P-wave ray velocities in a weak-anisotropy approximation

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Outline

Introduction

Exact velocity formulae

Velocity approximations

Tests of velocity approximations

Conclusions

Introduction

Applicability of analytic expressions for phase and ray velocities

- moveout expressions
- prestack time migration
- finite-difference computations of traveltimes
- tomography

Desirable: ray velocity as a function of ray angle

Introduction

Use of weak-anisotropy approximation

- expansion in terms of weak-anisotropy (WA) parameters

- no additional assumptions

- important role of the relation of $phase$ $\mathbf{n}$ and ray $\mathbf{N}$ vectors:

$$\mathbf{n} \cdot \mathbf{N} = c/v$$

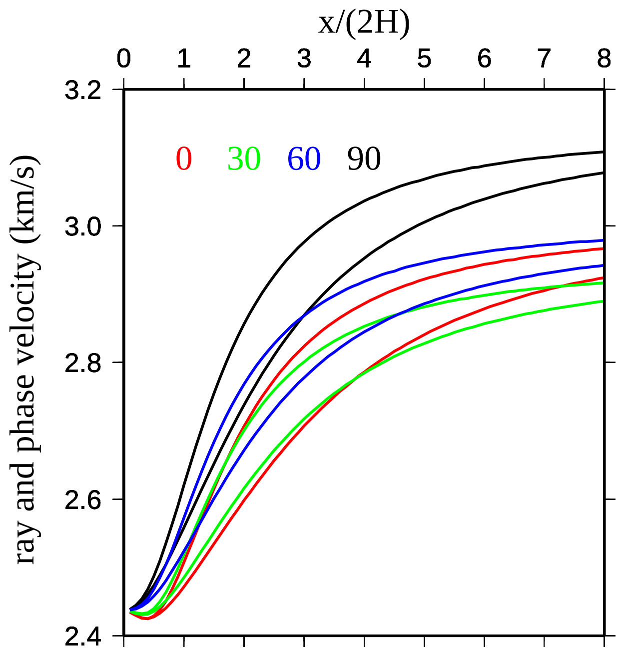
$\Rightarrow$ deviation of $\mathbf{n}$ and $\mathbf{N}$ controlled by the deviation of c and v

c - phase velocity, v - ray velocity

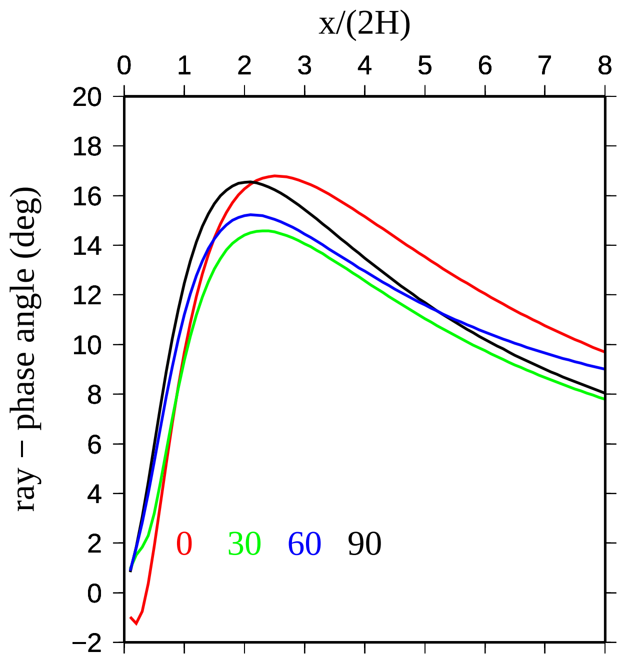
Introduction

ORT model - Schoenberg & Helbig (1997)

c versus v



$\mathbf{n}$ versus $\mathbf{N}$



Introduction

P-wave weak-anisotropy (WA) parameters

$$\epsilon_x = \frac{A_{11}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_y = \frac{A_{22}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_z = \frac{A_{33}-\alpha_0^2}{2\alpha_0^2}$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha_0^2}{\alpha_0^2}, \quad \delta_y = \frac{A_{13}+2A_{55}-\alpha_0^2}{\alpha_0^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha_0^2}{\alpha_0^2}$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha_0^2}, \quad \chi_y = \frac{A_{25}+2A_{46}}{\alpha_0^2}, \quad \chi_z = \frac{A_{36}+2A_{45}}{\alpha_0^2}$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha_0^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha_0^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha_0^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha_0^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha_0^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha_0^2}$$

α_0 - reference velocity

Exact velocity formulae

Phase velocity

$$c^2(\mathbf{n}) = a_{ijkl} n_j n_l g_i g_k$$

Ray velocity

$$v_i(\mathbf{n}) = a_{ijkl} p_l g_j g_k$$

a_{ijkl} - density-normalized stiffness tensor

p_i - i-th component of slowness vector $\mathbf{p} = \mathbf{n}/c$

g_i - i-th component of polarization vector $\mathbf{g}$

n_i - i-th component of phase vector $\mathbf{n}$

Phase-velocity approximations

First-order approximation

$$\tilde{c}^2(\mathbf{n}) = B_{33}(\mathbf{n})$$

Second-order approximation

$$\tilde{\tilde{c}}^2(\mathbf{n}) = \tilde{c}^2(\mathbf{n}) + [B_{13}^2(\mathbf{n}) + B_{23}^2(\mathbf{n})]/(\alpha_0^2 - \beta_0^2)$$

$\mathbf{n}$ - phase vector

α_0, β_0 - reference velocities

Ray-velocity approximations

1 Ignore the difference between $\mathbf{n}$ and $\mathbf{N}$ and use 1st-order approximation of c^2

$$\tilde{v}^2(\mathbf{N}) = \tilde{c}^2(\mathbf{N})$$

2 Consider the difference between $\mathbf{n}$ and $\mathbf{N}$ and use 1st-order approximation of c^2

$$\tilde{v}^2(\mathbf{N}) = \tilde{c}^2(\mathbf{N}) - 4[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})]/\tilde{c}^2(\mathbf{N})$$

3 Consider the difference between $\mathbf{n}$ and $\mathbf{N}$ and use 2nd-order approximation of c^2

$$\tilde{\tilde{v}}^2(\mathbf{N}) = \tilde{c}^2(\mathbf{N}) + 4a[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})]/\tilde{c}^2(\mathbf{N})$$

Velocity approximations

$$B_{mn}(\mathbf{n}) = a_{ijkl} n_j n_l e_i^{[m]} e_k^{[n]}$$

$B_{mn}(\mathbf{n})$ - Christoffel matrix rotated to vector basis $\mathbf{e}^{[i]}$

$$\mathbf{e}^{[1]} \equiv D^{-1}(n_1 n_3, n_2 n_3, n_3^2 - 1), \quad \mathbf{e}^{[2]} \equiv D^{-1}(-n_2, n_1, 0) \ ,$$

$$\mathbf{e}^{[3]} = \mathbf{n} \equiv (n_1, n_2, n_3) \qquad D = (n_1^2 + n_2^2)^{1/2}$$

a_{ijkl} - stiffness tensor, $\mathbf{n}$ - unit vector, $\mathbf{N}$ - ray vector

B_{33}, $B_{13}^2 + B_{23}^2$ - independent of the choice of $\mathbf{e}^{[1]}$, $\mathbf{e}^{[2]}$

$a = (r^2 - 3/4)/(1 - r^2)$, $\qquad r = \beta_0/\alpha_0$

Velocity approximations

Orthorhombic medium (6 WA parameters)

$$B_{13}(\mathbf{n}) = \alpha_0^2 D^{-1}\{n_3^3(\eta_y n_1^2 + \eta_x n_2^2) + n_3[(2\eta_z - \eta_x - \eta_y)n_1^2 n_2^2 - \eta_y n_1^4 - \eta_x n_2^4 + (\epsilon_x - \epsilon_z)n_1^2 + (\epsilon_y - \epsilon_z)n_2^2]\}$$

$$B_{23}(\mathbf{n}) = \alpha_0^2 D^{-1} n_1 n_2[(\eta_x - \eta_y)n_3^2 + \eta_z n_1^2 - \eta_z n_2^2 + \epsilon_y - \epsilon_x]$$

$$B_{33}(\mathbf{n}) = \alpha_0^2[1 + 2n_3^2(\eta_y n_1^2 + \eta_x n_2^2 + \epsilon_z) + 2\epsilon_x n_1^2 + 2\epsilon_y n_2^2 + 2\eta_z n_1^2 n_2^2]$$

$$\eta_x = \delta_x - \epsilon_y - \epsilon_z \,, \qquad \eta_y = \delta_y - \epsilon_z - \epsilon_x \,, \qquad \eta_z = \delta_z - \epsilon_x - \epsilon_y$$

