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Comment to "qS-waves in a vicinity of the axis of symmetry of homogeneous transversely isotropic media", by M. Popov, G.F. Passos, and M.A. Botelho [Wave Motion 42 (2005) 191–201]

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Abstract

A new asymptotic formula for S waves propagating near the symmetry axis in transversely isotropic elastic media derived by Popov et al. [Wave Motion 42 (2005) 191–201] is discussed and commented. It is shown that the formula is a modification of the previously published formula by Vavryčuk [Geophys. J. Int. 138 (1999) 581–589]. The formula of Popov et al. is less accurate and valid under more restrictive conditions than that of Vavryčuk.

A high-frequency modelling of wavefields in transversely isotropic (TI) media is more complicated in directions near the symmetry axis than in other directions, because we observe a so-called S-wave kiss singularity along the axis. The S-wave singularity causes anomalies in the field of polarization vectors (see Fig. 1) and possibly also in the shape of the slowness and wave surfaces. For these reasons, the standard zeroth-order ray theory is inapplicable and more sophisticated methods must be used to model the wavefield correctly. For example, Vavryčuk [1] proposed to apply higher-order ray theory, and Gridin [2] developed a high-frequency asymptotics using a uniform stationary phase method. Other approaches valid for kiss singularities in any type of anisotropy are presented in [3,4].

Recently, this topic was also studied by Popov et al. [5], where the authors claim that the previous results presented in [1,2] are either questionable or incorrect, and they propose another high-frequency asymptotics valid for directions near the singularity. In my comment, I will show that the assertion of Popov et al. [5] regarding the previously published results is not justified and that the results in [5] are not in contradiction with the previously published results. Moreover, I will show that the formulas in [5] are valid under stronger restrictions than those derived in [1,2].

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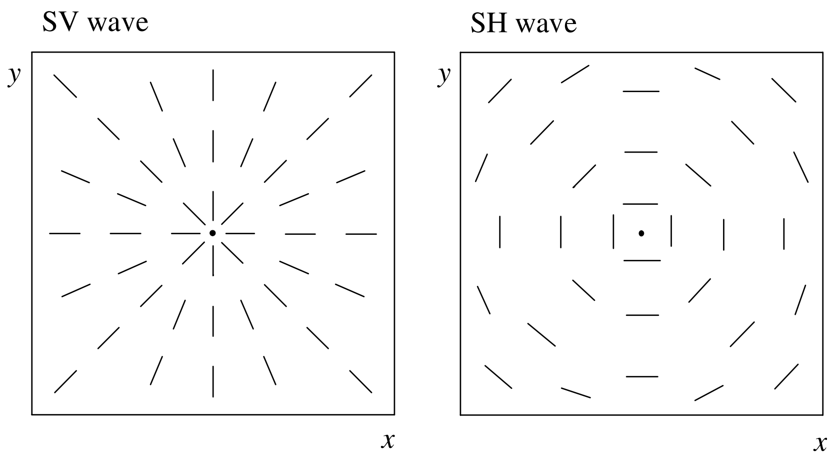

Fig. 1. Horizontal projections of polarization vectors of the SV and SH waves near the kiss singularity.

1. Exact solution

The exact elastodynamic Green tensor $G_{ij}(\mathbf{x}, t)$ for unbounded, homogeneous, anisotropic, elastic media can be expressed as follows (Burridge [6], Eq. 4.6; Wang and Achenbach [7], Eq. 13):

$$G_{kl}(\mathbf{x}, t) = -\frac{H(t)}{8\pi^2\rho}\sum_{m=1}^{3}\int_{S(\mathbf{N})}\frac{g_k^m g_l^m}{(c^m)^3}\dot{\delta}\left(t - \frac{\mathbf{N}\cdot\mathbf{x}}{c^m}\right)\mathrm{d}S(\mathbf{N}). \tag{1}$$

Superscript $m = 1,2,3$ denotes the type of wave (P, $S1$ and $S2$), $\mathbf{x}$ is the position vector of an observation point, t is time, $\mathbf{g} = \mathbf{g}(\mathbf{N})$ denotes the unit polarization vector, $c = c(\mathbf{N})$ is the phase velocity, ρ is the density of the medium, $H(t)$ is the Heaviside step function, $\dot{\delta}(t)$ is the time derivative of the Dirac delta function, and $\mathbf{N}$ is the direction of the slowness vector. The integration is over unit sphere $S(\mathbf{N})$. Formula (1) represents the exact solution for homogeneous, weakly as well as strongly anisotropic media, containing far-field as well as near-field waves, and is valid at all distances and directions including the shear-wave singularities. For an isotropic medium, integral (1) can be evaluated analytically to yield the well-known Stokes solution (see Mura [8], pp. 61–63). For a *transversely isotropic medium*, integral (1) has also been evaluated analytically, but only for the symmetry axis direction [9,10]. For other directions, the integral has to be either evaluated numerically, or expanded asymptotically.

