# Superpositions of Gaussian beams and column Gaussian packets in heterogeneous anisotropic media

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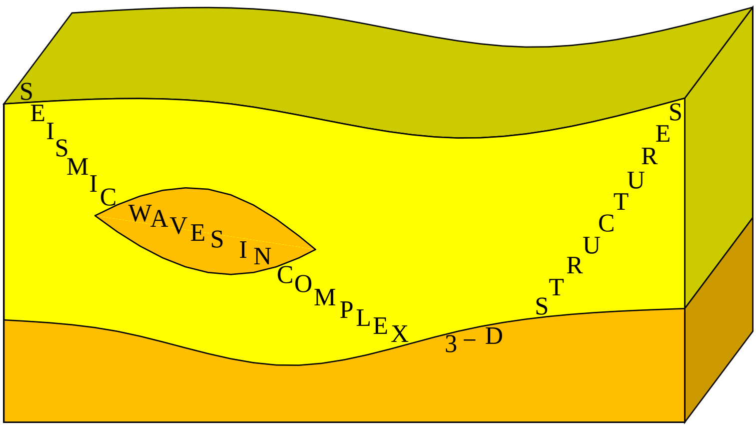


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The three-parametric integral superposition of Gaussian packets was proposed for isotropic media by Klimeš (1984), and was specified for anisotropic media by Klimeš (2014).

The two-parametric integral superposition of Gaussian beams is calculated along the reference surface. The paraxial Gaussian beams are expanded at the points of intersection of the reference rays with the reference surface.

The two-parametric integral superposition of Gaussian beams can be obtained from the three-parametric integral superposition of Gaussian packets by analytic asymptotic quadrature along the reference rays (Klimeš, 1984; 2015).

We consider a general system of reference lines instead of the reference rays for the one-parametric analytic asymptotic quadrature of the three-parametric integral superposition of Gaussian packets (Klimeš, 2015). We then obtain the two-parametric integral superposition of column Gaussian packets instead of the two-parametric integral superposition of Gaussian beams.

The two-parametric integral superposition of column Gaussian packets is performed along the reference surface, and represents a generalization of the two-parametric integral superposition of paraxial Gaussian beams.

Two-parametric integral superposition of column Gaussian packets along reference surface $\tilde{x}^m = \tilde{x}^m(\xi^K)$ parametrized by ray parameters $\xi^1$ and $\xi^2$:

$$u_{i[j]}(x^m,\omega) = \frac{\omega}{2\pi}\iint \mathrm{d}\xi^1\mathrm{d}\xi^2 \, A_{i[j]} \sqrt{\det}[\mathrm{i}(M_{AB} - \tilde{F}_{AB})] \exp\bigl(\mathrm{i}\omega\tilde{\theta}\bigr) \qquad (33)$$

$A_{i[j]}$... complex-valued vectorial or tensorial amplitude corresponding to the ray theory under consideration at point $\tilde{x}^m$,
$\omega$... circular frequency,
$M_{AB}$... 2×2 matrix of the second-order partial derivatives of the reference travel time along the reference surface with respect to $\xi^1$ and $\xi^2$,
$\tilde{F}_{AB}$... 2×2 complex-valued matrix with positive imaginary part describing the shape of the column Gaussian packet centred at point $\tilde{x}^i$.
Function $\sqrt{\det}\bigl(B_{AB}\bigr)$ is the product of the square roots of the eigenvalues of matrix $B_{AB}$. The individual square roots are taken with positive real parts.

Paraxial complex-valued travel time:

$$\tilde{\theta} = \tau + (x^k - \tilde{x}^k)\, p_k + \tfrac{1}{2}(x^k - \tilde{x}^k)\, \tilde{f}_{kl}\, (x^l - \tilde{x}^l) \tag{17}$$

$\tau$... travel time corresponding to the ray theory under consideration at point $\tilde{x}^i$ (need not be equal to the reference travel time),
$p_k$... reference slowness vector at point $\tilde{x}^i$.

