P-wave weak-anisotropy moveout approximations

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Outline

Introduction

Weak-anisotropy parameters

Exact traveltime formula

Ray-velocity approximations

Traveltime approximations

Inversion tests

NMO velocity, quartic coefficient

Conclusions

Possible extensions

Introduction

Moveout approximations

common

- expansion of T^2 in terms of the squared offset

hyperbolic, non-hyperbolic, ...

alternative

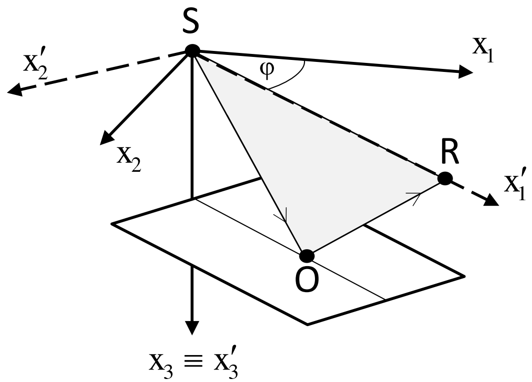
- expansion of T^2 in terms of the deviations

of anisotropy from isotropy

Introduction

Monoclinic or higher symmetry anisotropic layer

Unconverted P wave



reflection from a plane reflector Σ

coinciding with a symmetry plane

$\Rightarrow$ ray of reflected P wave in a plane $\perp$ to reflector Σ,

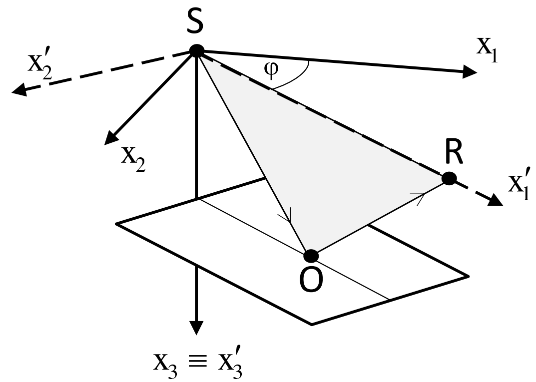
symmetric with respect to the normal to Σ,

$\Rightarrow$ 2D problem in the (x'_1, x'_3) plane of local x'_i coordinate system

Introduction

Monoclinic or higher symmetry anisotropic layer

Unconverted P wave



Two-step formulation:

1) solution of the 2D problem in the local coordinates x_i';
formulae expressed in terms of local medium parameters

2) transformation of the solution to 3D by using expressions
of local medium parameters in terms of global ones

Weak-anisotropy parameters

- 21 weak-anisotropy (WA) parameters

  slightly modified *Mensch & Rasolofosaon (1997)*

- an alternative to stiffness tensor or $C_{\alpha\beta}$ or $A_{\alpha\beta}$

  linear relation of WA to $C_{\alpha\beta}$ or $A_{\alpha\beta}$ parameters

- applicable to anisotropy of any type, strength and orientation

- may describe exactly any wave attribute

- natural combinations of $C_{\alpha\beta}$ or $A_{\alpha\beta}$ taken into account

- generalization of *Thomsen's (1986)* parameters

Weak-anisotropy parameters

- represent deviation from an isotropic reference

- freedom in the choice of the reference velocity

- specified in coordinate systems independent of symmetry elements
  of studied anisotropy symmetry

- simple treatment when coordinate system rotates

- all 21 WA parameters dimensionless, of comparable size

- first-order P-wave attributes depend on only **15 WA parameters**

Weak-anisotropy parameters

P-wave WA parameters

$$\epsilon_x = \frac{A_{11}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_y = \frac{A_{22}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_z = \frac{A_{33}-\alpha_0^2}{2\alpha_0^2}$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha_0^2}{\alpha_0^2}, \quad \delta_y = \frac{A_{13}+2A_{55}-\alpha_0^2}{\alpha_0^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha_0^2}{\alpha_0^2}$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha_0^2}, \quad \chi_y = \frac{A_{25}+2A_{46}}{\alpha_0^2}, \quad \chi_z = \frac{A_{36}+2A_{45}}{\alpha_0^2}$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha_0^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha_0^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha_0^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha_0^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha_0^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha_0^2}$$

α_0 - reference velocity

Weak-anisotropy parameters

Orthorhombic symmetry planes parallel to coordinate planes

$$\epsilon_x = \frac{A_{11}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_y = \frac{A_{22}-\alpha_0^2}{2\alpha_0^2}, \quad \epsilon_z = \frac{A_{33}-\alpha_0^2}{2\alpha_0^2}$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha_0^2}{\alpha_0^2}, \quad \delta_y = \frac{A_{13}+2A_{55}-\alpha_0^2}{\alpha_0^2}, \quad \delta_z = \frac{A_{12}+2A_{66}-\alpha_0^2}{\alpha_0^2}$$

α_0 - reference velocity

Relations between local and global WA parameters

$$\epsilon'_x(\varphi) = \epsilon_x \cos^4\varphi + \delta_z \cos^2\varphi \sin^2\varphi + \epsilon_y \sin^4\varphi$$

$$\delta'_y(\varphi) = \delta_y \cos^2\varphi + \delta_x \sin^2\varphi \text{ ,} \qquad \chi'_z(\varphi) = (\delta_x - \delta_y)\sin\varphi\cos\varphi$$

$$\epsilon'_{16}(\varphi) = -2\epsilon_x \cos^3\varphi \sin\varphi + 2\epsilon_y \sin^3\varphi \cos\varphi + \delta_z \cos\varphi \sin\varphi \cos 2\varphi$$

φ - angle between x_1 and x'_1

Exact formulae

Moveout formula

$$T^2(x) = (4H^2 + x^2)/v^2(\mathbf{N})$$

$T(x)$ - traveltime at the offset x

$v(\mathbf{N})$ - ray velocity along the ray direction $\mathbf{N}$;
the same on down- and up-going leg of ray

$\mathbf{N} = \mathbf{N}(\mathbf{n})$ - *ray vector*, unit vector in the direction of the ray

$\mathbf{n}$ - *phase vector*, unit vector parallel to the slowness vector;
$\mathbf{n}$ may deviate from (x_1', x_3') plane

H - depth of the plane horizontal reflector

Exact formulae

Deviation of vectors $\mathbf{n}$ and $\mathbf{N}$

$$\mathbf{p} \cdot \mathbf{v} = 1 \qquad \Rightarrow \qquad \mathbf{n} \cdot \mathbf{N} = v/c$$

$\mathbf{p}$ - slowness vector

$\mathbf{v}$ - ray-velocity vector

v, c - ray and phase velocities

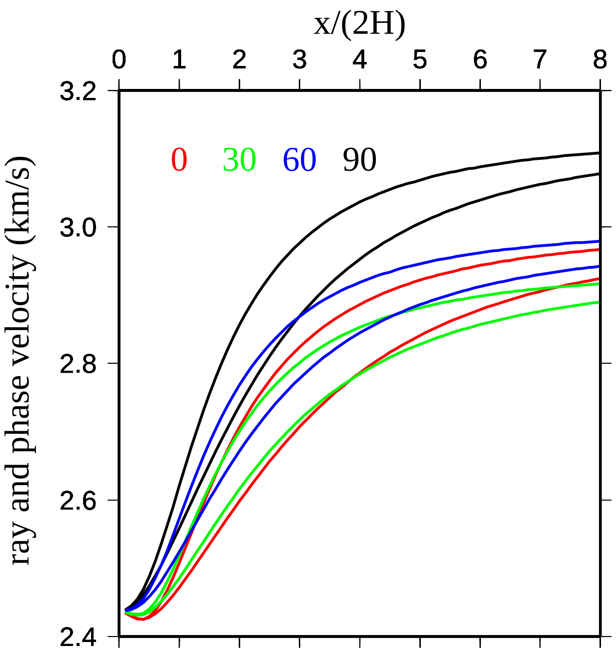
Deviation of $\mathbf{n}$ and $\mathbf{N}$ controlled by the deviation of c and v

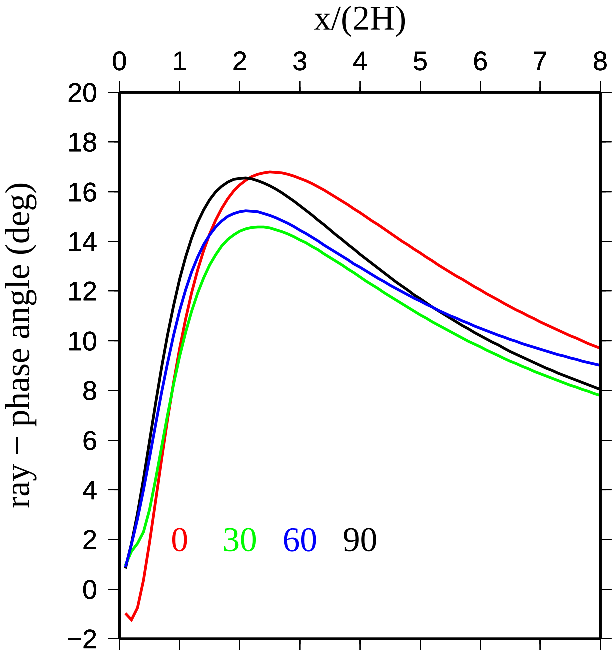
Exact formulae

ORT model - Schoenberg & Helbig (1997)