2. Asymptotic solution of Vavryčuk [1]

Since the polarization field is singular along the symmetry axis in TI, the standard asymptotics of (1), which is equivalent to the zeroth-order ray theory, fails along the axis and in its vicinity. Therefore, Vavryčuk [1,11] proposed to apply a more general approach and derived an asymptotic formula using higher-order ray theory. The formula consists of the zeroth- and the first-order terms of the ray series and it is expressed as follows (Vavryčuk [1], Eqs. 2 and 5):

$$\begin{aligned} G_{kl}^{S\,\mathrm{far}}(\mathbf{x}, t) = {} & \frac{1}{4\pi\rho}\frac{1}{v^{\mathrm{SV}}\sqrt{K^{\mathrm{SV}}}}\frac{g_k^{\mathrm{SV}} g_l^{\mathrm{SV}}}{r}\delta(t - \tau^{\mathrm{SV}}) + \frac{1}{4\pi\rho}\frac{1}{v^{\mathrm{SH}}\sqrt{K^{\mathrm{SH}}}}\frac{g_k^{\mathrm{SH}} g_l^{\mathrm{SH}}}{r}\delta(t - \tau^{\mathrm{SH}}) \\ & + \frac{1}{4\pi\rho}\frac{1}{\sqrt{a_{44}}}\frac{g_k^{\mathrm{SH}} g_l^{\mathrm{SH}} - g_k^{\mathrm{SH}\perp} g_l^{\mathrm{SH}\perp}}{r^2\sin^2\vartheta}[H(t - \tau^{\mathrm{SH}}) - H(t - \tau^{\mathrm{SV}})], \end{aligned} \tag{2}$$

where $\mathbf{x}$ is the position vector, t is time, ρ is the density of the medium, r is the distance from the source to the receiver, $\mathbf{g}$ is the polarization vector,

$$\mathbf{g}^{\mathrm{SH}} = \begin{bmatrix} \sin\varphi \\ -\cos\varphi \\ 0 \end{bmatrix}, \quad \mathbf{g}^{\mathrm{SH}\perp} = \begin{bmatrix} \cos\varphi \\ \sin\varphi \\ 0 \end{bmatrix}, \quad \mathbf{g}^{\mathrm{SV}} = \begin{bmatrix} \cos\phi\cos\varphi \\ \cos\phi\sin\varphi \\ -\sin\phi \end{bmatrix}, \tag{3}$$

a_{ij} are the density-normalized elastic parameters in the Voigt notation, v is the group velocity, $\tau = r/v$ is the traveltime, $K = K(\mathbf{p})$ is the Gaussian curvature of the slowness surface, $\mathbf{p} = p\mathbf{N}$ is the slowness vector, $\mathbf{N}$ is the slowness direction, $p = 1/c$ is slowness, and c is the phase velocity. The superscripts in (2) denote the type of wave.

The Gaussian curvature K can be calculated in TI media as follows:

$$K = \frac{1}{p^3 v} \frac{\sin\vartheta}{\sin\theta} \frac{\mathrm{d}\vartheta}{\mathrm{d}\theta}, \tag{4}$$

where angles ϑ and θ define the inclination of ray vector $\mathbf{n}$ and of slowness direction $\mathbf{N}$ from the vertical axis (see Fig. 2)

$$\mathbf{n} = \begin{bmatrix} \sin\vartheta\cos\varphi \\ \sin\vartheta\sin\varphi \\ \cos\vartheta \end{bmatrix}, \quad \mathbf{N} = \begin{bmatrix} \sin\theta\cos\varphi \\ \sin\theta\sin\varphi \\ \cos\theta \end{bmatrix}. \tag{5}$$

Angle ϕ is the polar angle of the ray defined in the $x_1 - x_2$ plane.