Tests of the velocity approximations

Specification of phase and ray vectors by polar angles

Phase vector $\mathbf{n}$

$$\mathbf{n} \equiv (\cos\phi \sin\theta,\ \sin\phi \sin\theta,\ \cos\theta)$$

Ray vector $\mathbf{N}$

$$\mathbf{N} \equiv (\cos\Phi \sin\Theta,\ \sin\Phi \sin\Theta,\ \cos\Theta)$$

Tests of the velocity approximations

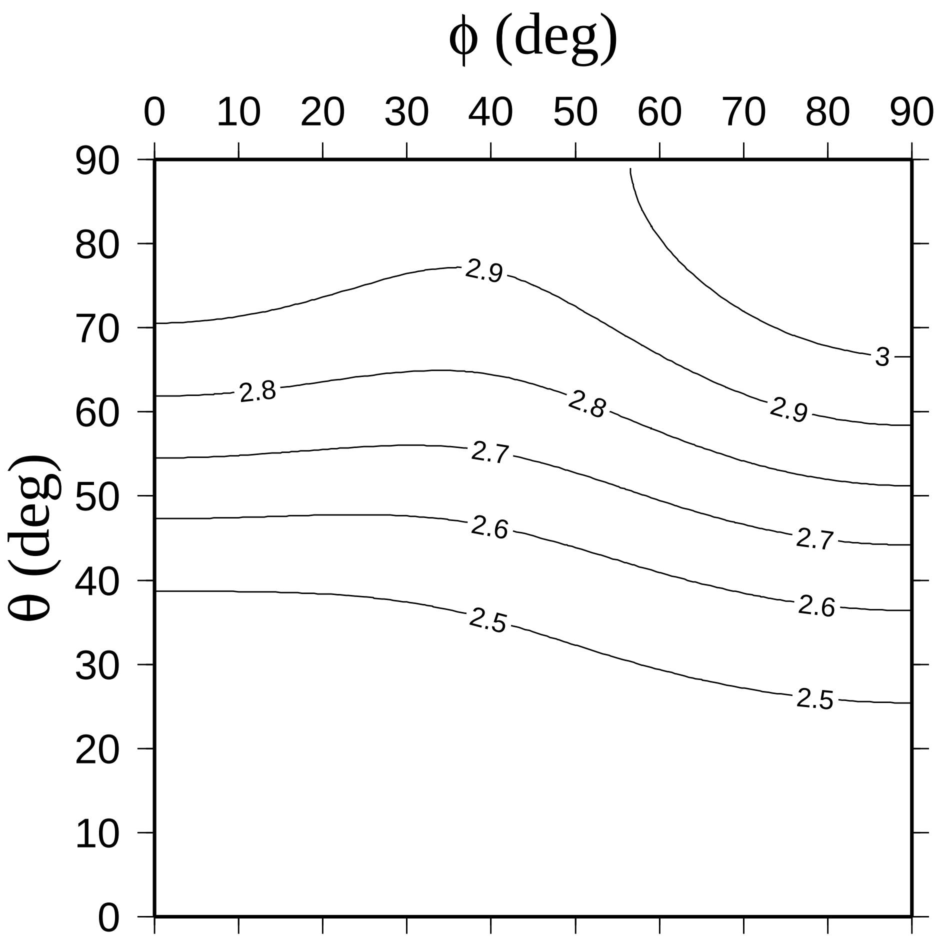
ORT model - Schoenberg & Helbig, 1997

planes of symmetry parallel to coordinate planes

$$\begin{pmatrix} 9.00 & 3.60 & 2.25 & 0 & 0 & 0 \\ & 9.84 & 2.40 & 0 & 0 & 0 \\ & & 5.9375 & 0 & 0 & 0 \\ & & & 2.00 & 0 & 0 \\ & & & & 1.60 & 0 \\ & & & & & 2.182 \end{pmatrix}$$

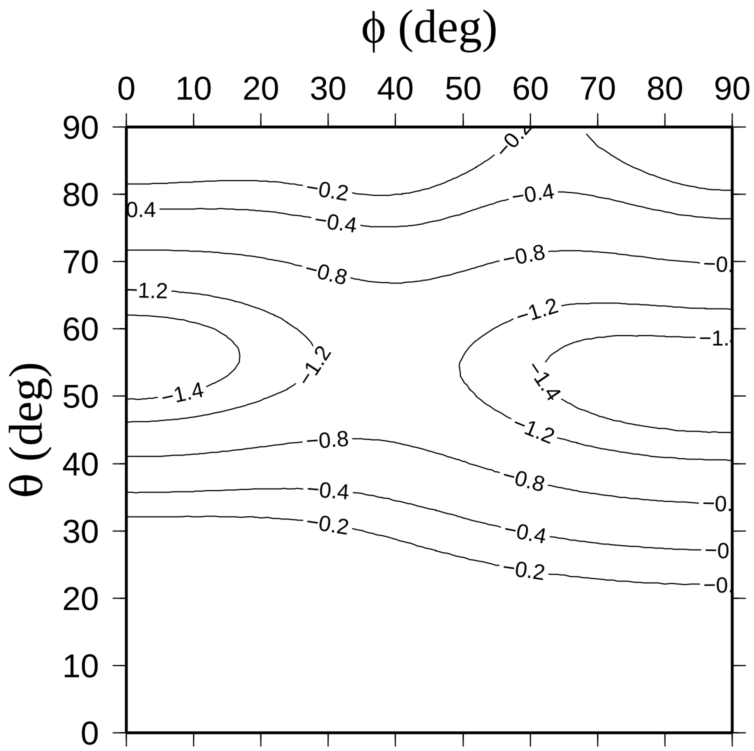
Tests of the velocity approximations

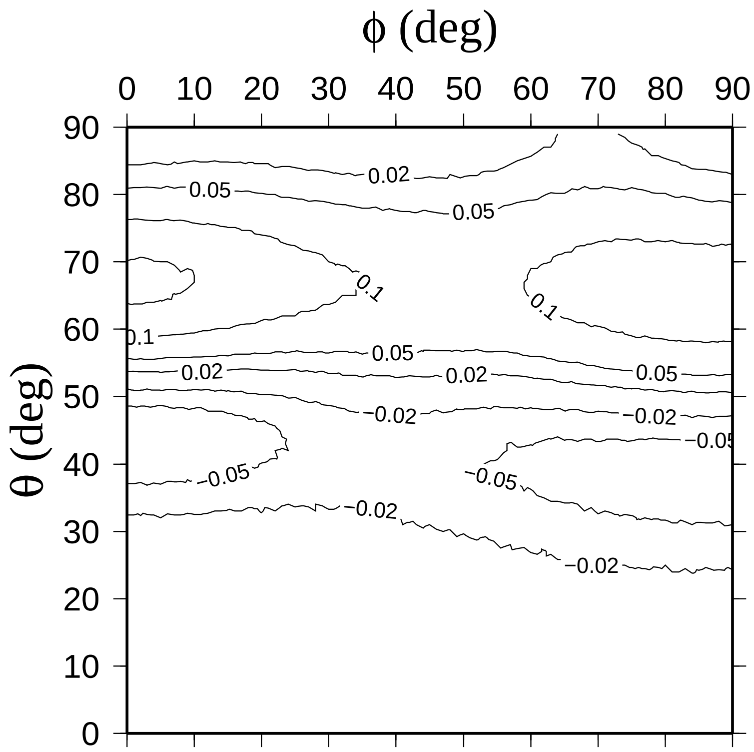
ORT model - phase velocity



Tests of the phase velocity approximations

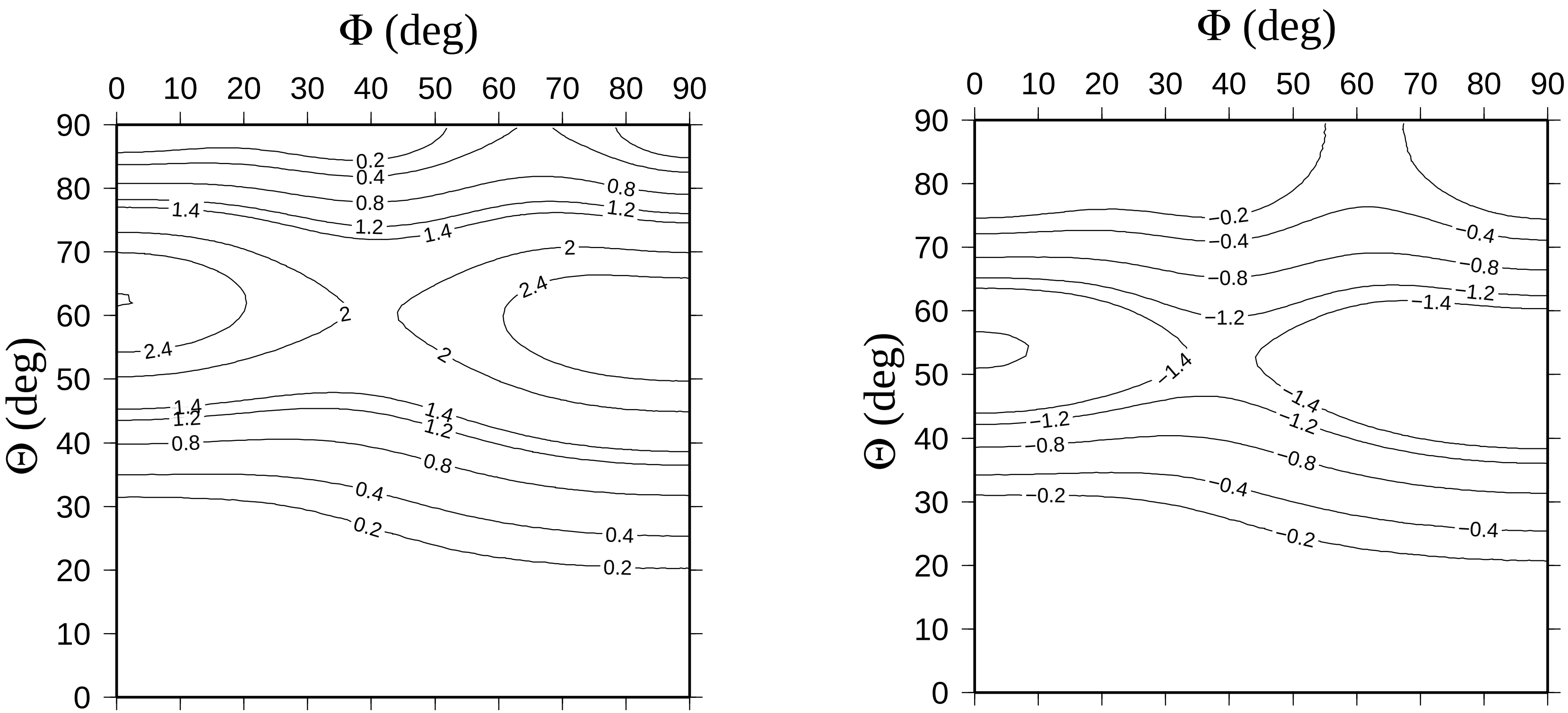
ORT model, 1-st order, 2-nd order, relative errors (%)





