Feasibility of anisotropic inversion based on P-wave travel–time curves

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Methodology

- We build velocity models
  - use realistic velocities simulating a well log
  - smooth the velocities to obtain model suitable for ray tracing
  - prepare several versions of possible models:
    - smooth continuous model
    - model with two layers separated by structural interface with velocity discontinuity
    - models with vertical cracks causing anisotropy
    - models with inclined interface
    - models with low-velocity channels
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- Trace P-wave refracted rays through the models, calculate the travel-time curves, and analyze the influence of the models on the travel-time curves in order to understand which structural features can be resolved from the surface measurement of the refracted P-wave travel-time curves.
Isotropic model

Travel times

Travel-time curve in the isotropic model

Isotropic smooth model

layered model ("well log")

smooth model used for ray tracing

Rays

triplication

Depth / km

Distance / km

P-wave velocity / km s⁻¹

Depth / km
Building of the anisotropic model from fracture density

- Overburden: isotropic
- Lower layer: contains cracks
  => anisotropic layer
Building of the anisotropic model from fracture density

- Overburden: isotropic
- Lower layer: contains cracks
  => anisotropic layer
- Cracks are oriented parallel to $x_2$ direction
Building of the anisotropic model from fracture density

Cracks are oriented parallel to the $x_2$ direction

$\Rightarrow$ propagation in the $x_1$ direction is slower, while propagation in the $x_2$ direction remains almost unchanged

$(\sqrt{A_{11}} < V_p^{\text{iso}}, \sqrt{A_{22}} \sim V_p^{\text{iso}})$

Stiffness tensor $A_{ij}$:

\[
\begin{bmatrix}
A_{11} & A_{12} & A_{13} & A_{14} & A_{15} & A_{16} \\
A_{22} & A_{23} & A_{24} & A_{25} & A_{26} \\
A_{33} & A_{34} & A_{35} & A_{36} \\
A_{44} & A_{45} & A_{46} \\
A_{55} & A_{56} \\
A_{66}
\end{bmatrix}
\]

In isotropic model $A_{11}=A_{22}=A_{33}=(v_p)^2$

$\sqrt{A_{22}}$ effective $V_p$

$\sqrt{A_{11}}$
Building of the anisotropic model from fracture density

Estimation of the material parameters of the anisotropic layer:

- isotropic background + dry penny-shaped cracks (described by radius $a$ and the normal vector $\mathbf{n}$)

Estimation of the material parameters of the anisotropic layer:

- isotropic background + dry penny-shaped cracks (described by radius $a$ and the normal vector $\mathbf{n}$)


- cracks understood as sources of extra strain in the medium, described by additional term $\Delta \mathbf{S}$ to the background compliance tensor $\mathbf{S}_b$:
  $$\mathbf{S}_e = \mathbf{S}_b + \Delta \mathbf{S}$$

where $\mathbf{S}_e$ is the effective compliance tensor of the medium, defined as inverse to the effective stiffness tensor $\mathbf{C}_e$:
$$\mathbf{S}_e = (\mathbf{C}_e)^{-1}$$
Anisotropic model 1 – crack density $e = 0.0003$

Travel times

Distance / km

Depth / km

Rays

Velocity model

effective Vp
sqrt(A11)

sqrt(A22)

P-wave velocity / km s$^{-1}$
Anisotropic model 1 – crack density $e = 0.0003$

Travel times

- Travel time / sec
- Distance / km
- P-wave velocity / km s$^{-1}$

Rays

- Depth / km
- Rays Velocity $\sqrt{A_{11}} \sqrt{A_{22}}$

Velocity model

- Change in model invisible
- Effective $V_p = \sqrt{A_{11}} \sqrt{A_{22}}$
- Depth / km
- P-wave velocity / km s$^{-1}$

Change in model invisible

Models effective $V_p$
Anisotropic model 1 – crack density $e = 0.0003$

change in model invisible
change in ray field noticeable
Anisotropic model 1 – crack density $e = 0.0003$

- Change in model invisible
- Change in ray field noticeable
- Almost no change in travel times
Anisotropic model 1 – crack density $e = 0.0003$

- Change in model invisible
- Change in ray field noticeable
- Almost no change in travel times

(compare next two slides)
Isotropic model
Anisotropic model 1 – crack density $e = 0.0003$
Anisotropic model 2 – crack density $e = 0.001$