The SH-wave quantities in (2) can be expressed explicitly in a closed form as a function of ray vector $\mathbf{n}$ [12]

$$\begin{aligned} c^{\mathrm{SH}} &= \left[\frac{\sin^2\vartheta}{a_{66}} + \frac{\cos^2\vartheta}{a_{44}}\right]^{\frac{1}{2}} \left[\frac{\sin^2\vartheta}{a_{66}^2} + \frac{\cos^2\vartheta}{a_{44}^2}\right]^{-\frac{1}{2}}, \quad v^{\mathrm{SH}} = \left[\frac{\sin^2\vartheta}{a_{66}} + \frac{\cos^2\vartheta}{a_{44}}\right]^{-\frac{1}{2}} \\ \tau^{\mathrm{SH}} &= r\sqrt{\frac{\sin^2\vartheta}{a_{66}} + \frac{\cos^2\vartheta}{a_{44}}}, \quad K^{\mathrm{SH}} = a_{66}^2 a_{44} \left[\frac{\sin^2\vartheta}{a_{66}} + \frac{\cos^2\vartheta}{a_{44}}\right]^2. \end{aligned} \tag{6}$$

However, determining the SV-wave quantities is more involved. First, we have to calculate the slowness vector $\mathbf{p}$ of the SV wave for a given ray vector $\mathbf{n}$. This can be done analytically by solving a system of two algebraic equations of the fourth-order in two unknowns (see Musgrave [13], Eq. 8.2.14; Vavryčuk [14], Eq. 5.6), or numerically by iterations. Then, the needed SV quantities in (2) can be calculated as a function of $\mathbf{p}$ or $\mathbf{N}$. In this way, we obtain for the phase velocity c^{SV},

$$c^{\mathrm{SV}} = \tfrac{1}{2}\left[(a_{11} + a_{44})\sin^2\theta + (a_{33} + a_{44})\cos^2\theta - \sqrt{A + B\cos^2\theta + C\cos^4\theta}\right], \tag{7}$$

where

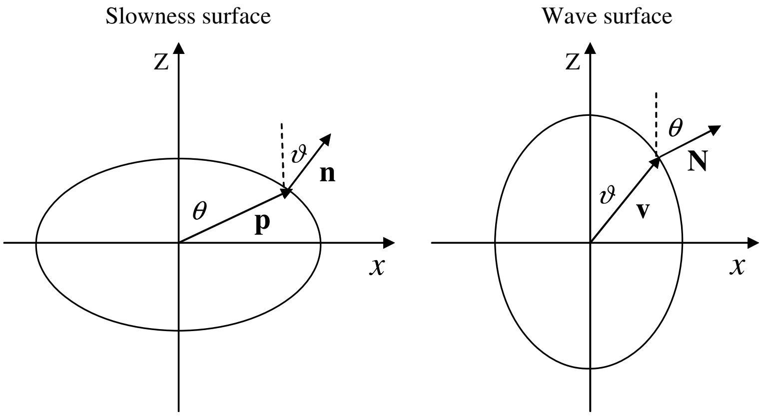

Fig. 2. Definitions of basic quantities. Vector **p** is the slowness vector, **N** is the direction of the slowness vector, **v** is the group velocity vector, and **n** is the ray vector.

$$
\begin{aligned}
&A = (a_{11} - a_{44})^2, \\
&B = 4(a_{13} + a_{44})^2 - 2(a_{11} - a_{44})(a_{11} + a_{33} - 2a_{44}), \\
&C = -4(a_{13} + a_{44})^2 + (a_{11} + a_{33})^2 - 4a_{44}(a_{11} + a_{33} - a_{44}),
\end{aligned} \tag{8}
$$

and for the angle ϕ defining the inclination of polarization vector $\mathbf{g}^{SV}$ from the horizontal plane,

$$
\begin{aligned}
&\sin 2\phi = 2D^{-1}(a_{13} + a_{44})p_3\sqrt{p_1^2 + p_2^2}, \\
&D = 2 - (a_{11} + a_{44})(p_1^2 + p_2^2) - (a_{33} + a_{44})p_3^2.
\end{aligned} \tag{9}
$$

The components of the group velocity v^{SV} read (see Musgrave [13], Eqs. 8.2.8–8.2.10; Červený et al. [15], Eqs. 5.34–5.35)

$$
\begin{aligned}
v_1^{SV} &= D^{-1}p_1\{a_{11} + a_{44} - 2a_{11}a_{44}(p_1^2 + p_2^2) + [(a_{13} + a_{44})^2 - a_{11}a_{33} - a_{44}^2]p_3^2\}, \\
v_2^{SV} &= D^{-1}p_2\{a_{11} + a_{44} - 2a_{11}a_{44}(p_1^2 + p_2^2) + [(a_{13} + a_{44})^2 - a_{11}a_{33} - a_{44}^2]p_3^2\}, \\
v_3^{SV} &= D^{-1}p_3\{a_{33} + a_{44} - 2a_{33}a_{44}p_3^2 + [(a_{13} + a_{44})^2 - a_{11}a_{33} - a_{44}^2](p_1^2 + p_2^2)\}.
\end{aligned} \tag{10}
$$

The most complicated quantity is the Gaussian curvature K^{SV}, which is expressed using Eq. (4).

