Weak-anisotropy moveout approximations for P waves in homogeneous TTI layers

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Outline

Introduction

Weak-anisotropy parameters

Approximate traveltime formulae

NMO velocity, quartic coefficient

Tests of formulae

Conclusions

Possible extensions

Introduction

Moveout approximations

standard

- expansion of T^2 in terms of the squared offset

hyperbolic, non-hyperbolic, ...

alternative

- expansion of T^2 in terms of the deviations

of anisotropy from isotropy;

weak anisotropy (WA) parameters

Introduction

Moveout approximations with WA parameters

- reflected P and SV waves in VTI media (Report 23)

- reflected P waves in upto monoclinic media
  with a plane of symmetry coinciding with the reflector (Report 24)

- **reflected P waves in anisotropy of arbitrary symmetry**
  (triclinic, but also tilted TI or ORT)

Weak-anisotropy parameters

- 21 weak-anisotropy (WA) parameters
- generalization of *Thomsen's (1986)* parameters
- an alternative to stiffness tensor $C_{\alpha\beta}$ or $A_{\alpha\beta}$
- applicable to anisotropy of any type, strength and orientation
- describe exactly any wave attribute
- linear relation of WA to $C_{\alpha\beta}$ or $A_{\alpha\beta}$ parameters
- natural combinations of $C_{\alpha\beta}$ or $A_{\alpha\beta}$ taken into account

Weak-anisotropy parameters

- simple transformation from one coordinate system to another

- represent deviation from an isotropic reference

- freedom in the choice of the reference velocity

- specified in coordinate systems independent of symmetry elements of studied anisotropy symmetry

- all 21 WA parameters dimensionless, of comparable size

- first-order P-wave attributes depend on only **15 WA parameters**

Weak-anisotropy parameters

P-wave WA parameters (global coordinates)

$$\epsilon_x = \frac{A_{11}-\alpha^2}{2\alpha^2}, \quad \epsilon_y = \frac{A_{22}-\alpha^2}{2\alpha^2}, \quad \epsilon_z = \frac{A_{33}-\alpha^2}{2\alpha^2}$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha^2}{\alpha^2}, \quad \delta_y = \frac{A_{13}+2A_{55}-\alpha^2}{\alpha^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha^2}{\alpha^2}$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha^2}, \quad \chi_y = \frac{A_{25}+2A_{46}}{\alpha^2}, \quad \chi_z = \frac{A_{36}+2A_{45}}{\alpha^2}$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2}$$

α - P-wave velocity in a reference isotropic medium

Weak-anisotropy parameters

Transformation relations: TI $\rightarrow$ TTI

$$\epsilon_x = \epsilon_x^{TI}(t_2^2+t_3^2)^2 + \epsilon_z^{TI}t_1^4 + \delta_y^{TI}t_1^2(t_2^2+t_3^2)$$

$$\epsilon_z = \epsilon_x^{TI}(t_1^2+t_2^2)^2 + \epsilon_z^{TI}t_3^4 + \delta_y^{TI}t_3^2(t_1^2+t_2^2)$$

$$\delta_y = 2\epsilon_x^{TI}(3t_1^2t_3^2+t_2^2) + 6\epsilon_z^{TI}t_1^2t_3^2 + \delta_y^{TI}[(t_2^2+t_3^2)t_3^2 + t_1^2(t_1^2+t_2^2) - 4t_1^2t_3^2]$$

ϵ_x^{TI}, ϵ_z^{TI}, δ_y^{TI} - WA parameters in ”crystal” coordinates

ϵ_x, ϵ_z, δ_y - WA parameters in global coordinates

t_i - components in global coordinates of a unit vector $\mathbf{t}$

$\mathbf{t}$ - vector parallel to the axis of symmetry

Approximate traveltime formulae

Reflection traveltime along a reference ray

$$T^2(x) = (T_d(x) + T_u(x))^2$$

$$T_d^2(x) = \frac{1}{4}(4H^2 + x^2)/v^2(\mathbf{N}^d) \qquad T_u^2(x) = \frac{1}{4}(4H^2 + x^2)/v^2(\mathbf{N}^u)$$

reference ray - a symmetric ray in a reference isotropic medium

x - offset, source-receiver distance

$T(x)$ - traveltime at the offset x

H - depth of the plane horizontal reflector

Approximate traveltime formulae

Reflection traveltime along a reference ray

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$T_d(x)$, $T_u(x)$ - traveltimes along down- and up-going legs
of the reference ray