α_0=2.437 km/s, β_0=1.414 km/s, ϵ_x=0.258, ϵ_y=0.328, ϵ_z=0,

δ_x=0.077, δ_y=–0.083, δ_z=0.340





Exact formulae

Normalized moveout formula

$$\bar{x} = x/2H \;, \qquad T_0 = 2H/\alpha_0$$

$$T^2(\bar{x}) = \alpha_0^2 T_0^2 \,(1 + \bar{x}^2)/v^2(\mathbf{N})$$

T_0 - two-way zero-offset traveltime

$\bar{x}$ - normalized offset

α_0 - vertical phase velocity ($\alpha_0^2 = A_{33}$) $\;\Rightarrow\; \epsilon_z = 0$

$A_{\alpha\beta}$ - density-normalized elastic moduli in the Voigt notation

Exact formulae

Problem: Sought: $v^2(\mathbf{N})$

Available: $\mathbf{N}$ and $\tilde{c}^2(\mathbf{N})$

$\tilde{c}^2(\mathbf{N})$ - first-order approximation of square of phase velocity

$v^2(\mathbf{N})$ - square of ray velocity

$\mathbf{N}$ - ray vector

$\Rightarrow$ need to find the relation between $v^2(\mathbf{N})$ and $\tilde{c}^2(\mathbf{N})$

Ray-velocity approximations

#1 Ignore the deviation of vectors $\mathbf{n}$ and $\mathbf{N}$,

use 1st-order approximation of phase velocity squared

$$\tilde{v}^2(\mathbf{N}) \sim \tilde{c}^2(\mathbf{N}) = B_{33}(\mathbf{N})$$

#2 Consider the deviation of vectors $\mathbf{n}$ and $\mathbf{N}$,

use 1st-order approximation of phase velocity squared

$$\tilde{v}^2(\mathbf{N}) \sim \tilde{c}^2(\mathbf{N}) - 4[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})]/\tilde{c}^2(\mathbf{N})$$

#3 Consider the deviation of vectors $\mathbf{n}$ and $\mathbf{N}$,

use 2nd-order approximation of phase velocity squared

$$\tilde{\tilde{v}}^2(\mathbf{N}) \sim \tilde{c}^2(\mathbf{N}) + 4a[B_{13}^2(\mathbf{N}) + B_{23}^2(\mathbf{N})]/\tilde{c}^2(\mathbf{N})$$

Ray-velocity approximations

$$B_{mn}(\mathbf{N}) = a_{ijkl} N_j N_l e_i^{[m]} e_k^{[n]}$$

$B_{mn}(\mathbf{N})$ - Christoffel matrix rotated to the vector basis $\mathbf{e}^{[i]}$

$$\mathbf{e}^{[1]} \equiv D^{-1}(N_1 N_3, N_2 N_3, N_3^2 - 1), \quad \mathbf{e}^{[2]} \equiv D^{-1}(-N_2, N_1, 0) \text{ ,}$$

$$\mathbf{e}^{[3]} = \mathbf{N} \equiv (N_1, N_2, N_3) \qquad D = (N_1^2 + N_2^2)^{1/2}$$

a_{ijkl} - stiffness tensor, $\mathbf{N}$ - a unit vector

$$a = (r^2 - 3/4)/(1 - r^2), \quad r^2 = \beta_0^2/\alpha_0^2, \quad \alpha_0^2 = A_{33}, \quad \beta_0^2 = A_{55}$$

Ray-velocity approximations

Specification in the plane (x'_1, x'_3): $N'_2 = 0$

$$B_{13}(\mathbf{N}) = \alpha_0^2 N'_1 N'_3 [\delta'_y - 2(\delta'_y - \epsilon'_x) N'^2_1]$$

$$B_{23}(\mathbf{N}) = \alpha_0^2 N'_1 (\chi'_z N'^2_3 + \epsilon'_{16} N'^2_1)$$

$$\tilde{c}^2(\mathbf{N}) = B_{33}(\mathbf{N}) = \alpha_0^2 \Big(1 + 2N'^2_1 [\epsilon'_x + (\delta'_y - \epsilon'_x) N'^2_3]\Big)$$

$$\epsilon'_z = 0$$

Traveltime approximations

Traveltime formulae - local coordinates

#1 $$T^2(\bar{x}) = T_0^2\ (1+\bar{x}^2)^3/P(\bar{x})$$

#2 $$T^2(\bar{x}) = T_0^2\ P(\bar{x})(1+\bar{x}^2)^3/\Big[P^2(\bar{x}) - Q_1^2(\bar{x}) - (1+\bar{x}^2)Q_2^2(\bar{x})\Big]$$

#3 $$T^2(\bar{x}) = T_0^2\ P(\bar{x})(1+\bar{x}^2)^3/\Big(P^2(\bar{x}) + a[Q_1^2(\bar{x}) + (1+\bar{x}^2)Q_2^2(\bar{x})]\Big)$$

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\delta'_y\bar{x}^2 + 2\epsilon'_x\bar{x}^4$$

$$Q_1(\bar{x}) = 2\bar{x}[2\epsilon'_x\bar{x}^2 + \delta'_y(1-\bar{x}^2)] \qquad Q_2(\bar{x}) = 2\bar{x}(\chi'_z + \epsilon'_{16}\bar{x}^2)$$

Traveltime approximations

Reference formula (Tsvankin & Grechka, 2011)

(Orthorhombic medium with symmetry planes
coinciding with coordinate planes)

$$T^2(\bar{x}) = T_0^2\left(1 + A_2(\varphi)\bar{x}^2 + \frac{A_4(\varphi)\bar{x}^4}{1+B(\varphi)\bar{x}^2}\right)$$

$$A_2(\varphi) = (1+2\delta_1)^{-1}\sin^2\varphi + (1+2\delta_2)^{-1}\cos^2\varphi$$

$$A_4(\varphi) = -2\eta(\varphi)A_2^2(\varphi)$$

$$B(\varphi) = [1+2\eta(\varphi)]A_2(\varphi)$$

$$\eta(\varphi) = \eta_1\sin^2\varphi - \eta_3\sin^2\varphi\cos^2\varphi + \eta_2\cos^2\varphi$$

δ_i, η_i - parameters of Tsvankin & Grechka (2011)

φ - angle of the source-receiver profile with x_1-axis

Traveltime approximations

ORT model - Schoenberg & Helbig (1997)

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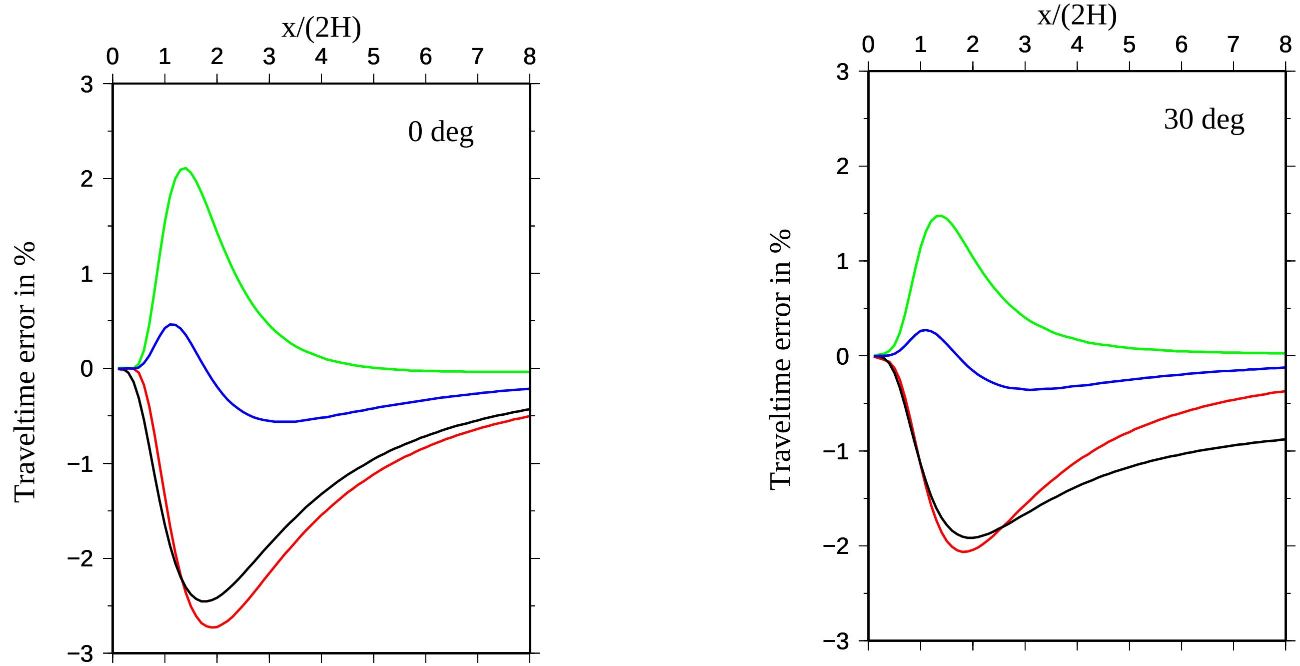