Since the dependence between the ray direction $\mathbf{n}$ and the slowness direction $\mathbf{N}$ is complicated, the quantities $\mathbf{g}^{SV}$, v^{SV}, K^{SV} and τ^{SV} standing in (2) cannot be expressed explicitly in terms of ray vector $\mathbf{n}$. Instead, they should be evaluated numerically or using approximate formulas.

3. Physical interpretation of the solution

Formula (2) consists of three terms. First two terms are the zeroth-order terms of the ray series and describe the standard far-field asymptotics of the SV and SH waves with convex slowness sheets at a given ray direction [6]. The third term is the first-order term of the ray series called the "coupling term" or the "near-singularity term". The coupling term describes the interaction between the SV an SH waves and it is significant near the symmetry axis [16]. For the direction along the axis, the amplitude of the coupling term diverges and its time duration goes to zero. Hence, the waveform of the coupling term becomes the Dirac delta function similarly as the standard far-field term (see [1], Fig. 2).

Formula (2) is valid for all directions of rays, assuming that the wavefront has no triplications and the receivers are far from the source. The essentially same formula has been obtained by Gridin [2], who derived the asymptotic Green function not only for directions near the symmetry axis but also for other difficult situations such as the directions near cuspidal edges at the wavefront due to triplications. The validity of (2) was furthermore confirmed directly by a numerical comparison of (1) and (2) assuming a smooth source-time function of a point source situated in two differently anisotropic homogeneous media (a cracked medium and sandstone, see [1]).

4. Taylor expansion near the symmetry axis

Assuming directions near the symmetry axis, $\vartheta \ll 1$, or exactly along the axis, $\vartheta = 0$, the wave quantities needed in Eq. (2) can further be simplified. Applying the Taylor expansion to (6–10) and neglecting the third- and higher-order terms in ϑ, we obtain for the phase and group velocities and for the traveltime of the SH wave,

$$
\begin{aligned}
&c^{SH} = \sqrt{a_{44}}\left[1 + \frac{1}{2}\frac{a_{44}}{a_{66}}\left(1 - \frac{a_{44}}{a_{66}}\right)\sin^2\vartheta\right], \quad v^{SH} = \sqrt{a_{44}}\left[1 + \frac{1}{2}\left(1 - \frac{a_{44}}{a_{66}}\right)\sin^2\vartheta\right], \\
&\tau^{SH} = \frac{r}{\sqrt{a_{44}}}\left[1 - \frac{1}{2}\left(1 - \frac{a_{44}}{a_{66}}\right)\sin^2\vartheta\right],
\end{aligned} \tag{11}
$$

and subsequently of the SV wave,

$$c^{\mathrm{SV}} = \sqrt{a_{44}}\left[1 - \frac{1-E}{2E^2}\sin^2\vartheta\right], \quad v^{\mathrm{SV}} = \sqrt{a_{44}}\left[1 - \frac{1-E}{2E}\sin^2\vartheta\right], \quad \tau^{\mathrm{SV}} = \frac{r}{\sqrt{a_{44}}}\left[1 + \frac{1-E}{2E}\sin^2\vartheta\right], \tag{12}$$

where

$$E = \frac{1}{a_{44}}\left(a_{11} - \frac{(a_{13}+a_{44})^2}{a_{33}-a_{44}}\right). \tag{13}$$

Quantity E must be positive, otherwise transverse isotropy displays an axial triplication and the slowness surface is not convex along and near the symmetry axis [13,17]. The directions of the slowness and polarization vectors of the SV wave are calculated as

$$\sin\theta = E^{-1}\sin\vartheta, \quad \sin\phi = \frac{a_{13}+a_{44}}{a_{33}-a_{44}}E^{-1}\sin\vartheta. \tag{14}$$