$\mathbf{N}^d$, $\mathbf{N}^u$ - ray vectors, unit vectors
parallel to the down- and up-going legs of the reference ray

$v(\mathbf{N})$ - approximate ray velocity along the reference ray direction $\mathbf{N}$;
$v(\mathbf{N})$ different on down- and up-going legs of the ray

Approximate traveltime formulae

Normalized reflection moveout formula

$$\bar{x} = x/2H \;, \qquad T_0 = 2H/\alpha$$

$$T^2(\bar{x}) = (T_d(\bar{x}) + T_u(\bar{x}))^2$$

$$T_d^2(\bar{x}) = \tfrac{1}{4}T_0^2\alpha^2(1+\bar{x}^2)/v^2(\mathbf{N}^d) \qquad T_u^2(\bar{x}) = \tfrac{1}{4}T_0^2\alpha^2(1+\bar{x}^2)/v^2(\mathbf{N}^u)$$

T_0 - two-way zero-offset traveltime in the reference isotropic medium

$\bar{x}$ - normalized offset

α - reference velocity

$v(\mathbf{N})$ - approximate ray velocity along the reference ray direction $\mathbf{N}$

Approximate traveltime formulae

Problem: We seek: $v^2(\mathbf{N})$

We have available: $\mathbf{N}$ and $\widetilde{c^{-1}}(\mathbf{N})$ or $\widetilde{c^2}(\mathbf{N})$

$\widetilde{c^{-1}}(\mathbf{N})$ - first-order approximation of phase slowness

$\widetilde{c^2}(\mathbf{N})$ - first-order approximation of square of phase velocity

$v(\mathbf{N})$ - ray velocity

$\mathbf{N}$ - ray vector

Assumption: I) $v^{-1}(\mathbf{N}) \sim \widetilde{c^{-1}}(\mathbf{N})$

II) $v^2(\mathbf{N}) \sim \widetilde{c^2}(\mathbf{N})$

Approximate traveltime formulae

Ray-velocity approximations in the plane (x_1, x_3)

ray vector $\mathbf{N} \equiv (N_1, 0, N_3)$

I) $v^{-1}(\mathbf{N}) \sim \alpha^{-1}[1 - (\epsilon_x N_1^4 + \delta_y N_1^2 N_3^2 + \epsilon_z N_3^4) - 2(\epsilon_{15} N_1^3 N_3 + \epsilon_{35} N_1 N_3^3)]$

dependent of the choice of α

II) $v^2(\mathbf{N}) \sim \alpha^2[1 + 2(\epsilon_x N_1^4 + \delta_y N_1^2 N_3^2 + \epsilon_z N_3^4) + 4(\epsilon_{15} N_1^3 N_3 + \epsilon_{35} N_1 N_3^3)]$

independent of the choice of α

Approximate traveltime formulae

I) $T^2(\bar{x}) = T_0^2 \ \frac{2(1+\bar{x}^2)^2 - P(\bar{x})}{1+\bar{x}^2}$ dependent of α

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x \bar{x}^4 + 2\delta_y \bar{x}^2 + 2\epsilon_z \qquad \text{(Report 24)}$$