1, # 2, # 3, Tsvankin & Grechka (2011)

Traveltime approximations

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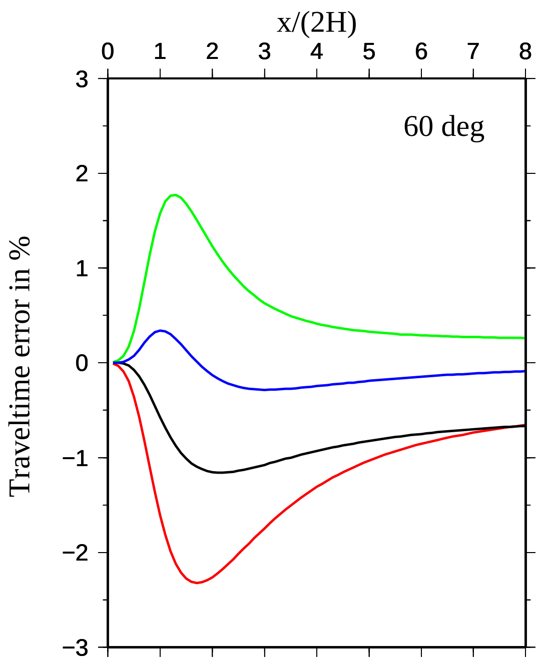
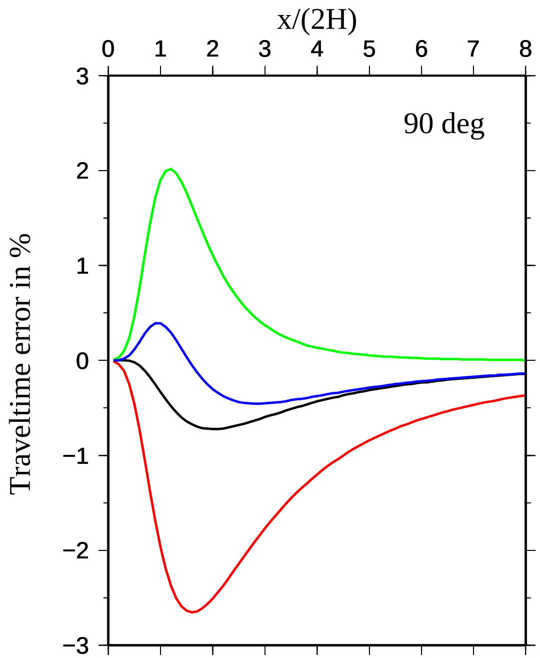




1, # **2**, # **3**, Tsvankin & Grechka (2011)

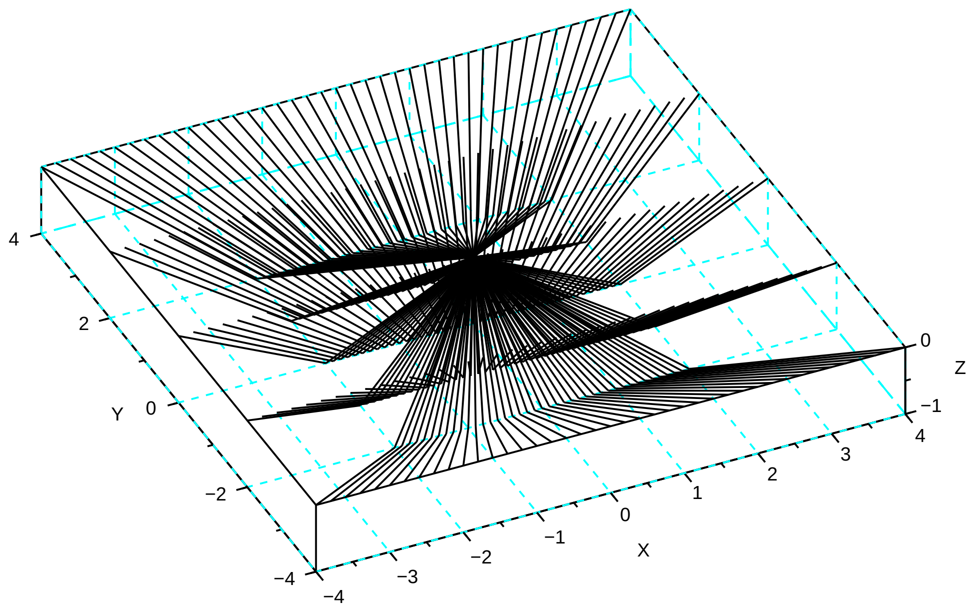
Inversion - B. Růžek

Weak ORT model - configuration

α_0=2.735 km/s, β_0=1.35 km/s, ϵ_x=0.046, ϵ_y=0.053, ϵ_z=0,

δ_x=0.008, δ_y=–0.008, δ_z=0.123, χ_z=0.003, ϵ_{16}=0.005, ϵ_{26}=–0.017

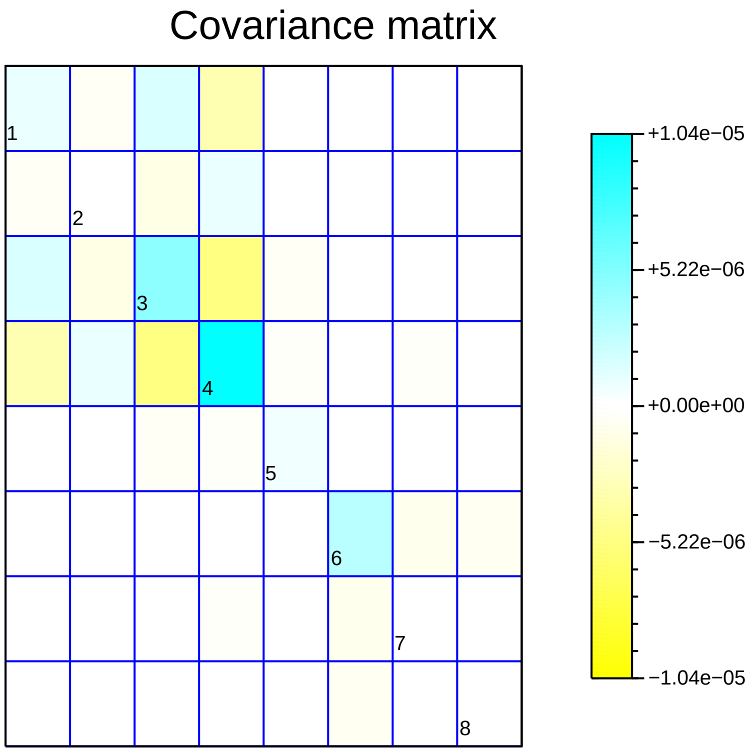
reflector at 1 km depth, coinciding with the symmetry plane