For the direction strictly along the symmetry axis, $\vartheta = 0$, the needed quantities in (2) can be evaluated exactly having the following form:

$$\begin{aligned} &\phi = 0, \quad p^{\mathrm{SH}} = p^{\mathrm{SV}} = \frac{1}{\sqrt{a_{44}}}, \quad c^{\mathrm{SH}} = c^{\mathrm{SV}} = v^{\mathrm{SH}} = v^{\mathrm{SV}} = \sqrt{a_{44}}, \\ &\tau^{\mathrm{SH}} = \tau^{\mathrm{SV}} = \frac{r}{\sqrt{a_{44}}}, \quad K^{\mathrm{SH}} = \frac{a_{66}^2}{a_{44}}, \quad K^{\mathrm{SV}} = \frac{1}{a_{44}}\left(a_{11} - \frac{(a_{13}+a_{44})^2}{a_{33}-a_{44}}\right)^2, \end{aligned} \tag{15}$$

and the far-field S-wave Green function reads [1]

$$G_{11}^{S\,\mathrm{far}}(z,t) = G_{22}^{S\,\mathrm{far}}(z,t) = \frac{1}{8\pi\rho}\left\{\frac{1}{a_{66}} + \left[a_{11} - \frac{(a_{13}+a_{44})^2}{a_{33}-a_{44}}\right]^{-1}\right\}\frac{1}{|z|}\delta\left(t - \frac{|z|}{\sqrt{a_{44}}}\right) \tag{16}$$

the other components of the Green function being zero. This formula coincides with the far-field approximation of the exact Green function along the symmetry axis published by Payton [9,10].

5. Asymptotic solution of Popov et al. [5]

First, let us complete the formula of Popov et al. [5] and transform it into the time domain to express the Green function in a form comparable with (2). Expressing Eqs. (12) and (13) of [5] valid for the force along the x_1 axis also for forces along the other axes, summing the contributions of the SV and SH waves, and transforming the equations into the time domain, we obtain the Green function in the following form:

$$\begin{aligned} G_{kl}^{S\,\mathrm{far}}(\mathbf{x},t) = {} & \frac{1}{4\pi\rho}\frac{1}{C^{\mathrm{SV}}}\frac{\hat{g}_k^{\mathrm{SV}}\hat{g}_l^{\mathrm{SV}}}{r}\delta(t-\hat{\tau}^{\mathrm{SV}}) + \frac{1}{4\pi\rho}\frac{1}{C^{\mathrm{SH}}}\frac{g_k^{\mathrm{SH}}g_l^{\mathrm{SH}}}{r}\delta(t-\hat{\tau}^{\mathrm{SH}}) \\ & + \frac{1}{4\pi\rho}\frac{1}{\sqrt{a_{44}}}\frac{g_k^{\mathrm{SH}}g_l^{\mathrm{SH}} - g_k^{\mathrm{SH}\perp}g_l^{\mathrm{SH}\perp}}{r^2\sin^2\vartheta}[H(t-\hat{\tau}^{\mathrm{SH}}) - H(t-\hat{\tau}^{\mathrm{SV}})], \end{aligned} \tag{17}$$

where

$$\hat{\mathbf{g}}^{\mathrm{SV}} = \begin{bmatrix}\cos\varphi \\ \sin\varphi \\ 0\end{bmatrix}, \tag{18}$$

$$\hat{\tau}^{\mathrm{SV}} = \frac{r}{\sqrt{a_{44}}}\left[1 - \frac{1}{2}\left(1 + \frac{a_{44}(a_{33}-a_{44})}{(a_{13}+a_{44})^2 - a_{11}(a_{33}-a_{44})}\right)\sin^2\vartheta\right], \tag{19}$$

$$\hat{\tau}^{\mathrm{SH}} = \frac{r}{\sqrt{a_{44}}}\left[1 - \frac{1}{2}\left(1 - \frac{a_{44}}{a_{66}}\right)\sin^2\vartheta\right], \tag{20}$$

$$C^{\mathrm{SV}} = a_{11} - \frac{(a_{13} + a_{44})^2}{a_{33} - a_{44}}, \tag{21}$$