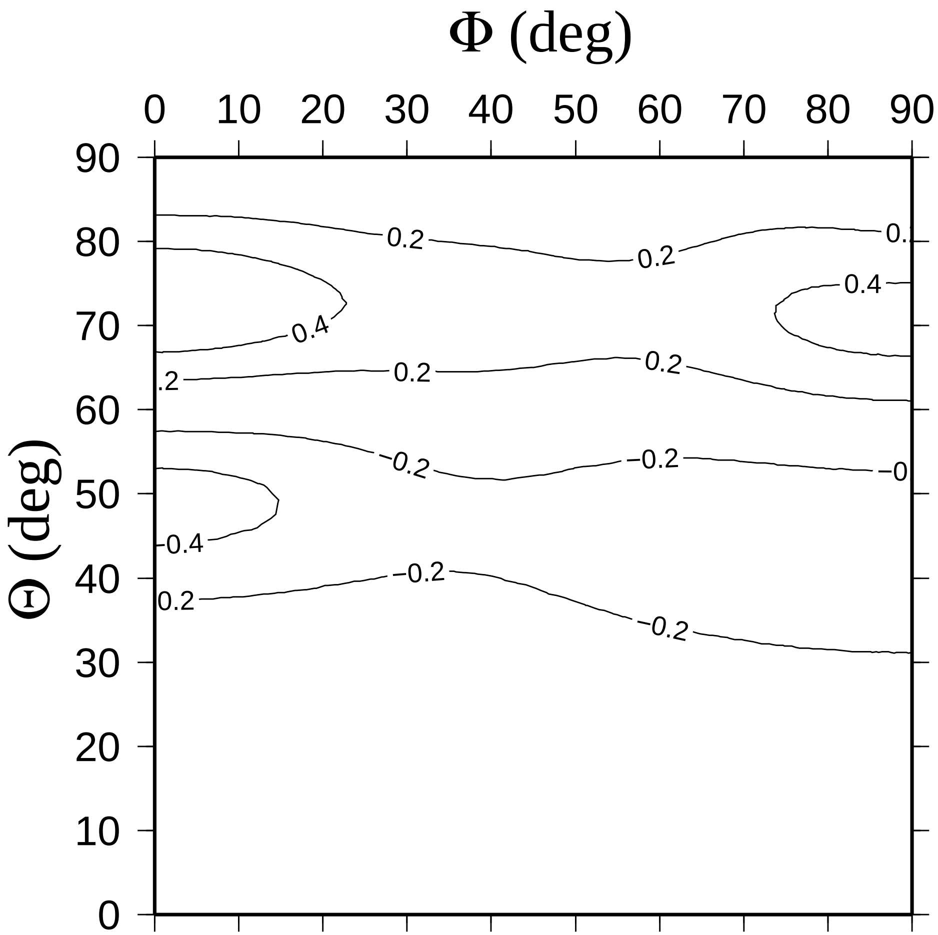
Tests of the ray velocity approximations

ORT model, approximations # 1, # 2, relative errors (%)



Tests of the ray velocity approximations

ORT model, approximation # 3, relative errors (%)



Tests of the velocity approximations

ORT model

P-wave anisotropy strength $[(A_{11} - A_{33})/(A_{11} + A_{33})] \sim 20\%$

Phase velocity - maximum relative errors

1st-order: $\sim$ 1.54%

2nd-order: $\sim$ 0.16%

Ray velocity - maximum relative errors

case # 1: $\sim$ 2.8%

case # 2: $\sim$ 2%

case # 3: $\sim$ 0.56%

case # 3 comparable with AA of Sripanich & Fomel (2014);
slightly worse than GMA of Hao & Stovas (2015)

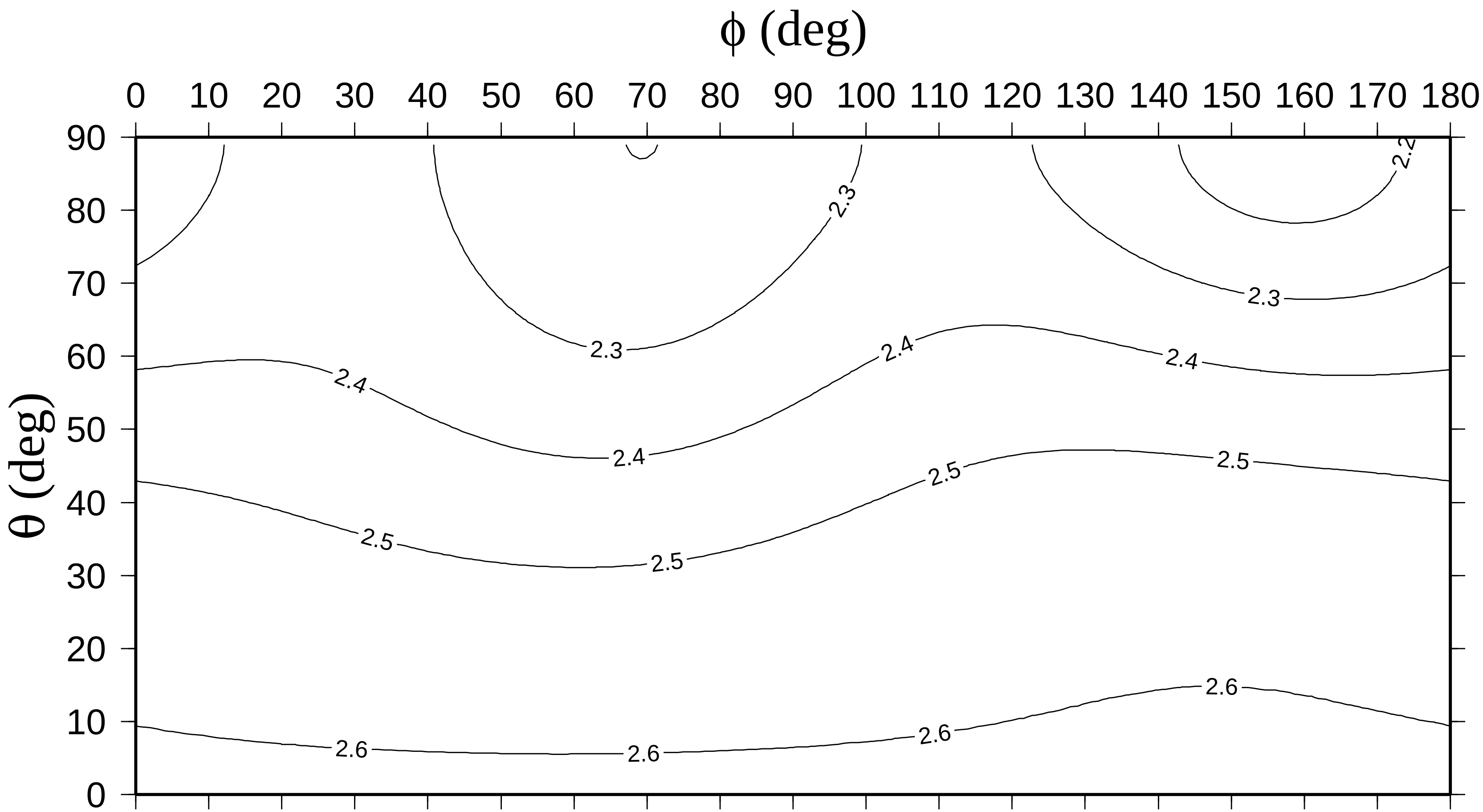
Tests of the velocity approximations

MONO model

$$\begin{pmatrix} 4.952 & 0.433 & 0.625 & 0 & 0 & 0.385 \\ & 5.096 & 1.010 & 0 & 0 & -0.288 \\ & & 6.779 & 0 & 0 & -0.481 \\ & & & 2.452 & 0 & 0 \\ & & & & 2.885 & 0 \\ & & & & & 2.356 \end{pmatrix}$$