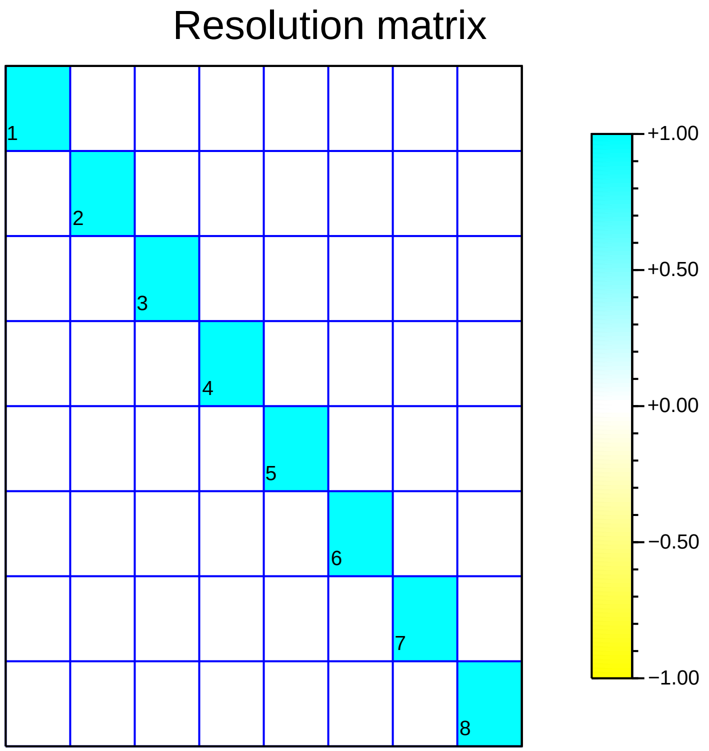


Inversion - moveout formula # 1

Weak ORT model - Gaussian noise 1 ms

1 - ϵ_x, 2 - ϵ_y, 3 - δ_x, 4 - δ_y, 5 - δ_z, 6 - χ_z, 7 - ϵ_{16}, 8 - ϵ_{26}

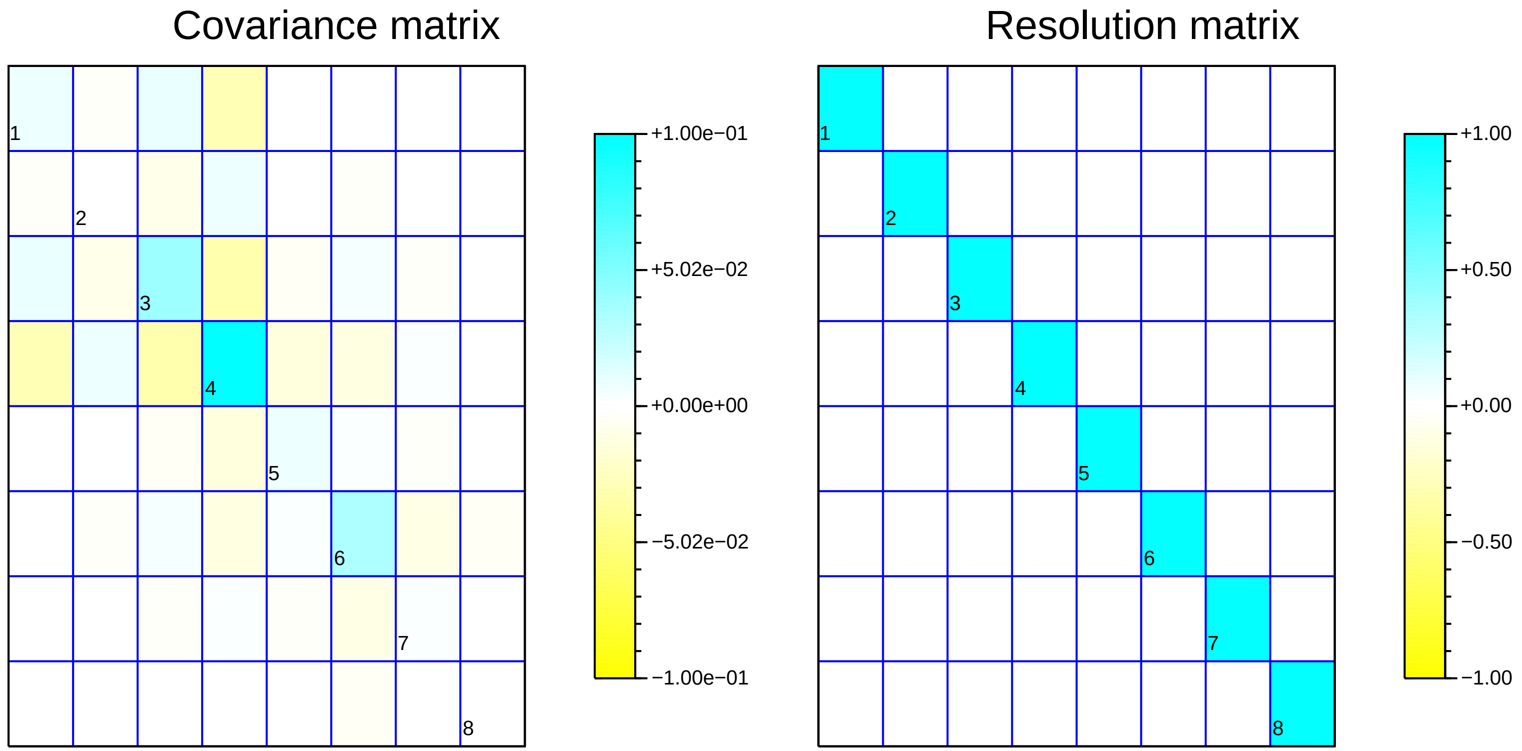




Inversion - moveout formula # 1

Weak ORT model - Gaussian noise 100 ms

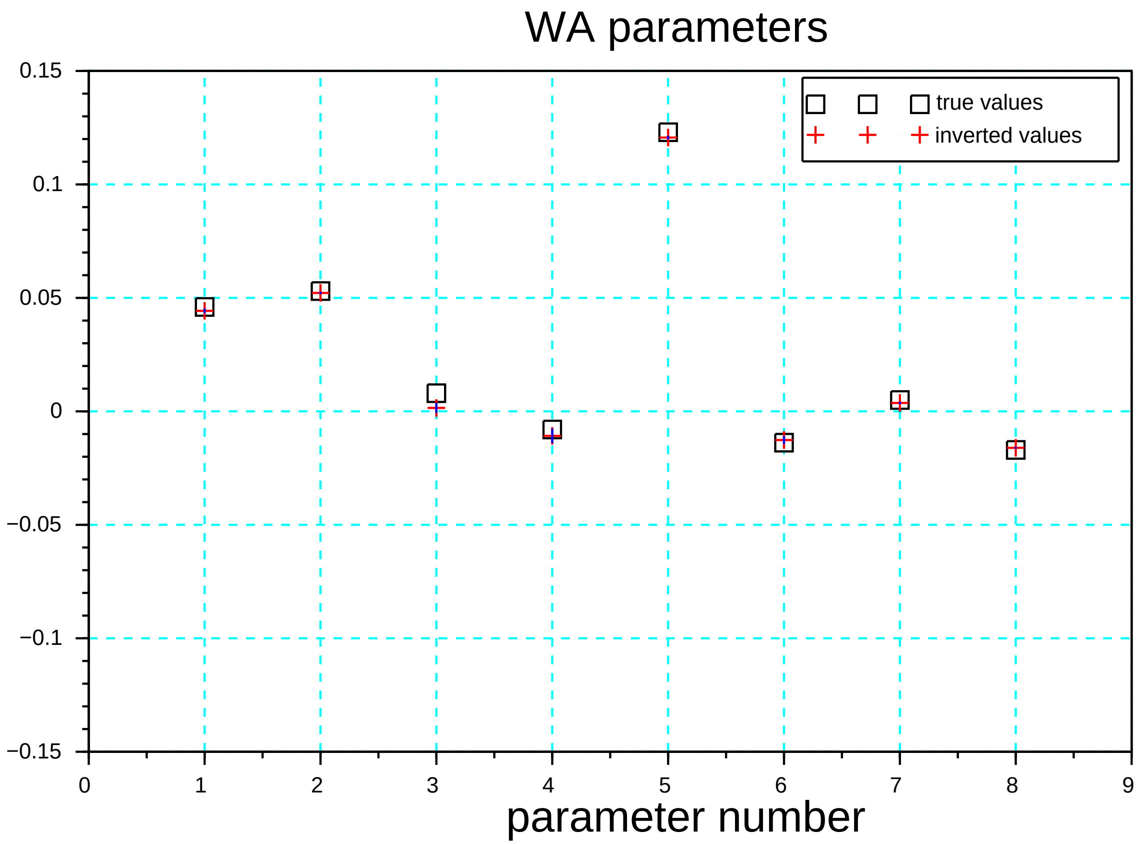
1 - ϵ_x, 2 - ϵ_y, 3 - δ_x, 4 - δ_y, 5 - δ_z, 6 - χ_z, 7 - ϵ_{16}, 8 - ϵ_{26}