$$C^{\mathrm{SH}} = a_{66}, \tag{22}$$

and the other quantities have the same meaning as in (2). Comparing formulas (2) and (17) we find that they are not very different. The only difference is that some quantities such as the traveltimes and the amplitude factors of the SV and SH waves, and the polarization vector of the SV wave have slightly different forms. A further comparison of Eqs. (18–22) with Eqs. (11–15) figures out that quantities $\hat{\tau}^{\mathrm{SV}}$, $\hat{\tau}^{\mathrm{SH}}$ standing in (17) are identical with the Taylor expansions of τ^{SV}, τ^{SH} expressed in (11) and (12). The quantities $\hat{\mathbf{g}}^{\mathrm{SV}}$, C^{SV} and C^{SH} standing in (17) are just values of g^{SV}, $v^{\mathrm{SV}}\sqrt{K^{\mathrm{SV}}}$ and $v^{\mathrm{SH}}\sqrt{K^{\mathrm{SH}}}$ in (2) taken along the symmetry axis, i.e., for $\vartheta = 0$.

This implies that Eq. (17) is not a solution inconsistent with Eq. (2), but it is rather its approximation. While Eq. (2) is valid for all rays, Eq. (17) is valid just for rays near the symmetry axis.

6. Numerical example

Here, I demonstrate a different range of validity of Eqs. (2) and (17) numerically. I use two transversely isotropic models: first, the Mesaverde immature sandstone [18] with the density-normalized elastic parameters (in km^2/s^2): $a_{11} = a_{22} = 22.36$, $a_{33} = 18.91$, $a_{13} = 8.49$, $a_{44} = a_{55} = 6.61$, $a_{66} = 8.00$ and $\rho = 2.46$ g cm^{-3}, and second, a theoretical model of anisotropy caused by thin layers (Baptie et al. [19], model PTL3) with the density-normalized elastic parameters (in km^2/s^2): $a_{11} = a_{22} = 10.99$, $a_{33} = 6.68$, $a_{13} = 2.56$, $a_{44} = a_{55} = 2.17$, $a_{66} = 3.54$ and $\rho = 2.60$ g cm^{-3}. The Mesaverde immature sandstone displays anisotropy of 8.3, 4.6 and 9.5% for the P, SV and SH waves; the model of the layered medium displays anisotropy of 24.8, 15.2 and 24.5% for the P, SV and SH waves. The wavefield is generated by a single point force $\mathbf{f} = (1,1,0)^{\mathrm{T}}$. The source-time function is a one-sided pulse defined by Eq. (10) of [1]. The wavefield is calculated by Eqs. (2) and (17) and is compared with the exact solution (1).

Fig. 3 shows waves propagating in the sandstone and recorded at two observation points that lie in the $x_1 - x_3$ plane at distances from the source of 25.7 km (a) and 257.1 km (b). The distance corresponds to 10 and 100 wavelengths of the S waves propagating in the direction of the symmetry axis, respectively. The rays from the source to observation points deviate from the symmetry axis by angles 10° and 30°, respectively. The waveforms at the observation points are calculated using a numerical integration of the exact solution (1) and using approximate solution (2) proposed by Vavryčuk [1]. The comparison of both approaches reveals that the coincidence of the S waveforms at large distance is perfect and within the width of the line (see Fig. 3b). However, visible differences are detected at the shorter distance (see Fig. 3a). Since the P and S waves are not fully separated in this case, we compare the complete waveforms. The exact waveform is calculated using Eq. (1), and the far-field approximation is the sum of the far-field S wave (2) and of the standard far-field P wave [6,12]. The discrepancy between the exact and approximate solutions at the shorter distance is expected and appears mainly at times between the P and S waves. At these times, the near-field waves are pronounced in the exact solution but neglected by the far-field approximation. Obviously, as the source-receiver distance decreases, the discrepancy between the exact and approximate solutions becomes more prominent.

Fig. 4 shows waveforms of the S waves propagating in the sandstone and in the layered medium. The source-receiver geometry is similar to that in the previous example. The distance of the observation points from the source is 257.1 km for the sandstone and 147.2 km for the layered medium and corresponds to 100 wavelengths of the S waves propagating along the symmetry axis. The waveforms are calculated using all three different approaches: first, using a numerical integration of the exact solution (1); second, using Eq. (2) proposed by Vavryčuk [1]; and third, using Eq. (17) derived from equations published by Popov et al. [5].