Zero offset $(\bar{x} = 0)$

$$T^2(0) = 4H^2(2\alpha^2 - A_{33})/\alpha^4$$

$\alpha^2 = A_{33} \Rightarrow \epsilon_z = 0$, $T^2(0) = 4H^2/A_{33}$

A_{33} - first-order approximation of c_v^2

c_v - vertical P-wave phase velocity

Approximate traveltime formulae

II) $T^2(\bar{x}) = \frac{1}{4}T_0^2(1+\bar{x}^2)^3[P_d^{-1/2}(\bar{x}) + P_u^{-1/2}(\bar{x})]^2$ independent of α

$$P_d(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x\bar{x}^4 + 4\epsilon_{15}\bar{x}^3 + 2\delta_y\bar{x}^2 + 4\epsilon_{35}\bar{x} + 2\epsilon_z$$

$$P_u(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x\bar{x}^4 - 4\epsilon_{15}\bar{x}^3 + 2\delta_y\bar{x}^2 - 4\epsilon_{35}\bar{x} + 2\epsilon_z$$

$$P_d(\bar{x}) \sim P_u(\bar{x}) \sim P(\bar{x})$$

Approximate traveltime formulae

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$$P_d(\bar{x}) \sim P_u(\bar{x}) \sim P(\bar{x})$$

III) $T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3/P(\bar{x})$ (Report 24)

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x\bar{x}^4 + 2\delta_y\bar{x}^2 + 2\epsilon_z$$

Zero offset $(\bar{x} = 0)$: $T^2(0) = 4H^2/A_{33}$

Approximate traveltime formulae

Transformation relations: TI $\rightarrow$ TTI

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ϵ_x^{TI}, ϵ_z^{TI}, δ_y^{TI} - WA parameters in ”crystal” coordinates