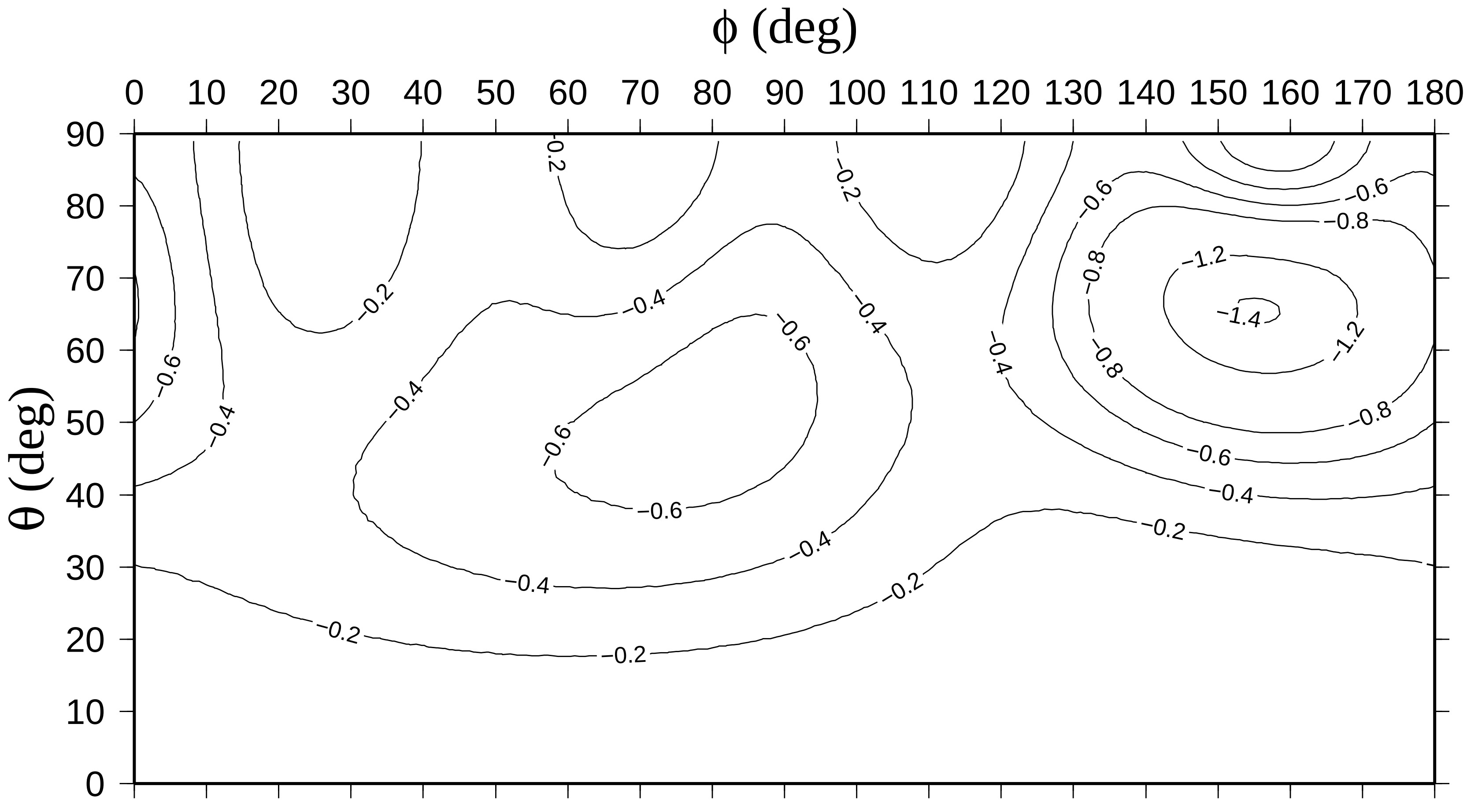
Tests of the velocity approximations

MONO model - phase velocity



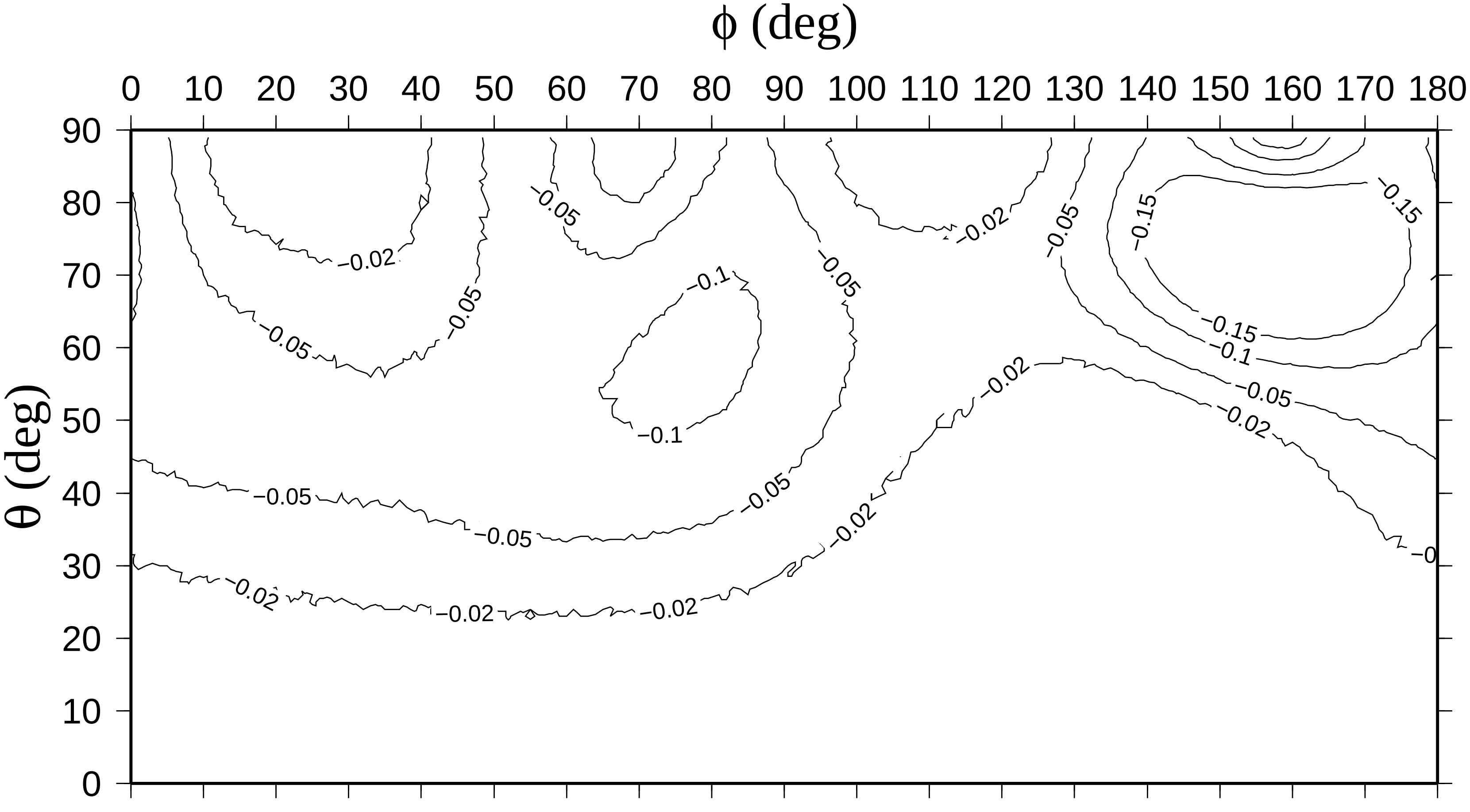
Tests of the phase-velocity approximation

MONO model, 1-st order, relative errors (%)



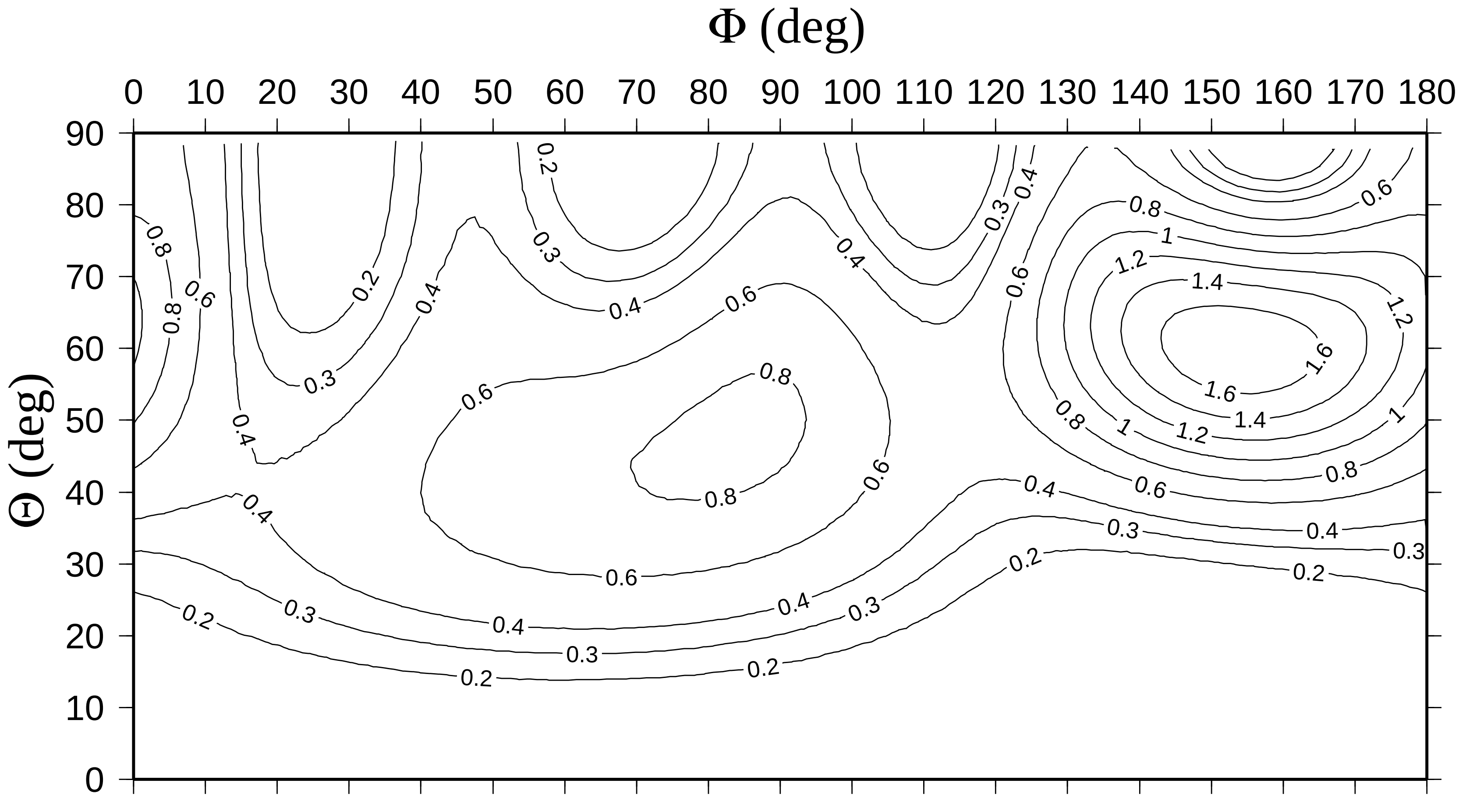
Tests of the phase-velocity approximation