Inversion - moveout formula # 1

Weak ORT model - Gaussian noise 1 ms

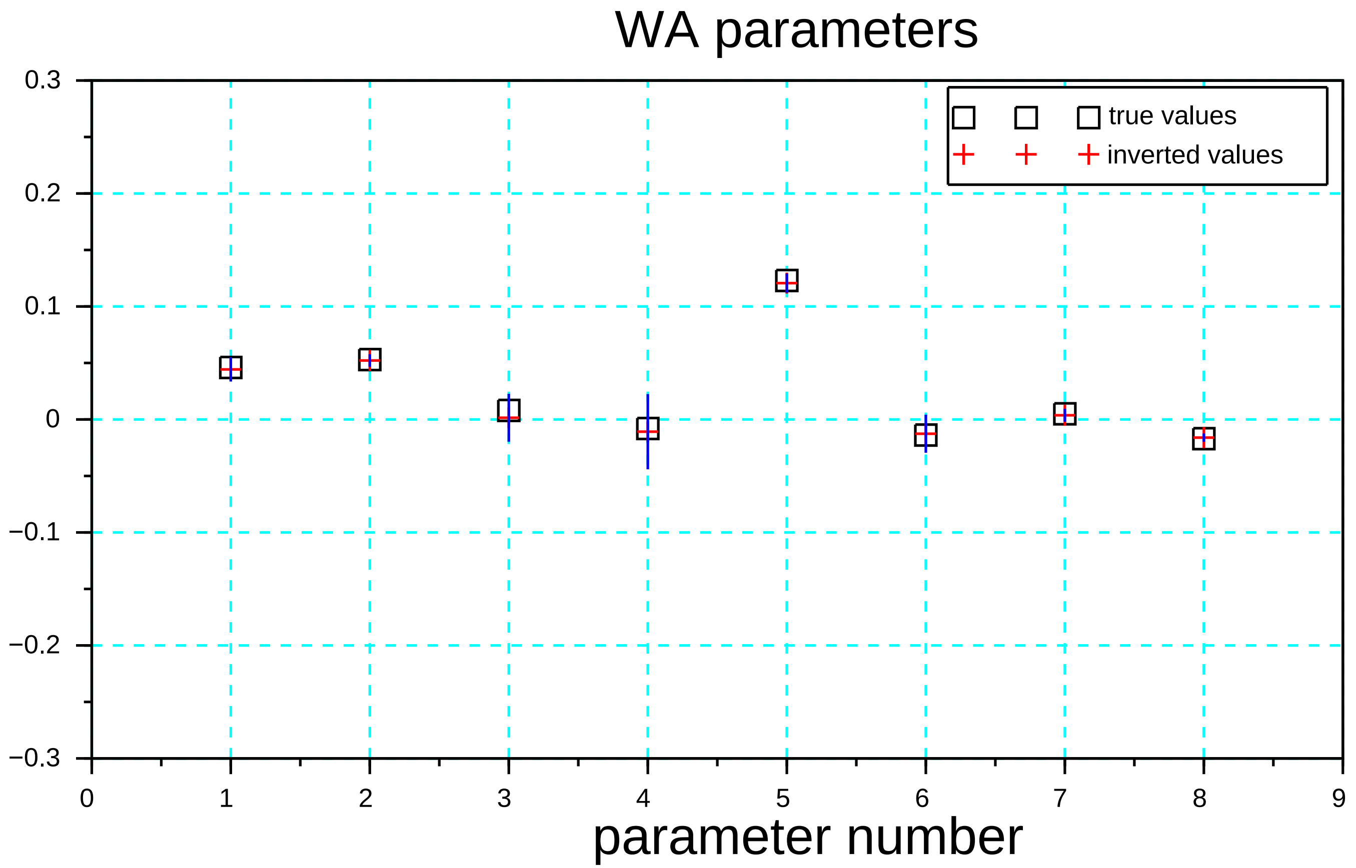
1 - ϵ_x, 2 - ϵ_y, 3 - δ_x, 4 - δ_y, 5 - δ_z, 6 - χ_z, 7 - ϵ_{16}, 8 - ϵ_{26}



Inversion - moveout formula # 1

Weak ORT model - Gaussian noise 10 ms

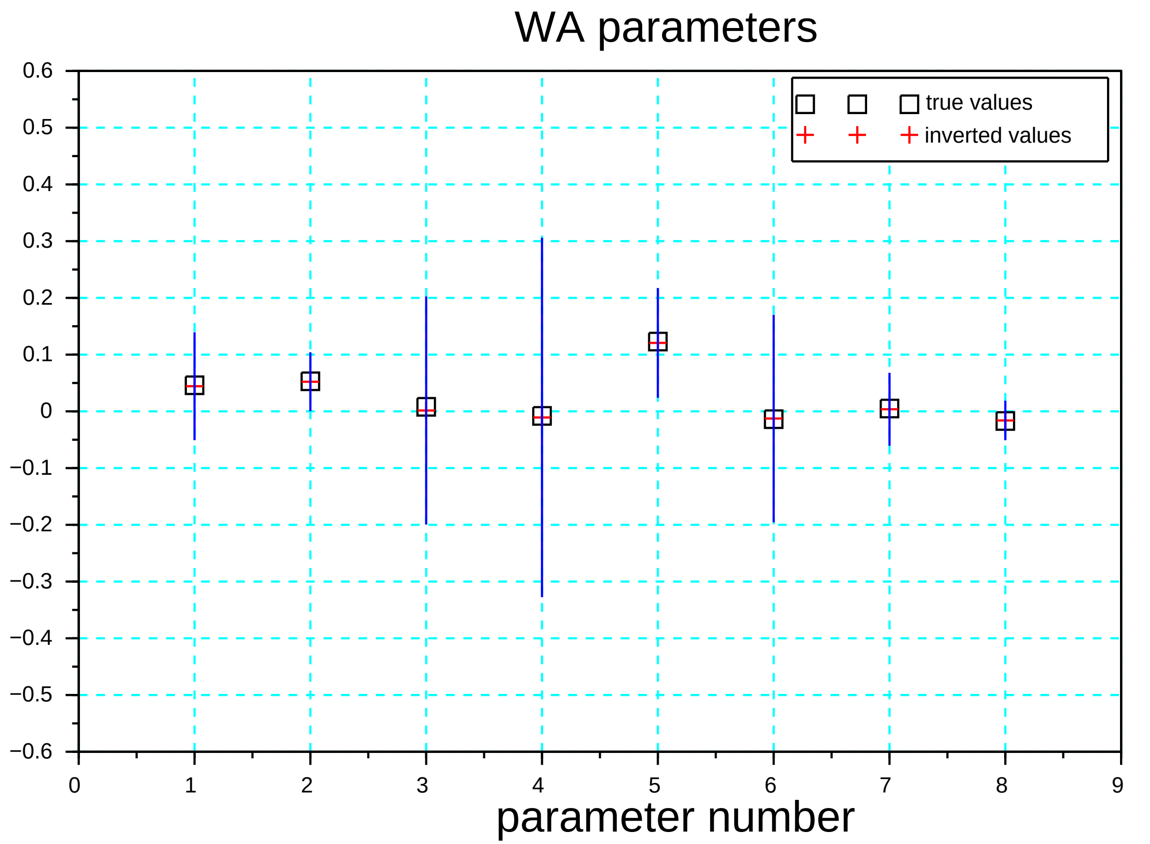
1 - ϵ_x, 2 - ϵ_y, 3 - δ_x, 4 - δ_y, 5 - δ_z, 6 - χ_z, 7 - ϵ_{16}, 8 - ϵ_{26}



Inversion - moveout formula # 1

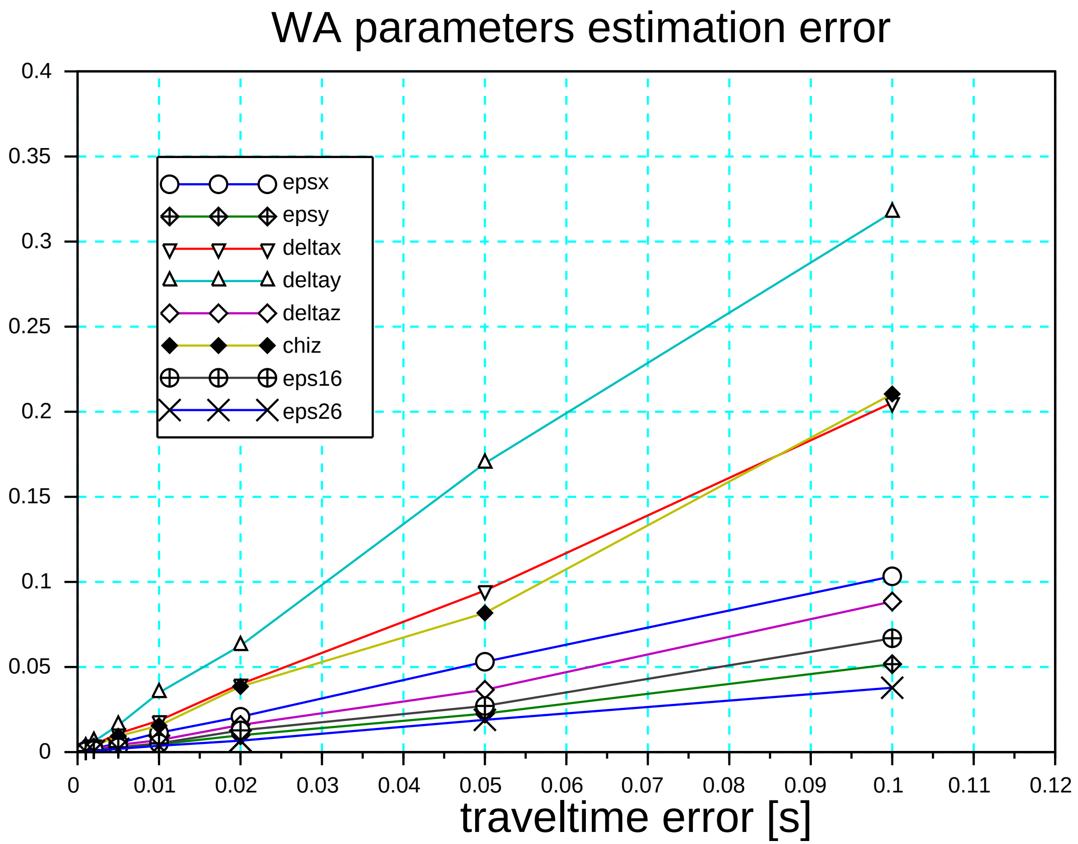
Weak ORT model - Gaussian noise 100 ms

1 - ϵ_x, 2 - ϵ_y, 3 - δ_x, 4 - δ_y, 5 - δ_z, 6 - χ_z, 7 - ϵ_{16}, 8 - ϵ_{26}



Inversion - moveout formula # 1

Weak ORT model - Gaussian noise



NMO velocity, quartic coefficient

$$T^2(x^2, \varphi) = T_0^2 + A_2(\varphi)x^2 + A_4(\varphi)x^4 + ...$$

NMO velocity

$$A_2(\varphi) = v_{NMO}^{-2}(\varphi)$$

Global coordinates - ORT, case #1

$$v_{NMO}^{-2}(\varphi) = W_{11}\cos^2\varphi + 2W_{12}\cos\varphi\sin\varphi + W_{22}\sin^2\varphi$$