ϵ_x, ϵ_z, δ_y - WA parameters in global coordinates

$\mathbf{t}$ - vector parallel to the axis of symmetry

NMO velocity, quartic coefficient

III) Formulae independent of the choice of α

For $\alpha^2 = A_{33} \Leftrightarrow \epsilon_z = 0$

NMO velocity

$$v_{NMO}^{-2} = \alpha^{-2}(1 - 2\delta_y)$$

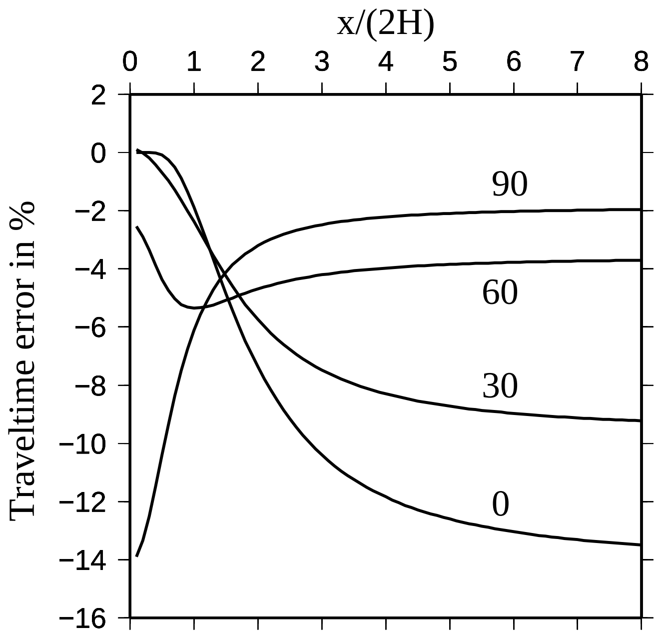
Quartic coefficient

$$A_4 = 2[\delta_y - \epsilon_x + 2(\delta_y)^2]/(\alpha^4 T_0^2)$$

Tests of formulae

Greenhorn shale; anisotropy $\sim$ 26%

α=3.094 km/s, ϵ_x^{TI}=0.256, δ_y^{TI}=–0.0523, ϵ_z^{TI}=0, $\Phi = 45°$



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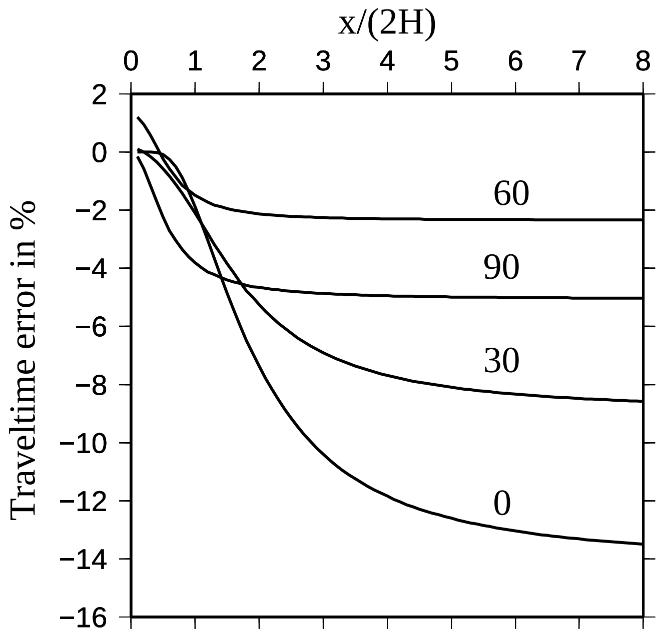

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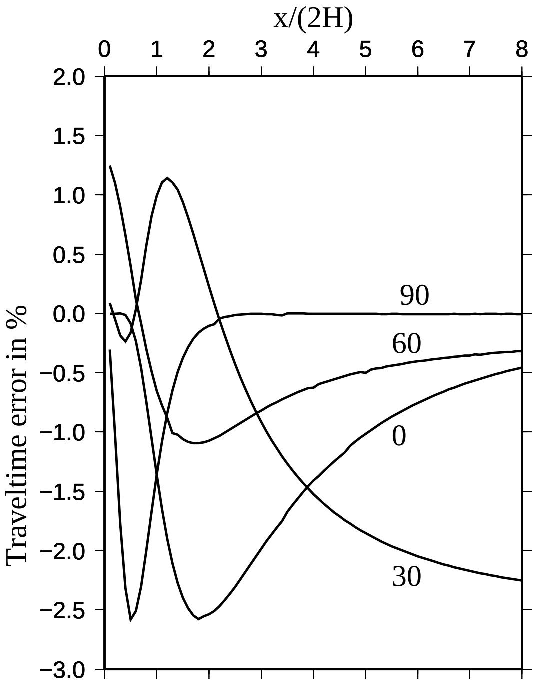

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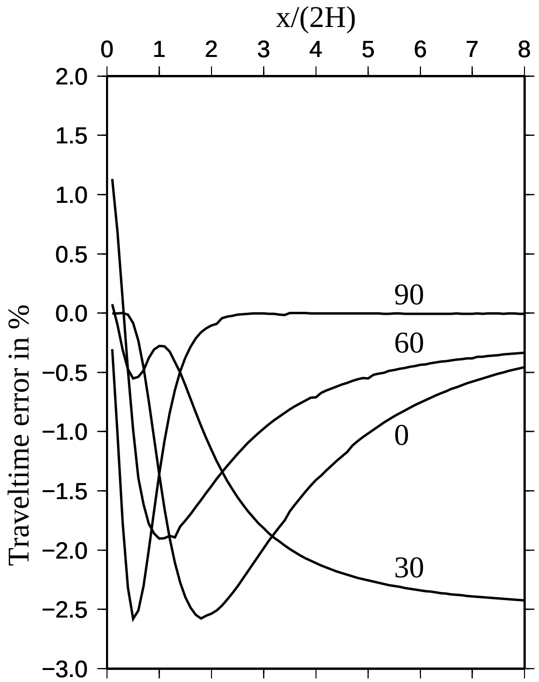