Second-order partial derivatives of the complex-valued travel time at point $\tilde{x}^i$:

$$\tilde{f}_{ij} = Z_{Mi} \tilde{F}_{MN} Z_{Nj} + Z_{3i} Z_3^k N_{kj} + N_{il} Z_3^l Z_{3j} - Z_{3i} Z_3^k N_{kl} Z_3^l Z_{3j} \tag{27}$$

$\tilde{F}_{AB}$... 2×2 complex-valued matrix with positive imaginary part describing the shape of the column Gaussian packet centred at point $\tilde{x}^i$,
$N_{kl}$... 3×3 matrix of the second-order partial derivatives of the reference travel time at point $\tilde{x}^i$,
$Z_3^i$... contravariant vector tangent to the reference line,
$Z_{3i}$... covariant vector perpendicular to the reference surface, $Z_{3i} Z_3^i = 1$,
$Z_{1i}$, $Z_{2i}$... covariant vectors perpendicular to the reference line, scaled as $Z_{Ki}\, \partial\tilde{x}^i / \partial\xi^L = \delta_{KL}$, $K, L = 1, 2$.

The column Gaussian packets infinitely extend along the reference lines, whether the Gaussian beams infinitely extend along the reference rays.

Unfortunately, the paraxial expansion of the column Gaussian packets is generally singular at caustics. The column Gaussian packets are not singular at caustics only if the reference lines are tangent to the reference rays at the reference surface. If we choose the reference lines along the reference rays, the two-parametric integral superposition of column Gaussian packets along the reference surface reduces to the two-parametric integral superposition of paraxial Gaussian beams along the reference surface.

The integral superpositions of column Gaussian packets and paraxial Gaussian beams may correspond to the anisotropic ray theory, to the frequency-dependent coupling ray theory for S waves or to the prevailing-frequency approximation of the coupling ray theory (Klimeš & Bulant, 2012) in anisotropic media, or to the isotropic ray theory in isotropic media. The equations can be used in both Cartesian and curvilinear coordinates.

**References:**


Klimeš, L. (1984): The relation between Gaussian beams and Maslov asymptotic theory. *Stud. geophys. geod.*, **28**, 237–247.

Klimeš, L. (2014): Superposition of Gaussian packets in heterogeneous anisotropic media. *Seismic Waves in Complex 3-D Structures*, **24**, 127–130, online at “http://sw3d.cz”.

Klimeš, L. (2015): Superpositions of Gaussian beams and column Gaussian packets in heterogeneous anisotropic media. *Seismic Waves in Complex 3-D Structures*, **25**, 103–108.

Klimeš, L. & Bulant, P. (2012): Single-frequency approximation of the coupling ray theory. In: *Seismic Waves in Complex 3-D Structures, Report 22*, pp. 143–167, Dep. Geophys., Charles Univ., Prague, online at “http://sw3d.cz”.

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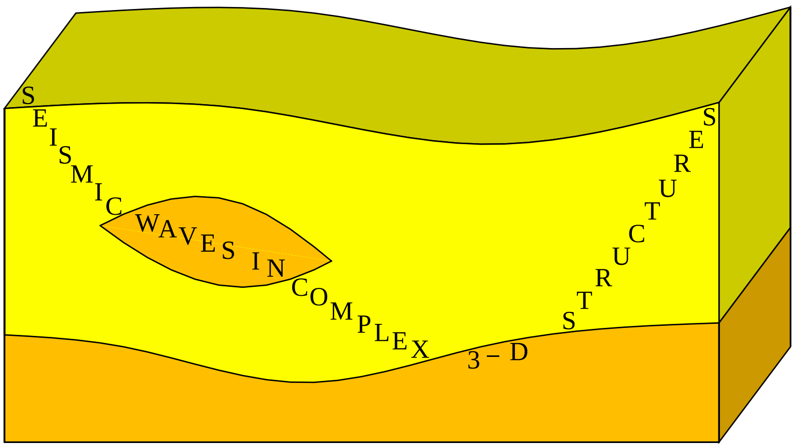


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