$$W_{11} = \alpha_0^{-2}(1 - 2\delta_y) \;\;, \quad W_{12} = 0 \;\;, \quad W_{22} = \alpha_0^{-2}(1 - 2\delta_x)$$

φ - angle of the source-receiver profile with x_1-axis

NMO velocity, quartic coefficient

$$T^2(x^2, \varphi) = T_0^2 + A_2(\varphi)x^2 + A_4(\varphi)x^4 + ...$$

Quartic coefficient

Global coordinates - ORT, case #1

$$A_4(\varphi) = A_4^{(1)} \sin^2 \varphi - A_4^{(x)} \sin^2 \varphi \cos^2 \varphi + A_4^{(2)} \cos^2 \varphi$$

$$A_4^{(1)} = -2(\epsilon_y - \delta_x - 2\delta_x^2)/\alpha_0^4 T_0^2 \, , \quad A_4^{(2)} = -2(\epsilon_x - \delta_y - 2\delta_y^2)/\alpha_0^4 T_0^2$$

$$A_4^{(x)} = -2[\epsilon_x + \epsilon_y - \delta_z - 2(\delta_x - \delta_y)^2]/\alpha_0^4 T_0^2$$

Conclusions

- based on WA approximation

- relatively simple formulae; no square roots

- no non-physical assumptions (no acoustic approximation)

- applicable to VTI, HTI, orthorhombic or even monoclinic media
  for the reflector $\|$ with a plane of symmetry

- reduced number of parameters: WA (incl. $\epsilon_z = 0$)/$C_{\alpha\beta}$:
  monoclinic - 9/12, orthorhombic - 6/9, TI - 3/5

- simple transformation from one coordinate system to another:
  could be simply used, e.g., in AVAZ

- physical insight

Conclusions

- inaccuracies for large deviations of $\mathbf{n}$ and $\mathbf{N}$,
  i.e., for large deviations of c and v

- vertical axis $\equiv$ longitudinal axis $\rightarrow$ $c = v$
  $\Rightarrow$ large accuracy for small offsets

- accuracy for large offsets within the order of used formulae
  for VTI, for example, $c = v$ along horizontal
  $\Rightarrow$ high accuracy for large offsets (no need for adjustment)

- high accuracy of 2nd-order formulae: $< 1\%$ for $\sim$ 25-33% anisotropy

- byproduct: simple expressions for ray velocities, NMO velocity and A_4

- formula # 1 sufficient for the inversion purposes

Possible extensions

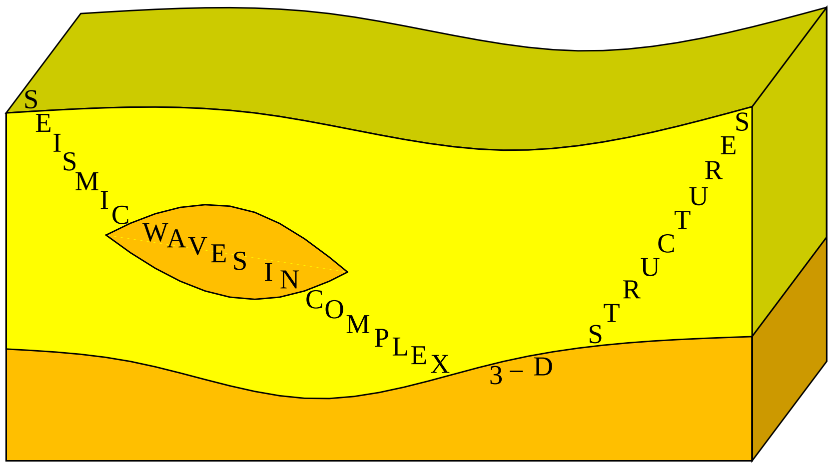
- dip-constrained media with an inclined reflector

  parallel to a symmetry plane

- unconverted coupled S wave along a common ray

- layered media

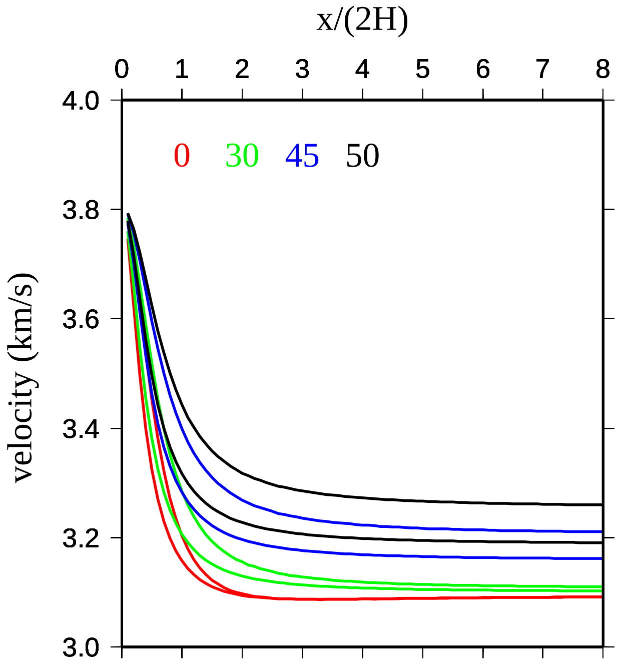
Acknowledgement

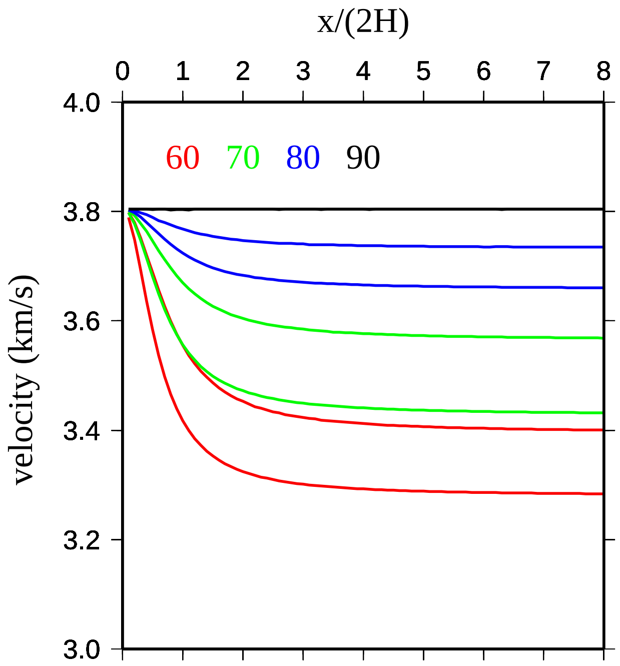


Tests of the formulae

HTI model, anisotropy $\sim$ 26%

α_0=3.805 km/s, β_0=1.510 km/s, ϵ_x=–0.169, δ_y=–0.373





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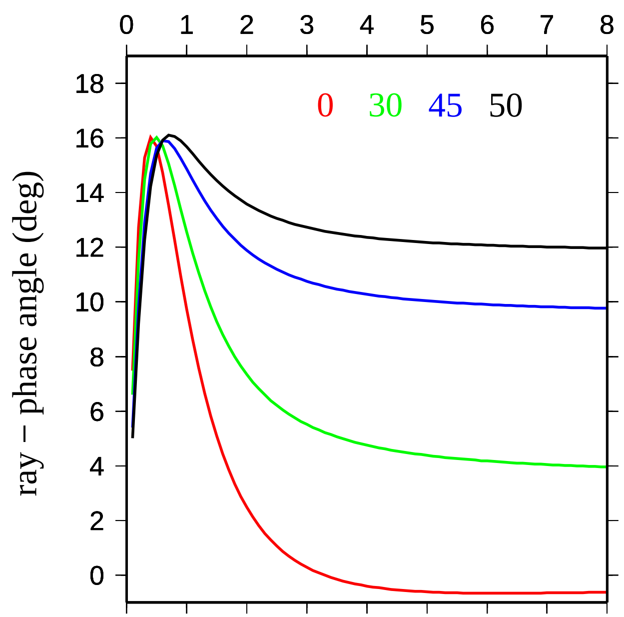