MONO model, 2-nd order, relative errors (%)



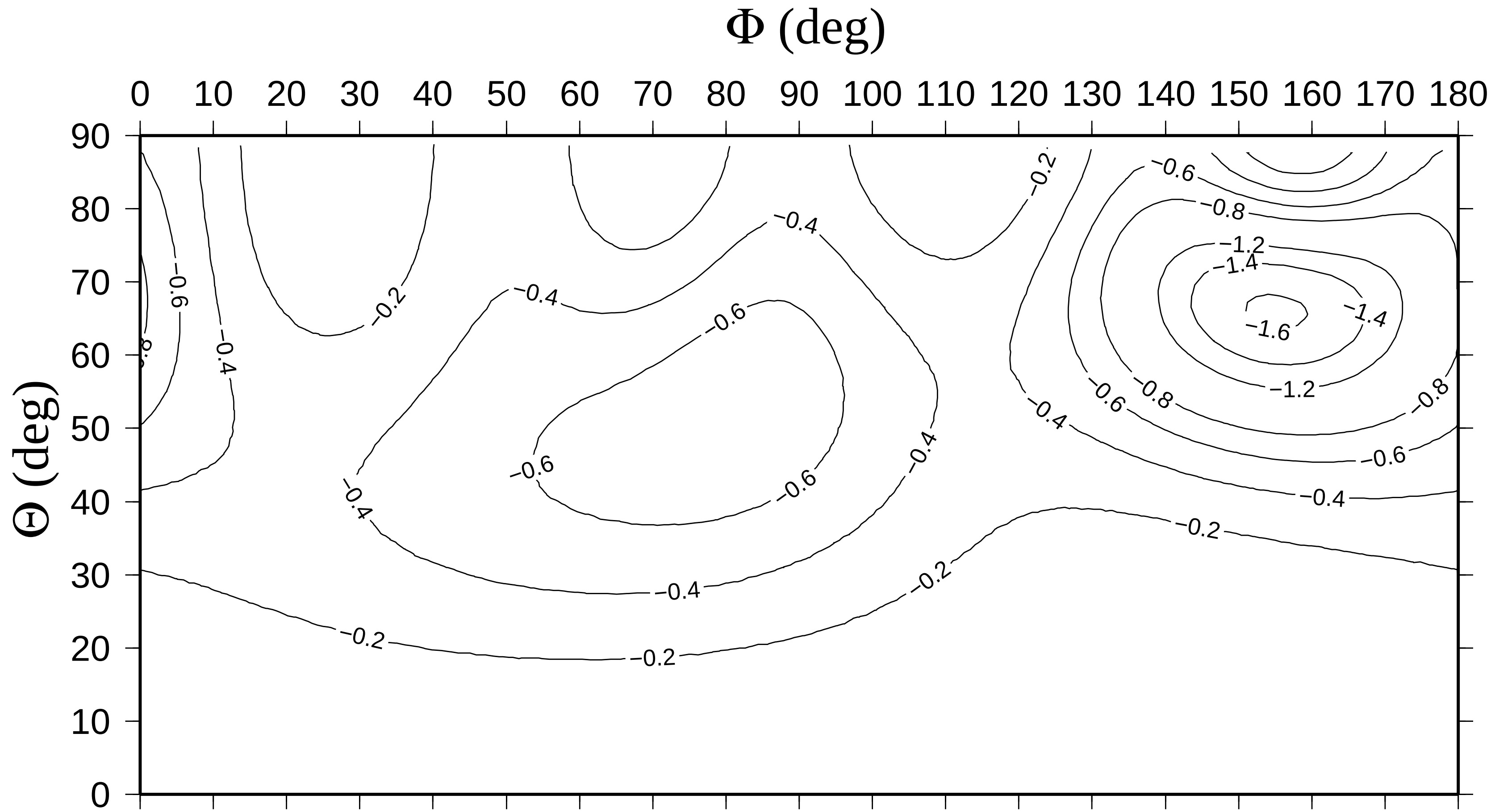
Tests of the ray-velocity approximation

MONO model, approximation # 1, relative errors (%)



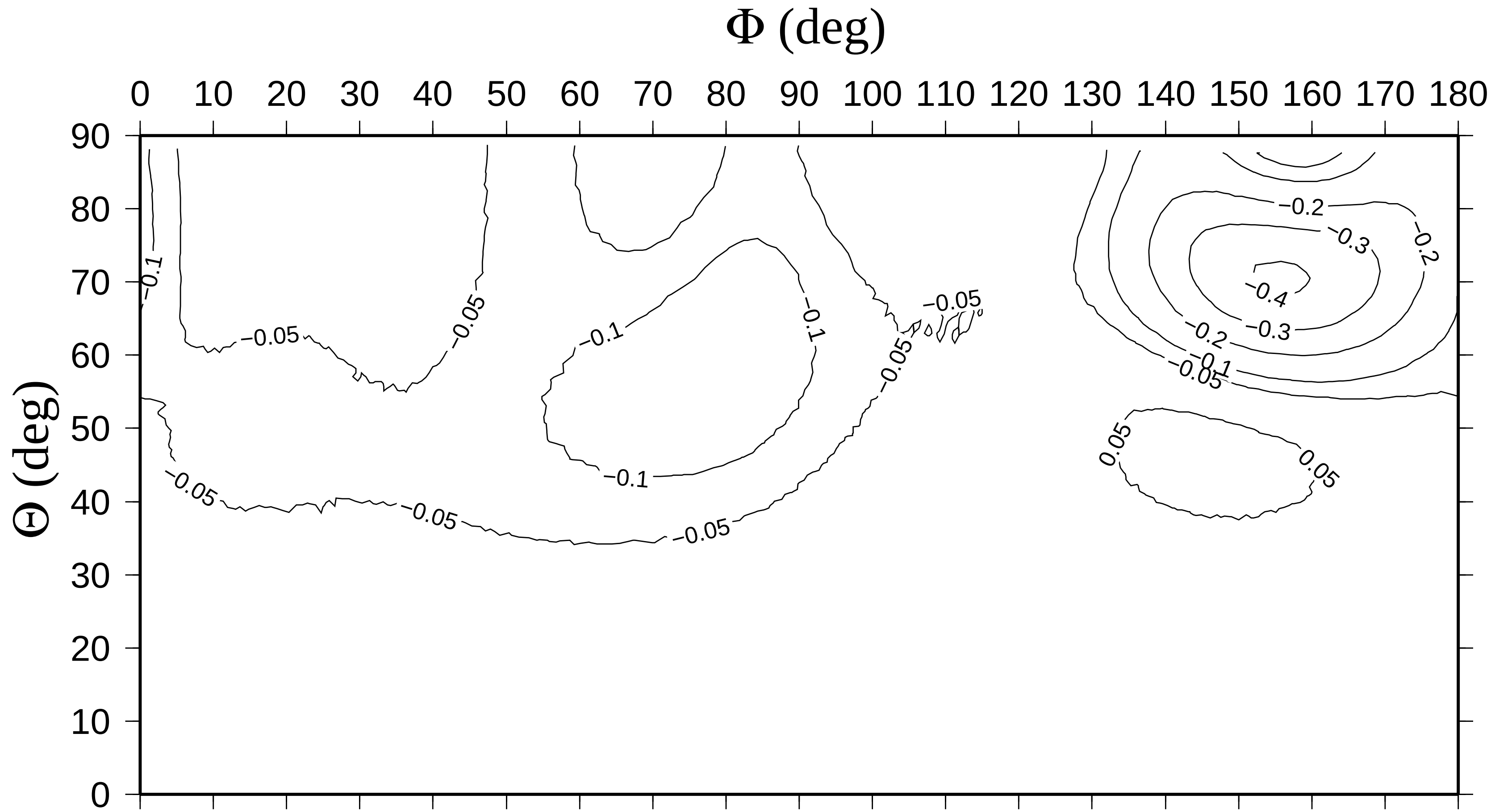
Tests of the ray-velocity approximation

MONO model, approximation # 2, relative errors (%)