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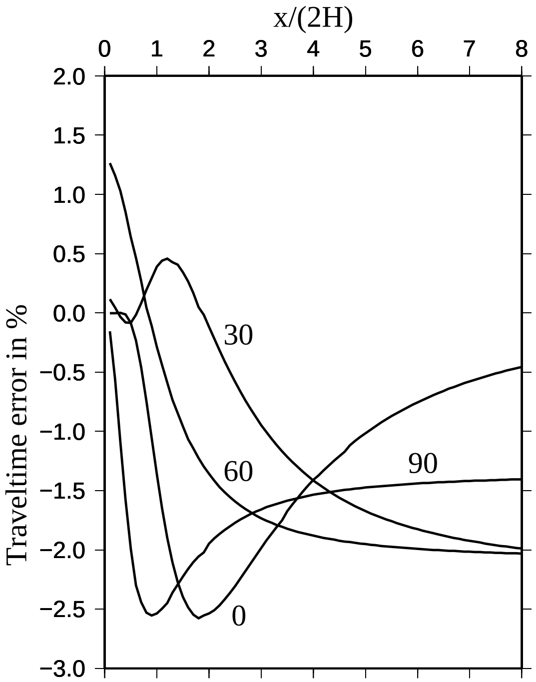

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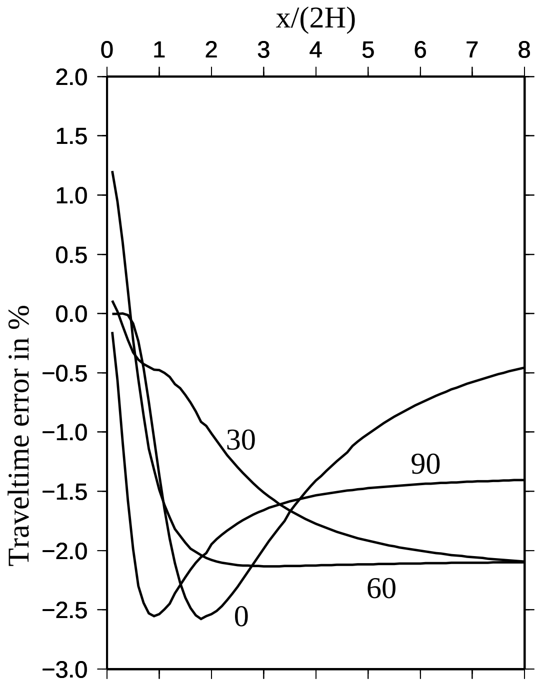


III)

Conclusions

- based on WA approximation

- weak or moderate anisotropy of arbitrary symmetry and orientation

- relatively simple formulae

- no non-physical assumptions (no acoustic approximation)

- reduced number of parameters

- simple transformation from one coordinate system to another

Conclusions

- insight

- accuracy for small offsets within the order of used approximation

- accuracy for large offsets within the order of used approximation

- inaccuracies for large deviations of $\mathbf{n}$ and $\mathbf{N}$,

- for higher-symmetry media reduction to previous formulae

Possible extensions

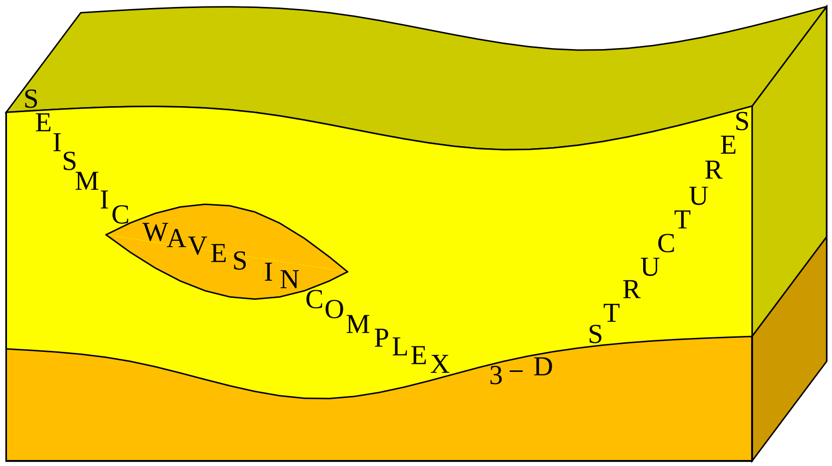
- tilted orthorhombic or monoclinic media

- an inclined reflector

- unconverted separate or coupled S waves

- layered media

Acknowledgement



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