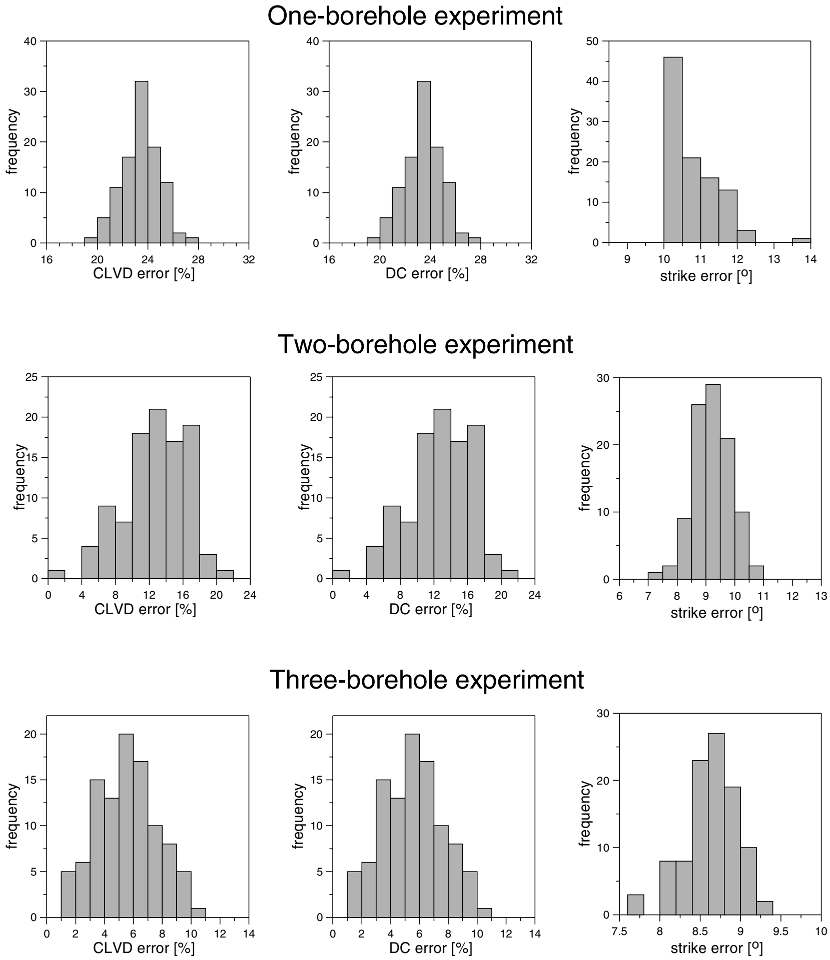

Figure 6 The histograms of errors in the CLVD and double-couple components and the strike. The true moment tensor is pure double-couple. The inversion is performed with noisy amplitudes and using the approximate medium model and the mislocated source.

measured in one borehole, from the P-wave amplitudes measured in two boreholes, or from the S-wave amplitudes measured in two or more boreholes. The zero-trace condition, however, may be applied legitimately only to observations of shear faulting on planar faults in isotropic media, which produces double-couple mechanisms. If the source area is anisotropic (see Rössler, Rümpker and Krüger 2004; Vavryčuk 2005, 2006) or if the source is more complex, containing a non-zero tensile component (see Vavryčuk 2001), the zero-trace condition is no longer valid. In this case, the source produces a general non-double-couple mechanism (Frohlich 1994; Julian, Miller and Foulger 1998; Miller, Foulger and Julian 1998) and application of the zero-trace constraint can distort the retrieved moment tensor and bias the fault-plane solution. The numerical modelling shows that when inverting tensile sources under the zero-trace constraint, the double-couple component can be significantly overestimated and the retrieved CLVD component can even have the wrong sign. The bias in the fault orientation is proportional to the amount of the tensile component in the source and can attain values up to 45° for pure tensile faulting. Obviously, to eliminate these errors, the moment tensors must be inverted without any constraint and the experiments must be designed to be sufficiently robust for doing this job. This can be achieved by an experiment with at least two monitoring boreholes and by processing both P- and S-waves. Optimally, the inversion for the complete moment tensor should be supplemented by estimates of their reliability and by error analysis (Šílený 1998; Trifu, Angus and Shumila 2000; Trifu and Shumila 2002; Šílený 2004).

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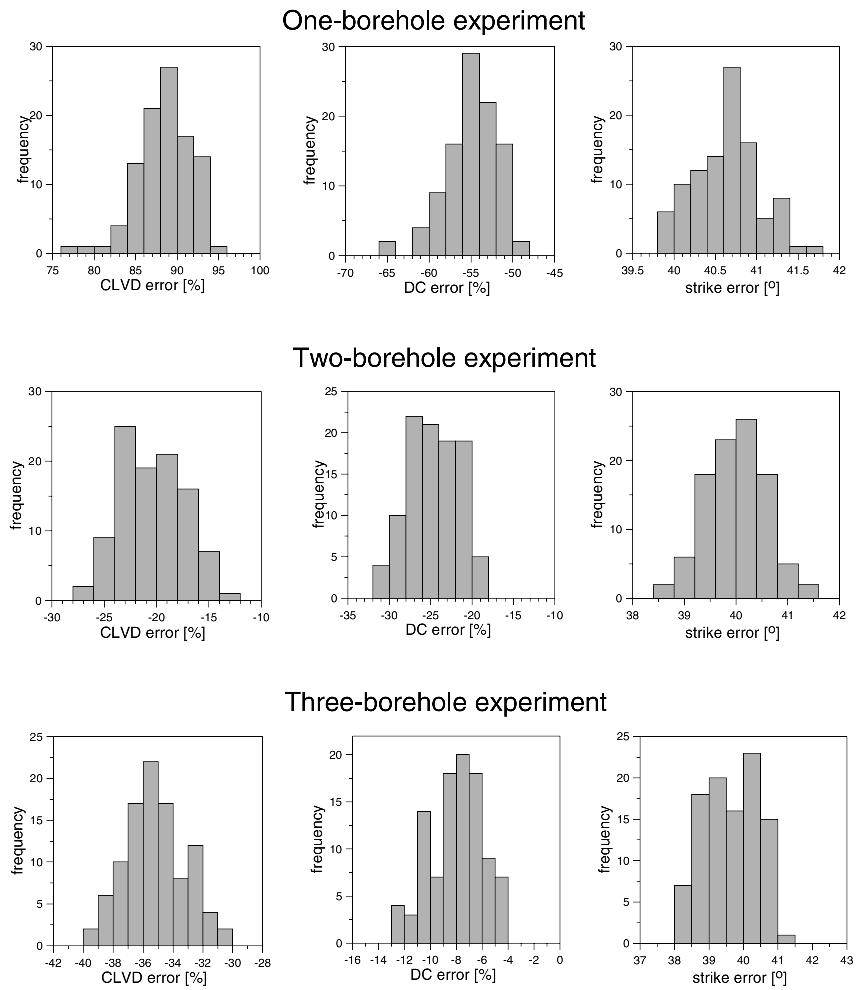

Figure 7 The histograms of errors in the CLVD and double-couple components and the strike. The true moment tensor is non-double-couple (the slip azimuth is 60°). The inversion is performed with noisy amplitudes and using the approximate medium model and the mislocated source.

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