Tests of the ray-velocity approximation

MONO model, approximation # 3, relative errors (%)



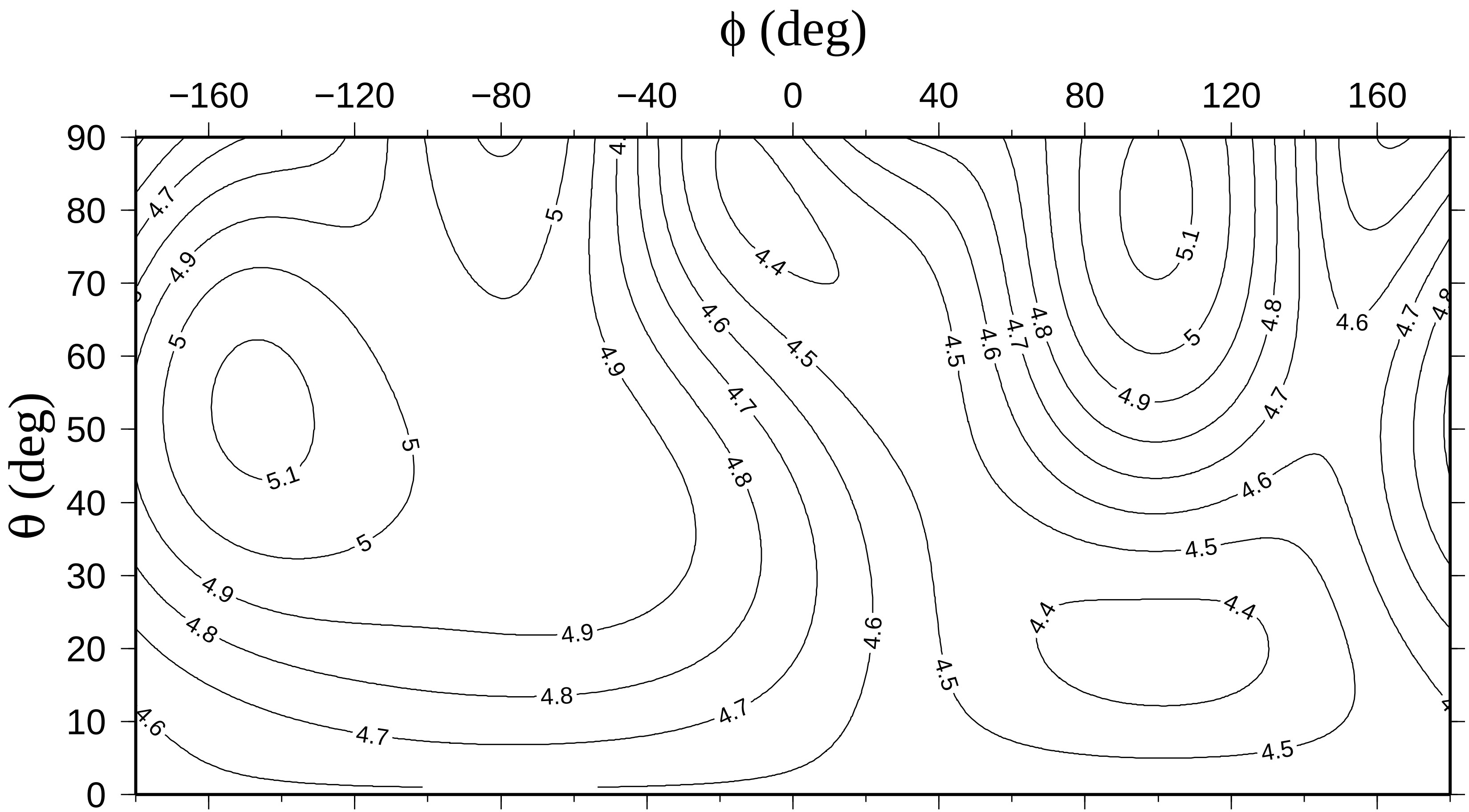
Tests of the velocity approximations

TRI model

$$\begin{pmatrix} 19.81 & 8.62 & 9.00 & -2.37 & -1.44 & 0.95 \\ & 25.79 & 9.09 & 0.57 & -0.99 & -0.89 \\ & & 20.68 & -2.10 & 0.43 & 0.49 \\ & & & 7.17 & -0.15 & -0.08 \\ & & & & 8.14 & -0.33 \\ & & & & & 6.49 \end{pmatrix}$$