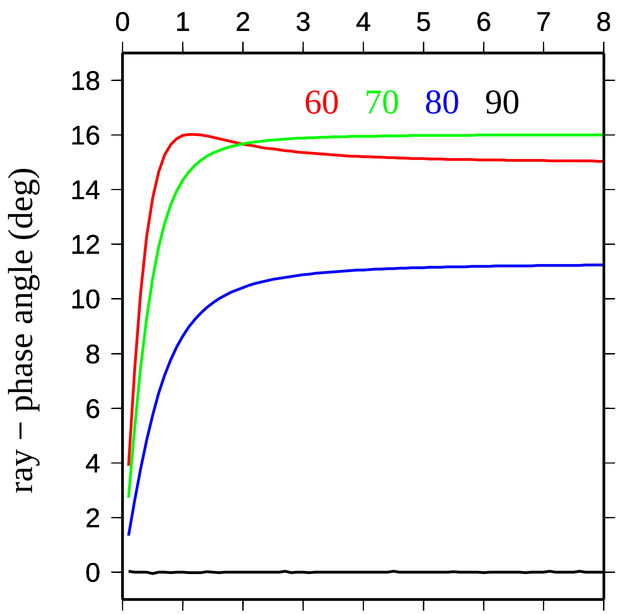

Tests of the formulae

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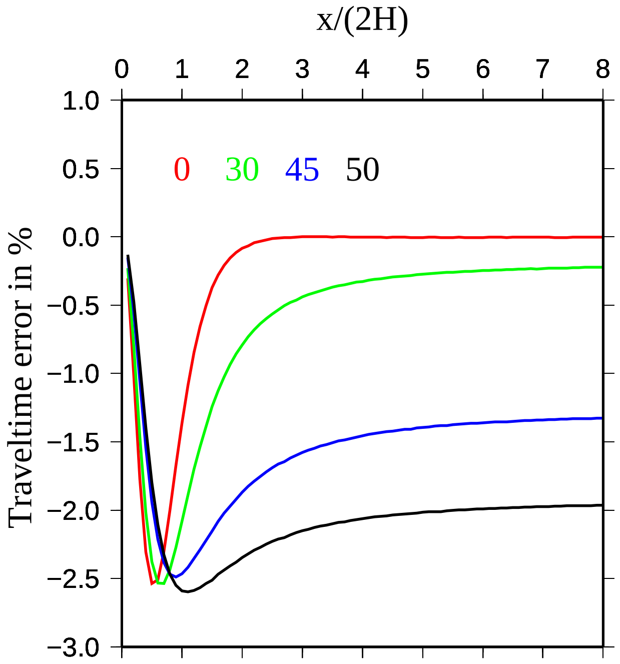

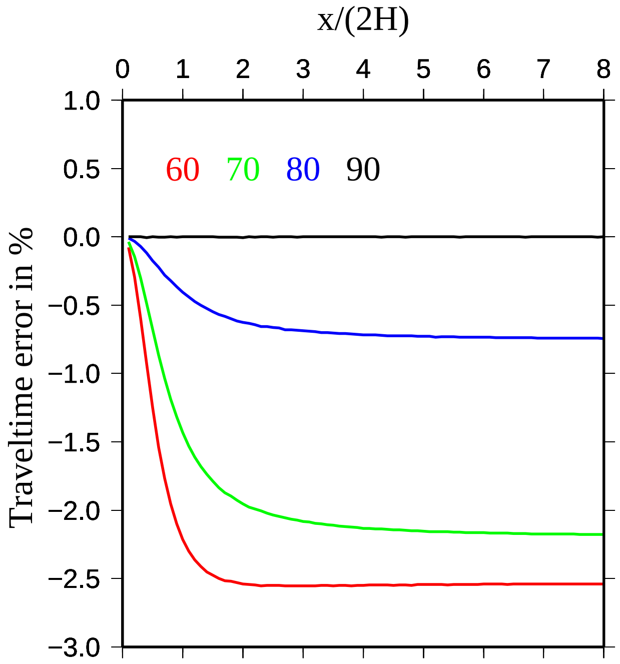

Tests of the formulae

HTI model, anisotropy $\sim$ 26%, formula # 2

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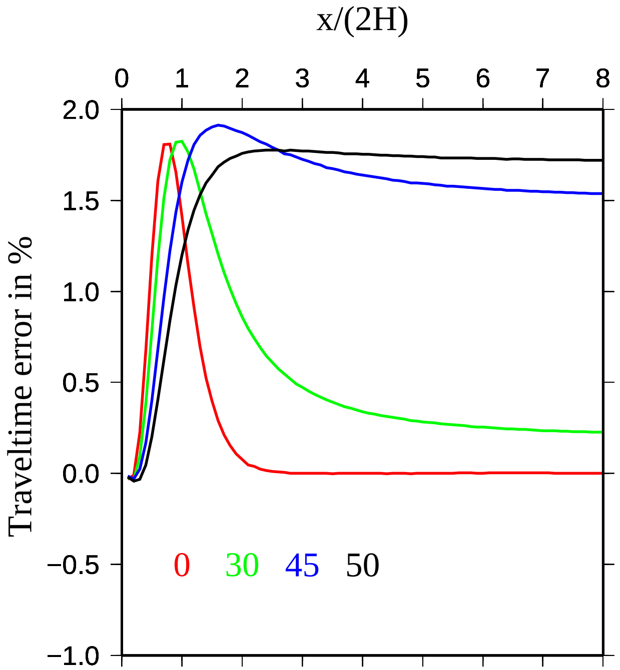