![Graph showing travel times, rays, and velocity model with effective Vp $\sqrt{A_{11}}$ and $\sqrt{A_{22}}$.]
Anisotropic model 3 – crack density $e = 0.03$

Travel times

Rays

Velocity model

effective $V_p\sqrt{A_{11}}\sqrt{A_{22}}$

Distance along $x_1$ / km

P-wave velocity / km s$^{-1}$

Depth / km
Anisotropic model 3 – crack density $e = 0.03$

- Anisotropy of the model about 7.5% 
  \[
  \frac{\sqrt{A_{11}} - \sqrt{A_{22}}}{\sqrt{A_{11}}}
  \]

- Noticeable changes in ray field and at the end of the travel-times curve

(Compare with the next slide)
Isotropic model

Travel times

Distance / km  P-wave velocity / km s⁻¹

Depth / km

Rays

Velocity model

layered model ("well log")

smooth model used for ray tracing
Anisotropic model 3 – crack density $e = 0.03$

5 profiles at the surface of the model:

- 0 degree
- 22.5 degree
- 45 degree
- 67.5 degree
- 90 degree (with respect to $x_1$ direction)
Anisotropic model 3 – crack density $e = 0.03$ – profile 0 degree
Anisotropic model 3 – crack density $e = 0.03$ – profile 22.5 degree
Anisotropic model 3 – crack density $e = 0.03$ - profile 45 degree
Anisotropic model 3 – crack density $e = 0.03$ - profile 67.5 degree
Anisotropic model 3 – crack density $\varepsilon = 0.03$ - profile 90 degree
Anisotropic model 3 – crack density $e = 0.03$ – all profiles
Anisotropic model 3 – crack density $e = 0.03$ - all profiles - detail

Travel time / sec

Distance / km

Travel-time difference $\sim 0.1$ sec
Isotropic versus anisotropic model - conclusions

• In the velocity model under investigation, the crack-induced anisotropy in the lower layer of the model starts to be visible on the surface travel-time curve from the anisotropy of 7.5 %

• The anisotropy induced by vertical cracks affects most the profile oriented perpendicularly to the cracks, its effect on the profile oriented parallel to the cracks is negligible
Isotropic model 2 - two layers

Travel times

Distance / km  | P-wave velocity / km s⁻¹
---|---

Rays

Velocity model

layered model (“well log”)

smooth model used for ray tracing
Anisotropic model 4 – two layers - crack density $e = 0.03$

max. anisotropy of the model about 8.1 % 

$$\frac{(\sqrt{A_{11}}-\sqrt{A_{22}})}{\sqrt{A_{11}}}$$

Distance along $x_1$ / km

Travel times

Rays

Travel time / sec

P-wave velocity / km s$^{-1}$

Depth / km

Velocity model

“well log”

effective Vp

$\sqrt{A_{11}}$ $\sqrt{A_{22}}$
Anisotropic model 4 – two layers - crack density $\epsilon = 0.03$

5 profiles at the surface of the model:
- 0 degree
- 22.5 degree
- 45 degree
- 67.5 degree
- 90 degree (with respect to $x_1$ direction)
Anisotropic model 4 – two layers - crack density $e = 0.03$

5 profiles at the surface of the model:
- 0 degree
- 22.5 degree
- 45 degree
- 67.5 degree
- 90 degree
Anisotropic model 4 – two layers - crack density $e = 0.03$

5 profiles at the surface of the model - detail:
- 0 degree
- 22.5 degree
- 45 degree
- 67.5 degree
- 90 degree

Travel time / sec

Distance / km

Travel-time difference $\sim 0.17$ sec
Isotropic model 2

Travel time / sec

Distance / km

Depth / km

Distance / km
Isotropic model 2

- Travel-time curves for 5 profiles
- x1-x3 P-wave velocity section
- Rays traced in the x1-x3 plane
Isotropic model 2
Anisotropic model 4
Anisotropic model 4

Travel-time difference ~ 0.17 sec
Travel time / sec

Distance / km

Depth / km

Anisotropic model 4

=> cracks in the x2-x3 plane slow down the profile in the x1 direction (red dots) do not influence the profile in the x2 direction (black dots)
Isotropic model 2
Isotropic model 2 – inclined interface

![Graph showing travel time, distance, and depth relationships with an inclined interface.]
Isotropic model 2 – inclined interface