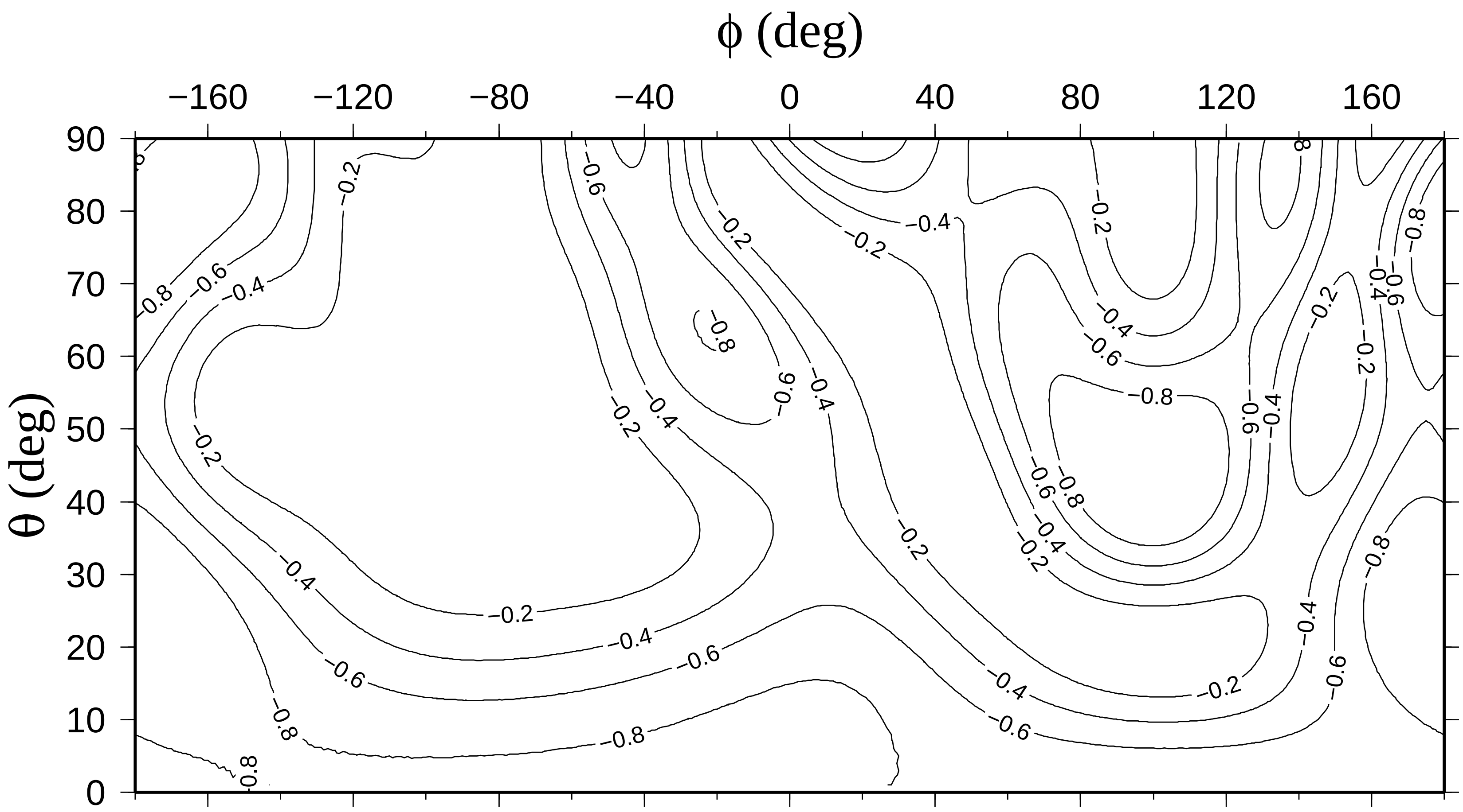
Tests of the velocity approximations

TRI model - phase velocity



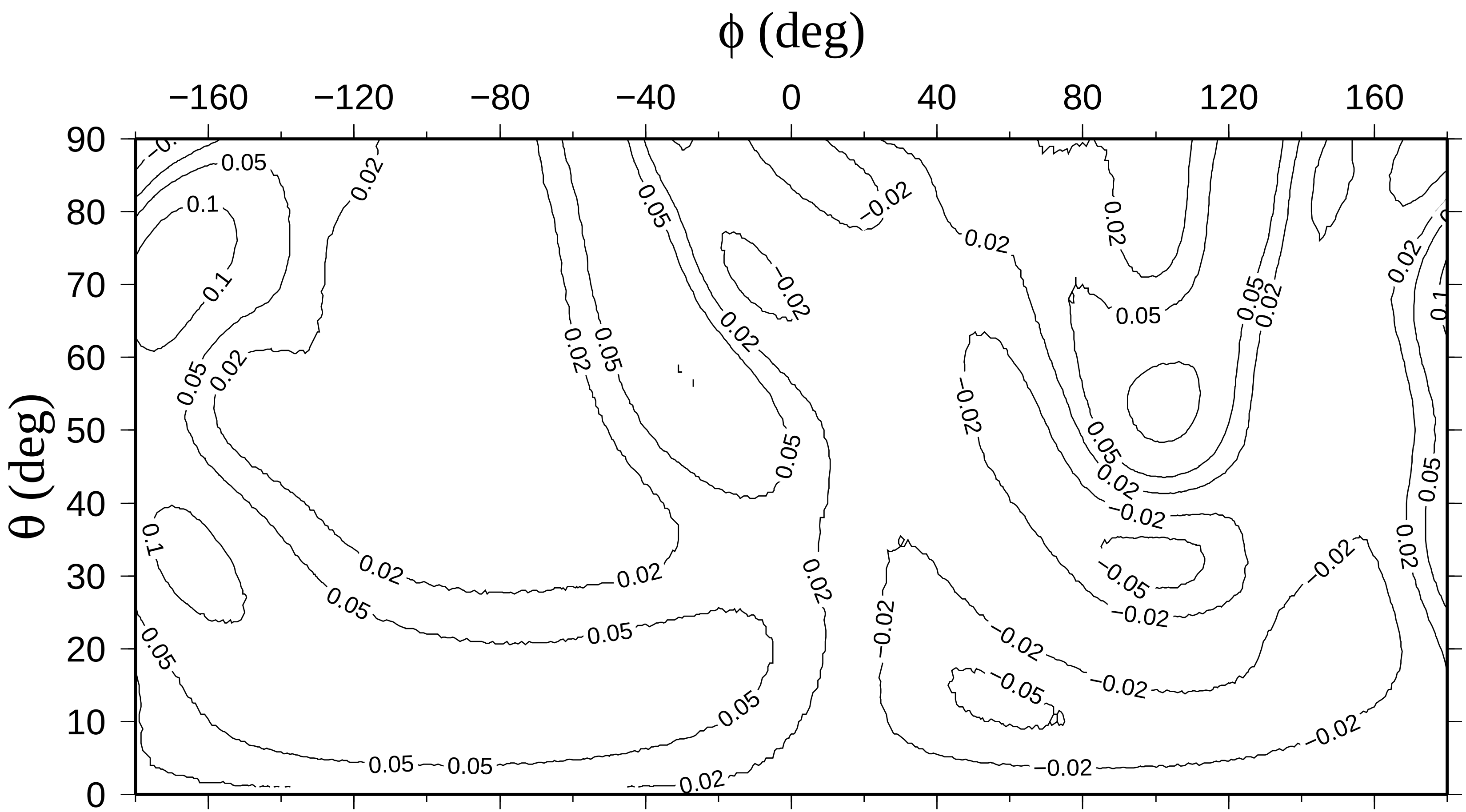
Tests of the phase velocity approximations

TRI model, 1-st order, relative errors (%)



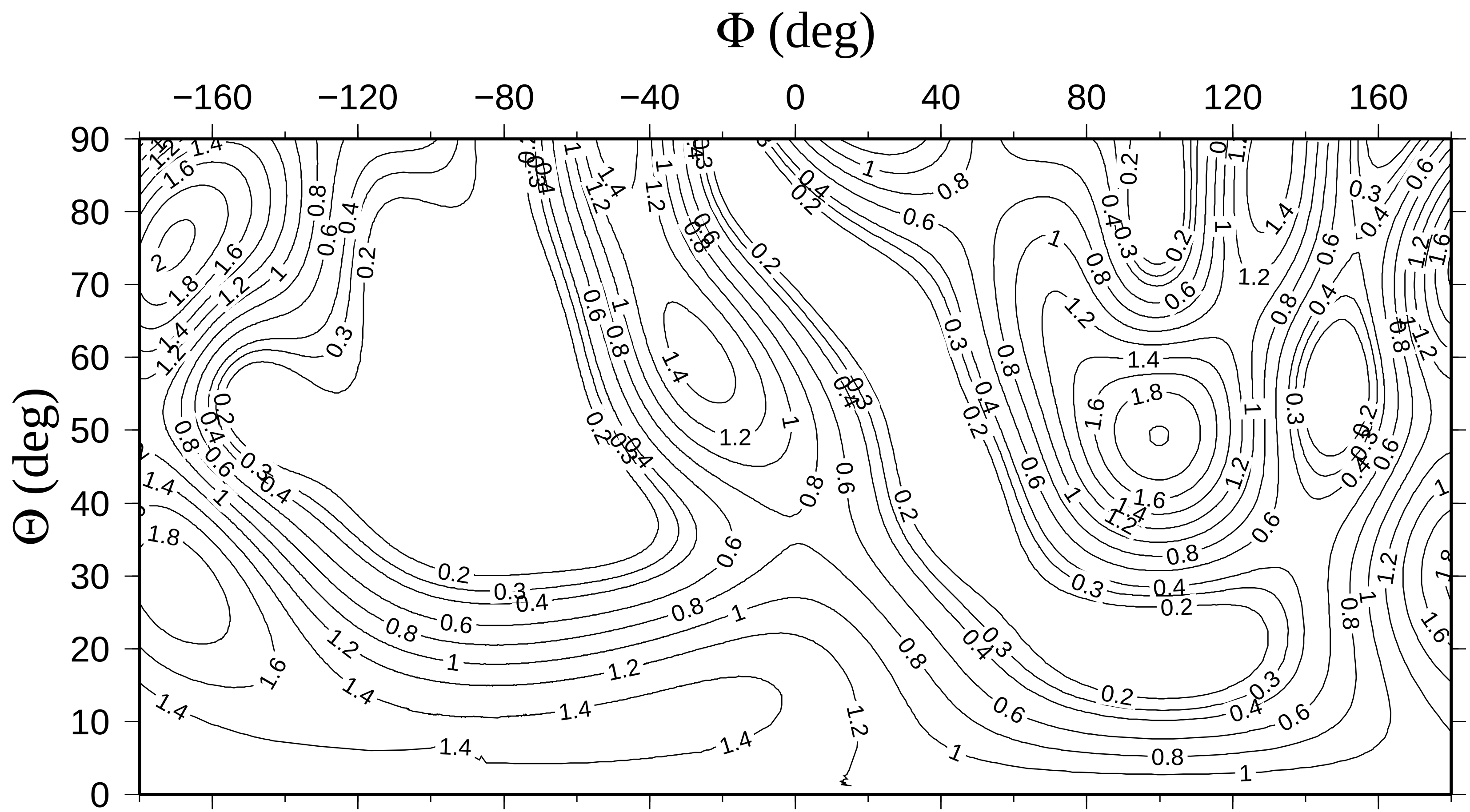
Tests of the phase velocity approximations

TRI model, 2-nd order, relative errors (%)



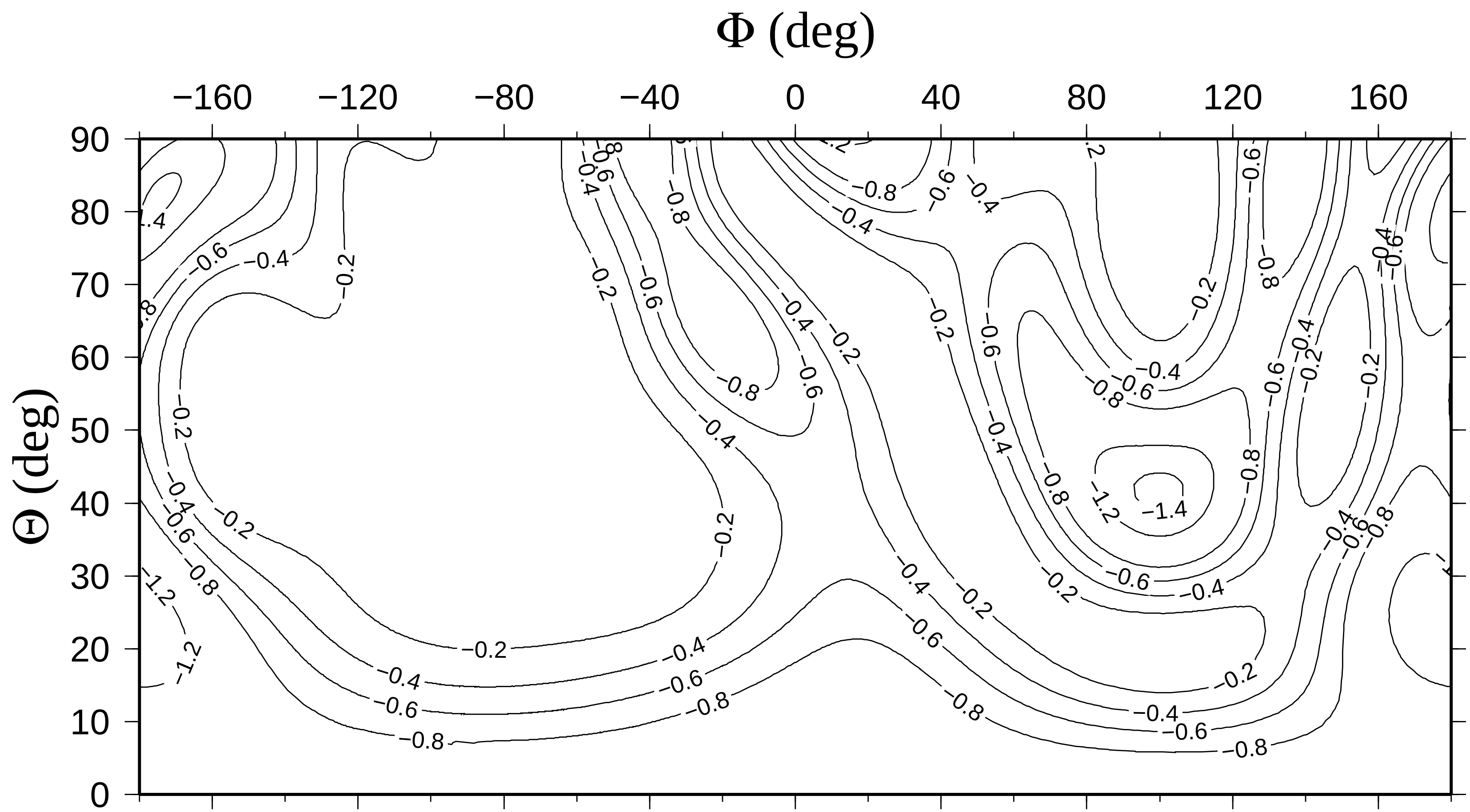
Tests of the ray velocity approximations