A comparison of the waveforms reveals that the approximate waveforms calculated using Eq. (2) coincide within the width of the line with the exact solution (1). The waveforms calculated using Eq. (17) approximate the exact solution well for rays deviating by 10°. However, we detect remarkable differences in the waveforms for the ray deviating by 30°. First, the approximation (17) erroneously predicts no signal at the x_3-component;

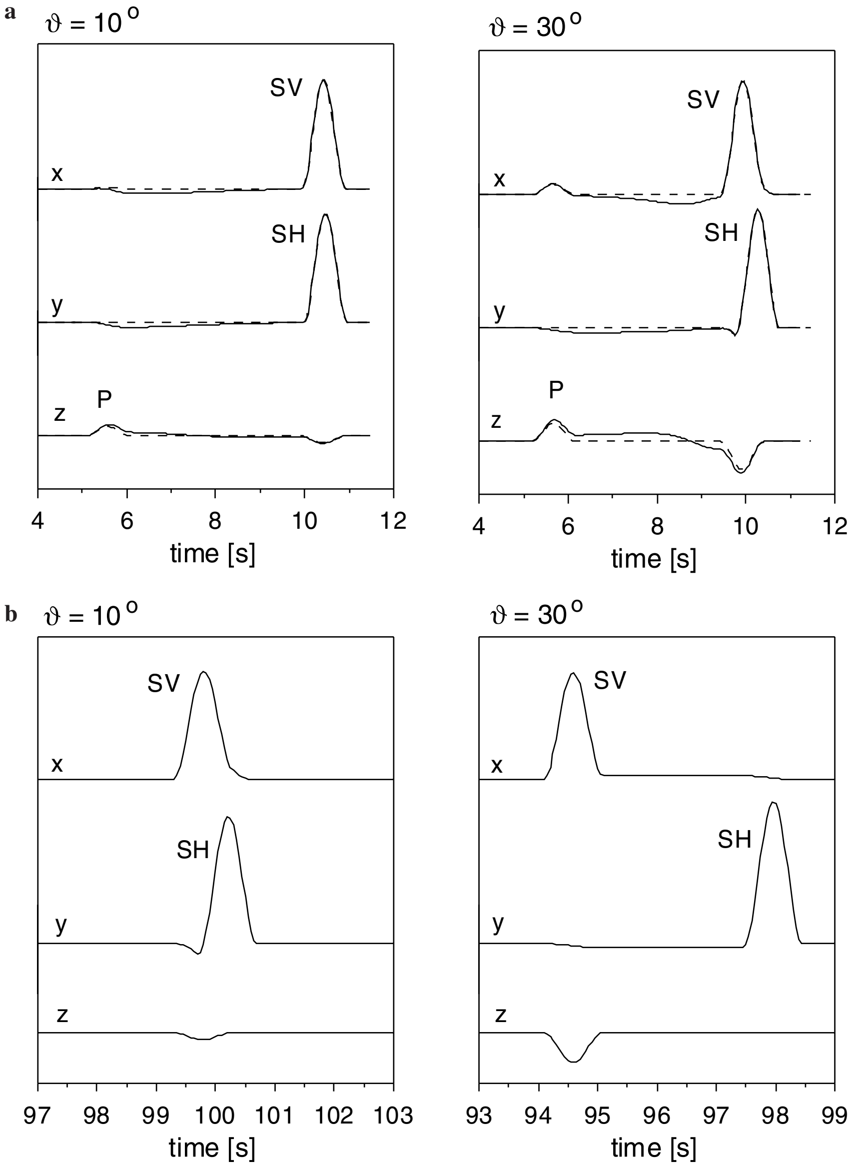

Fig. 3. The waves propagating in the Mesaverde immature sandstone. The source-receiver distance is 10 (a) and 100 (b) wavelengths of the S wave propagating along the symmetry axis. Solid line – waveforms calculated using the exact formula (1), dashed line – waveforms calculated using the approximate formula (2) proposed by Vavryčuk [1]. Angle ϑ defines the deviation of a ray from the symmetry axis. For parameters of the source and of the medium, see the text.

second, the traveltimes of the SV and SH waves are shifted; and third, the amplitudes of the SV and SH waves are distorted at the horizontal components. These differences increase with increasing strength of anisotropy and with increasing deviation between the ray and the symmetry axis. Obviously, the lower accuracy of Eq. (17) compared with Eq. (2) is produced by using Taylor expansions of polarization vectors, traveltimes and amplitudes instead of using the exact quantities.

