Integral superposition
of paraxial Gaussian beams
in inhomogeneous anisotropic layered structures
in Cartesian coordinates

Vlastislav Červený\textsuperscript{1)} and Ivan Pšenčík\textsuperscript{2)}

\textsuperscript{1)} Charles University, Faculty of Mathematics and Physics, Praha, Czech Republic
\textsuperscript{2)} Institute of Geophysics, Acad. Sci., Praha, Czech Republic

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Outline

Introduction

Integral superposition

Conclusions
Introduction

Wave modelling in inhomogeneous anisotropic media

- ray method
- coupling ray method
- paraxial ray approximations, paraxial Gaussian beams
- weighted summation of paraxial ray approximations or paraxial Gaussian beams
Integral superposition

\[ G_{ij}(R, S, \omega) = \frac{\omega}{2\pi} \int_D \int \mathbf{G}^{\text{ray}}_{ij}(R_{\gamma}, S)[- \det \mathbf{N}(R_{\gamma})]^{1/2} \exp[i\omega T(R, R_{\gamma})] d\gamma_1 d\gamma_2 \]

\( \mathbf{G}(R, S, \omega) \) - Green function

\( \omega \) - circular frequency

\( \gamma_1, \gamma_2 \) - ray parameters defined on \( D \), specifying ray \( \Omega \)

\( R, S, R_{\gamma} \) - receiver, source and a point on \( \Omega \), in a vicinity of \( R \)

\( \mathbf{G}^{\text{ray}}(R_{\gamma}, S) \) - elementary ray-theory Green function

\([- \det \mathbf{N}(R_{\gamma})]^{1/2} \) - the weighting function; \( \mathbf{N} \) - \( 2 \times 2 \) matrix

\( T(R, R_{\gamma}) \) - paraxial travel time at receiver \( R \)
Integral superposition

Elementary ray-theory Green function

\[ G_{ij}^{\text{ray}}(R_\gamma, S) = \frac{g_i(R_\gamma)g_j(S)}{4\pi[\rho(S)\rho(R_\gamma)C(S)C(R_\gamma)]^{1/2}} \frac{\exp[iT^G(R_\gamma, S)]}{\mathcal{L}(R_\gamma, S)} \mathcal{R}^C \]

\( g(S), g(R_\gamma) \) - polarization vectors at \( S \) and \( R_\gamma \)

\( C(S), C(R_\gamma), \rho(S), \rho(R_\gamma) \) - phase velocities and densities at \( S \) and \( R_\gamma \)

\( \mathcal{L}(R_\gamma, S) \) ... the relative geometrical spreading

\( T^G(R_\gamma, S) \) ... complete phase shift due to caustics

\( \mathcal{R}^C \) ... complete reflection/transmission coefficient
Integral superposition

Weighting function

\[ \mathcal{N}(R_\gamma) = -Q^{(x)T}P^{(x)} + Q^{(x)T}\mathcal{E}M(R_\gamma)\mathcal{E}^TQ^{(x)} \]

\[ Q^{(x)}(R_\gamma), P^{(x)}(R_\gamma) - 3 \times 2 \text{ parts of } \hat{Q}^{(x)}, \hat{P}^{(x)} \]

\[ \hat{Q}^{(x)}(R_\gamma), \hat{P}^{(x)}(R_\gamma) - 3 \times 3 \text{ paraxial matrices obtained from DRT} \]

\[ M(R_\gamma) - 2 \times 2 \text{ matrix of Gaussian-beam parameters (given)} \]

\[ \mathcal{E}(R_\gamma) - 3 \times 2 \text{ matrix } \mathcal{E} = (e_1, e_2) \text{ (given)} \]

\[ e_i - \text{ unit vectors, } \quad e_1^T e_2 = 0, \quad e_1^T p = 0, \quad p - \text{ slowness vector} \]
Integral superposition

Travel time

\[ T(R, R_\gamma) = T(R_\gamma) + x^T(R, R_\gamma)p(R_\gamma) + \frac{1}{2}x^T(R, R_\gamma)\hat{M}^{(x)}(R_\gamma)x(R, R_\gamma) \]

\[ T(R_\gamma) \text{ - travel time at } R_\gamma, \quad x(R, R_\gamma) = x(R) - x(R_\gamma) \]

\[ \hat{M}^{(x)}(R_\gamma) = \mathcal{F}M\mathcal{F}^T + p\eta^T + \eta p^T - p(\mathcal{U}^T\eta)p^T; \quad \hat{M}^{(x)} \text{ - } 3 \times 3 \text{ matrix} \]

\[ M(R_\gamma) \text{ - } 2 \times 2 \text{ - matrix of Gaussian-beam parameters (given)} \]

\[ \mathcal{F}(R_\gamma) \text{ - } 3 \times 2 \text{ matrix } \mathcal{F} = (f_1, f_2) \quad f_1 = C^{-1}(e_2 \times \mathcal{U}) \quad f_2 = C^{-1}(\mathcal{U} \times e_1) \]

\[ p, \eta, \mathcal{U} \text{ - slowness, eta, ray-velocity vector, } e_I \text{ vectors given} \]
Conclusions

- applicable to 3D inhomogeneous anisotropic media with curved structural interfaces
- applicable to separate P, S1 and S2 waves
- applicable to coupled S waves in weak anisotropy or around S-wave singularities
- applicable to summation of paraxial ray approximations, including Maslov-Chapman integrals
- applicable to moment-tensor point sources
Conclusions

- DRT performed in Cartesian coordinates
- $3 \times 2$ parts of $3 \times 3$ paraxial matrices sufficient
- no need for two-point ray tracing
- removes or smoothes singularities of standard ray theory
- no need for computation of vector bases along ray $\Omega$
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