Travel-time difference ~ 0.35 sec
Isotropic model 2 – inclined interface

=> inclined interface speeds up the profile in the x1 direction
does not influence the profile in the x2 direction
Isotropic model 2 – inclined interface
Anisotropic model 4 – inclined interface

![Graph showing travel time vs distance and depth](Graphical representation of travel time and distance with depth as a parameter for an anisotropic model 4 with an inclined interface.)
Anisotropic model 4 – inclined interface

=> effects of inclined interface and cracks in the x2-x3 plane act against each other
Isotropic model 2
Isotropic model 2 – declined interface

![Graph showing travel time, distance, and depth relationships in an isotropic model with a declined interface.](image-url)
Isotropic model 2 – declined interface

=> declined interface slows down the profile in the x1 direction does not influence the profile in the x2 direction
Isotropic model 2 – declined interface
Anisotropic model 4 – declined interface
Anisotropic model 4 – declined interface

=> effects of declined interface and cracks in the x2-x3 plane sum up
Isotropic model 2 inclined and declined – compilation of figures
Anisotropic model 4 inclined and declined – compilation of figures
Anisotropic model 5 – lower layer VTI

max. vertical anisotropy about 10 %
\[(\sqrt{A_{11}} - \sqrt{A_{33}}) / \sqrt{A_{11}}\]
Anisotropic model 6 – lower layer VTI + cracks in the x2-x3 plane

max. vertical anisotropy about 10 %  
(sqrt(A11)-sqrt(A33)) / sqrt(A11)

max. horizontal anisotropy about 8 %  
(sqrt(A11)-sqrt(A22)) / sqrt(A11)
Isotropic model 2
Anisotropic model 5 – lower layer VTI
Anisotropic model 5 – lower layer VTI

=> VTI slightly affects the R/T coefficients, its effect on travel-time curves is negligible
Anisotropic model 5 – lower layer VTI
Anisotropic model 6 – lower layer VTI + cracks in the x2-x3 plane
Anisotropic model 6 – lower layer VTI + cracks in the x2-x3 plane

=> adding cracks in the x2-x3 plane to the VTI layer has similar effects as adding them to the isotropic layer
Isotropic model 3 – thin low velocity channel
Isotropic model 2

Distance / km

Depth / km

Travel time / sec

Distance / km
Isotropic model 3 – thin low velocity channel
Isotropic model 3 – thin low velocity channel

=> thin low velocity layer affects ray paths, its effect on travel-time curves is negligible
Isotropic model 4 – thin layer of a very low velocity
Isotropic model 2

Distance / km

Depth / km

Travel time / sec
Isotropic model 4 – thin layer of a very low velocity
Isotropic model 4 – thin layer of a very low velocity

=> thin layer of a very low velocity affects ray paths, its effect on travel-time curves starts to be visible
Isotropic model 5 – thicker layer of a very low velocity

![Graph showing rays and velocity model with depth and distance along x1 on the x-axis and P-wave velocity on the y-axis.]
Isotropic model 2

![Graph showing travel time vs. distance and depth.](image-url)
Isotropic model 5 – thicker layer of a very low velocity
Isotropic model 5 – thicker layer of a very low velocity

=> thicker layer of a very low velocity affects ray paths, its effect on travel-time curves is well visible
Isotropic model 5 – thicker layer of a very low velocity

=> layers of a low velocity slow down the affected part of the travel-time curve, but they do not produce any qualitative effects on the travel-time curve which would enable to identify them.
Isotropic model 6 – low velocity halfspace
Isotropic model 6 – low velocity halfspace
Isotropic model 6 – low velocity halfspace

=> low velocity halfspace considerably affects ray paths and travel-time curves
Conclusions

• In the models considered in this study, the effects of the vertical cracks on the P-wave travel-time curves start to be visible from the anisotropy of 7.5%.

• The vertical cracks slow down the rays propagating perpendicularly to the cracks, while their influence on the rays propagating parallel to the cracks is negligible. This effect appears similarly either when we introduce vertical cracks to the isotropic layer, or when we introduce them to the VTI layer.

• Structural interface declined away from the source in the isotropic model has similar effects on the travel-time curves as the vertical cracks, while the interface inclined towards the source has opposite effects. Effects of dipping interface and of vertical cracks sum up.

• Low-velocity channels slow down the affected part of the travel-time curve, but do not produce any qualitative effects which would enable to identify them from the travel-time curves.
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