7. Conclusions

The assertion of Popov et al. [5] that the asymptotic formulas presented by Vavryčuk [1] and Gridin [2] are either questionable or erroneous is unjustified and misleading. The solution published in [5] is not in contra-

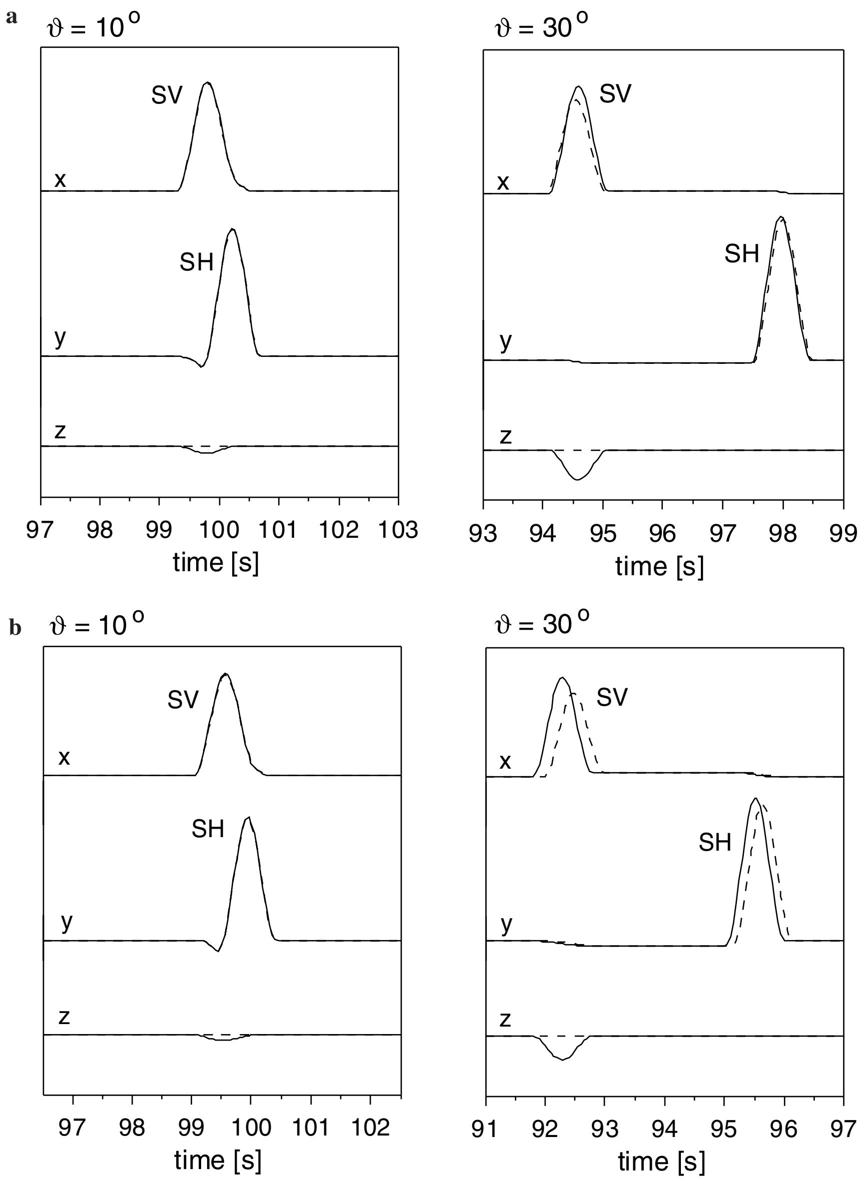

Fig. 4. Waveforms of S waves propagating in the Mesaverde immature sandstone (a) and in the layered medium (b). Solid line – waveforms calculated using the exact formula (1) and using the approximate formula (2) proposed by Vavryčuk [1]. Dashed line – waveforms calculated using the approximate formula (17) proposed by Popov et al. [5]. Angle ϑ defines the deviation of a ray from the symmetry axis. For parameters of the source and for distances of observation points from the source, see the text.

diction with the previously published results. The solution is just a modification of the formula presented in [1]. The modification consists in substituting some quantities standing in the formula of Vavryčuk [1] by their Taylor expansions valid for directions near the symmetry axis. Some quantities are substituted directly by the values along the symmetry axis. These substitutions are justified for small deviations of a ray from the symmetry axis and are advantageous in cases, when we prefer to avoid calculating such quantities as the SV-wave traveltime or the Gaussian curvature of the SV slowness surface. These quantities stand in the previously published formulas and evaluating them is more involved than evaluating the other quantities needed. On the other hand, to approximate formulas for the SH-wave traveltime or the SH-wave Gaussian curvature as done by Popov et al. [5] brings no advantage, because these quantities can be calculated using simple exact formulas.

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