TRI model, approximation # 1, relative errors (%)



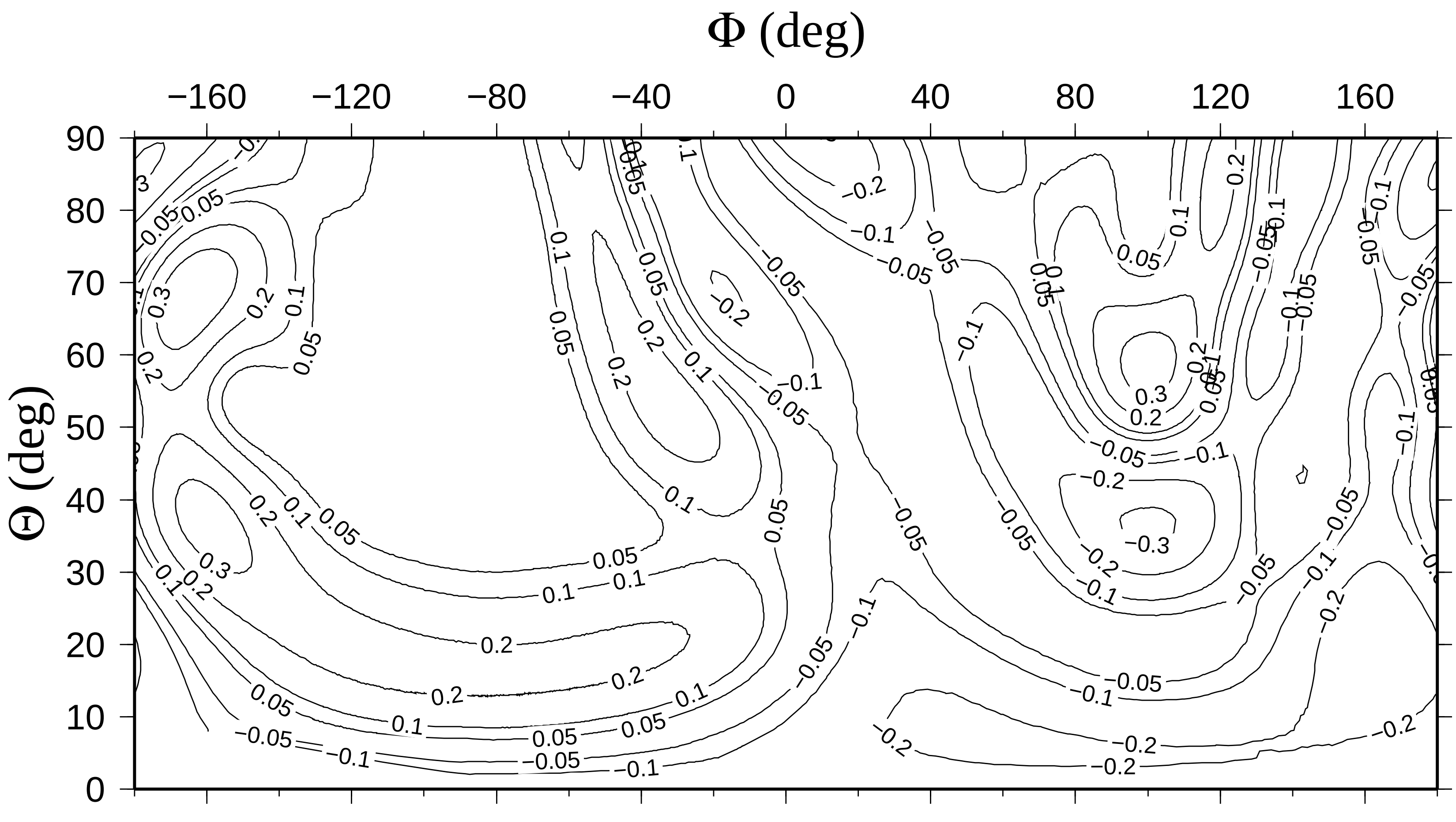
Tests of the ray velocity approximations

TRI model, approximation # 2, relative errors (%)



Tests of the ray velocity approximations

TRI model, approximation # 3, relative errors (%)



Tests of the velocity approximations

TRI model

P-wave anisotropy strength $[(A_{22} - A_{11})/(A_{11} + A_{22})] \sim 26\%$

Phase velocity - maximum relative errors

1st-order: $\sim$ 1%

2nd-order: $\sim$ 0.15%

Ray velocity - maximum relative errors

case # 1: $\sim$ 2%

case # 2: $\sim$ 1.5%

case # 3: $\sim$ 0.39%

Conclusions

- based on WA approximation

- expressed in terms of WA parameters

- relatively simple formulae; no square roots

- no non-physical assumptions

- dependence on 3 elements of rotated Christoffel matrix

- elements of Christoffel matrix linear functions of WA parameters

- independent or weakly dependent on the reference medium

- applicable to weak or moderate anisotropy of any symmetry

- applicable to weak or moderate anisotropy of any orientation

Conclusions

- inaccuracies for large deviations of $\mathbf{n}$ and $\mathbf{N}$,
  i.e., for large deviations of c and v

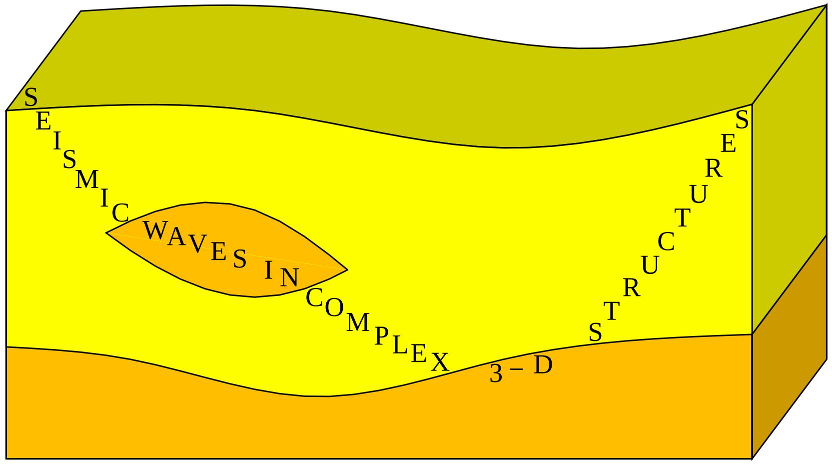
- accuracy of second-order formulae:
  relative errors $<$ 0.5% for $\sim$ 26% anisotropy

- accuracy of second-order formulae close to anelliptic approximation

- reference of WA formulae - spherical velocity surface

- reduced number of WA parameters (WA/exact):
  triclinic - 15/21, monoclinic - 9/12, orthorhombic - 6/9, TI - 3/5

Acknowledgement



Consortium project SW3D: http://sw3d.cz/

Weak-anisotropy parameters

$$\epsilon_x = \frac{A_{11}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_y = \frac{A_{22}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_z = \frac{A_{33}-\alpha_0^2}{2\alpha_0^2}$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha_0^2}{\alpha_0^2}, \quad \delta_y = \frac{A_{13}+2A_{55}-\alpha_0^2}{\alpha_0^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha_0^2}{\alpha_0^2}$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha_0^2}, \quad \chi_y = \frac{A_{25}+2A_{46}}{\alpha_0^2}, \quad \chi_z = \frac{A_{36}+2A_{45}}{\alpha_0^2}$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha_0^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha_0^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha_0^2}$$

$$\epsilon_{26} = \frac{A_{26}}{\alpha_0^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha_0^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha_0^2}$$

$$\epsilon_{46} = \frac{A_{46}}{\alpha_0^2}, \quad \epsilon_{56} = \frac{A_{56}}{\alpha_0^2}, \quad \epsilon_{45} = \frac{A_{45}}{\beta_0^2}$$

$$\gamma_x = \frac{A_{44}-\beta_0^2}{2\beta_0^2}, \quad \gamma_y = \frac{A_{55}-\beta_0^2}{2\beta_0^2}, \quad \gamma_z = \frac{A_{66}-\beta_0^2}{2\beta_0^2}$$

α_0, β_0 - reference velocities