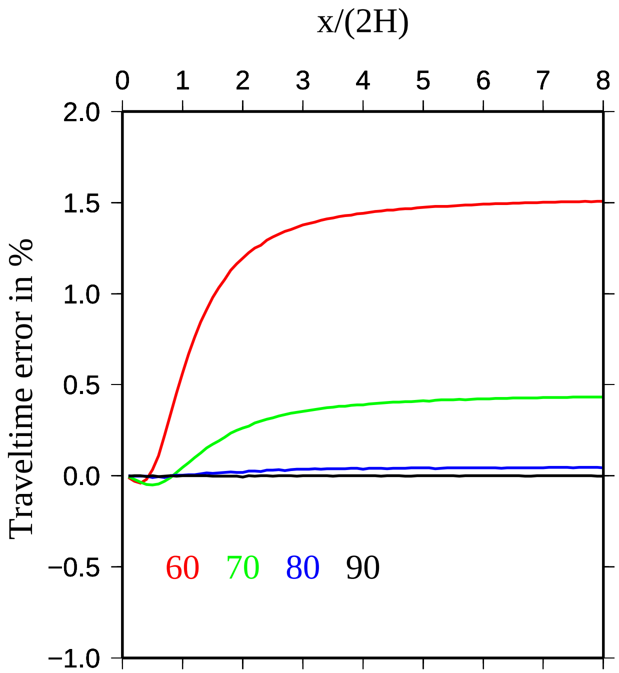

Tests of the formulae

HTI model, anisotropy $\sim$ 26%, formula # 3

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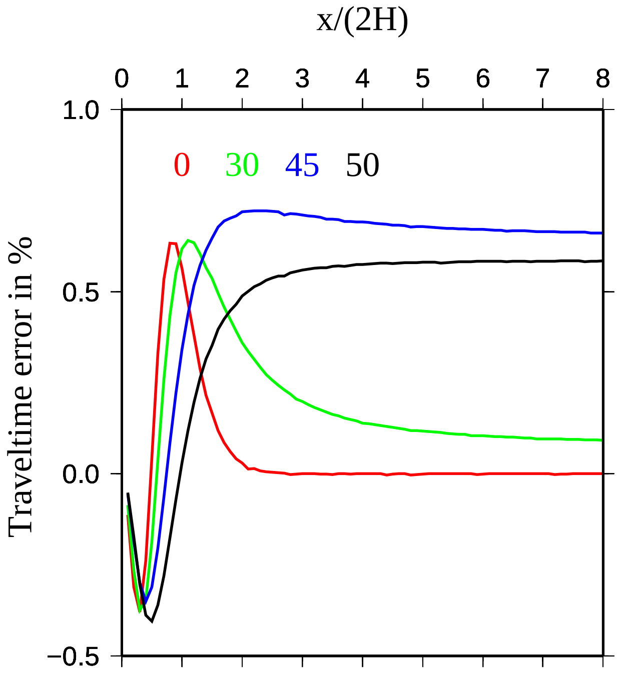
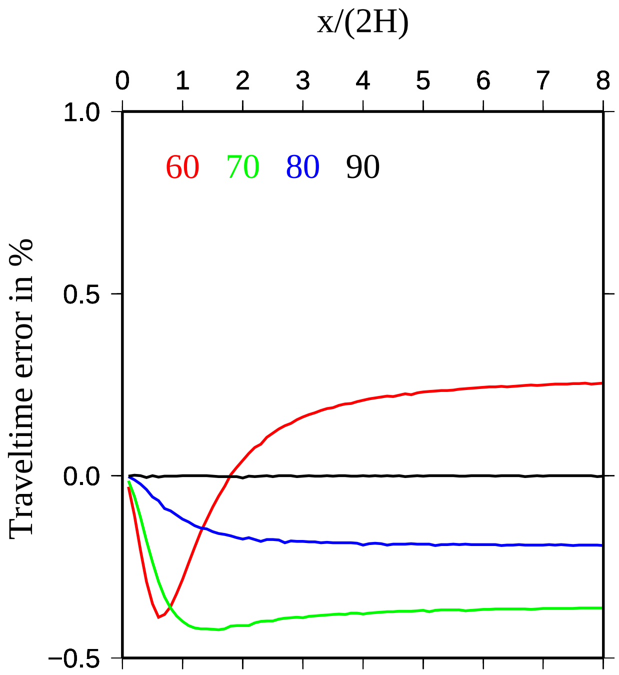
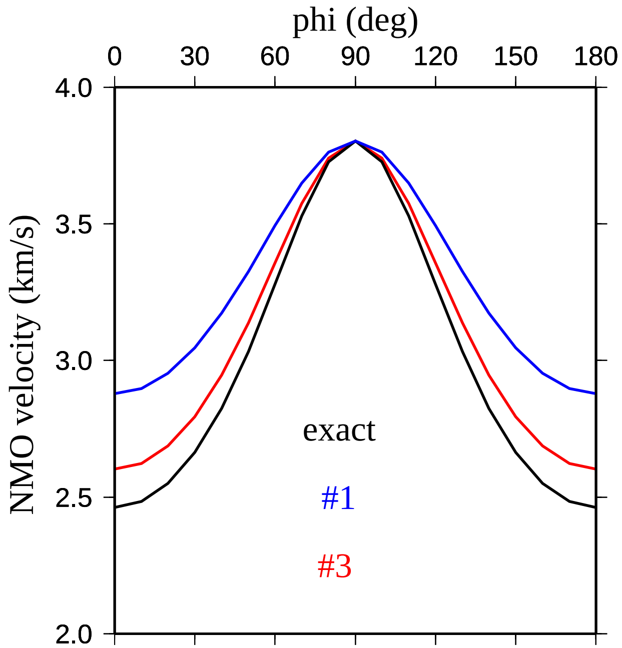




Tests of the formulae

HTI model, anisotropy $\sim$ 26%

α_0=3.805 km/s, β_0=1.510 km/s, ϵ_x=–0.169, δ_y=–0.373

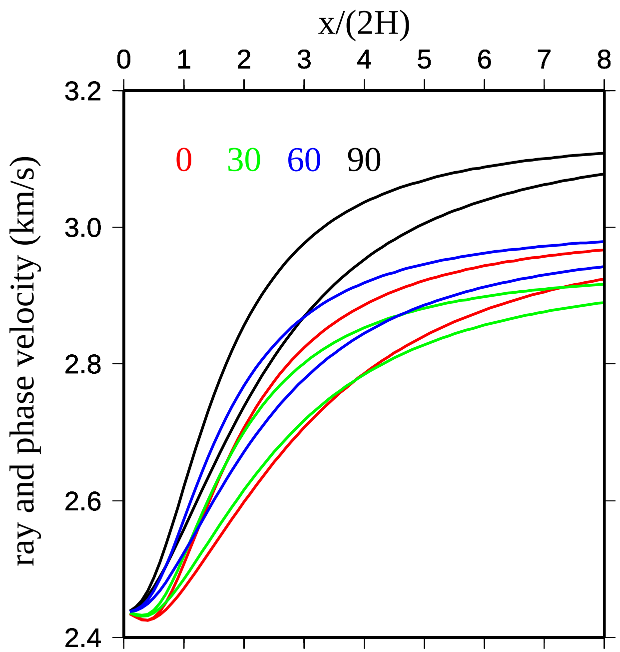


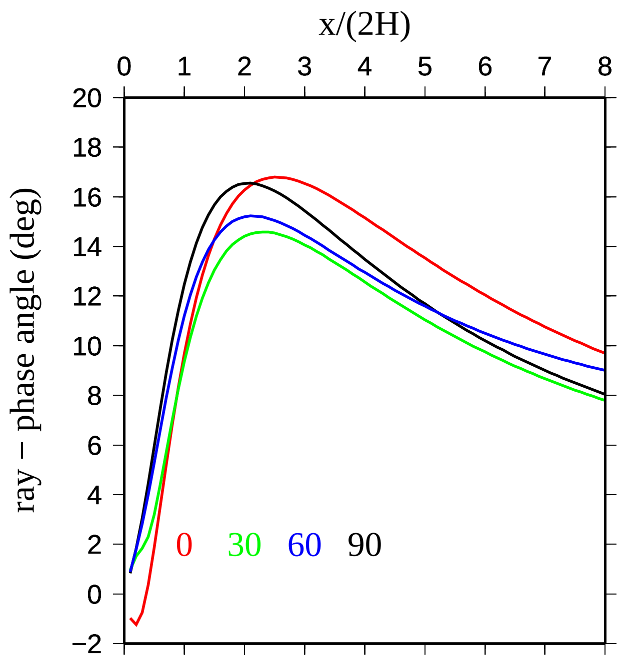
Tests of the formulae

ORT model, anisotropy ~ 25%

α_0=2.437 km/s, β_0=1.414 km/s, ϵ_x=0.258, ϵ_y=0.328

δ_x=0.077, δ_y=–0.083, δ_z=0.340



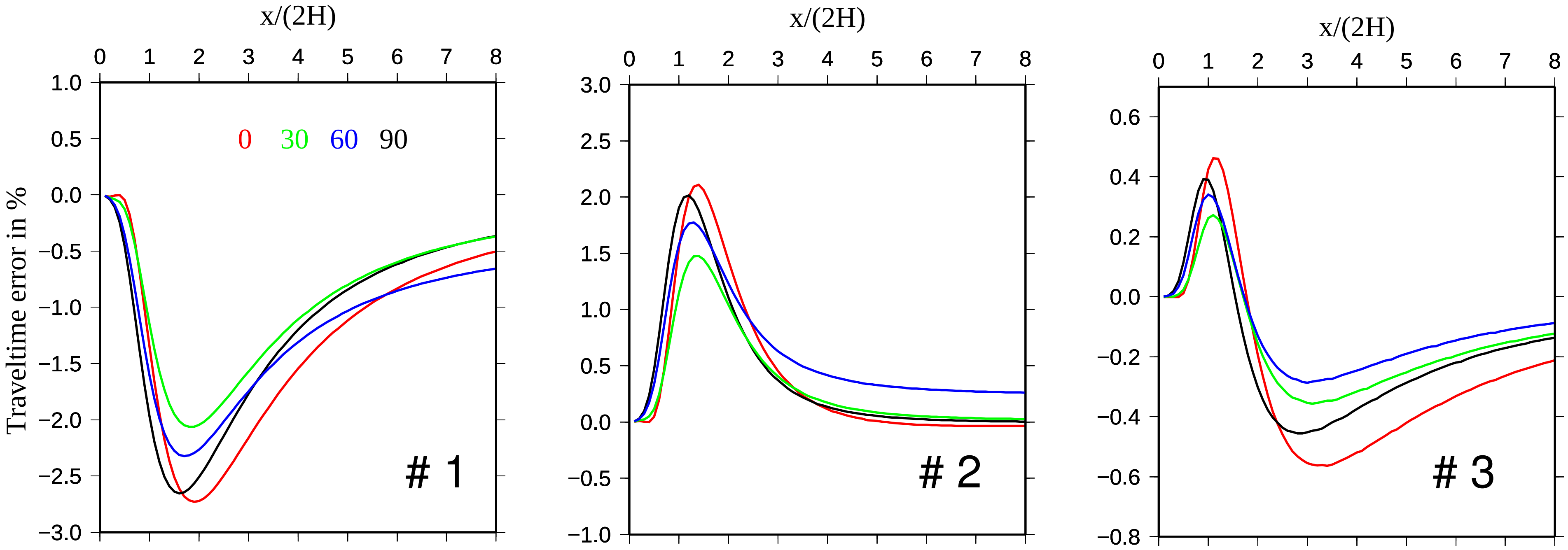


Tests of the formulae

ORT model, anisotropy ~ 25%

α_0=2.437 km/s, β_0=1.414 km/s, ϵ_x=0.258, ϵ_y=0.328

δ_x=0.077, δ_y=–0.083, δ_z=0.340

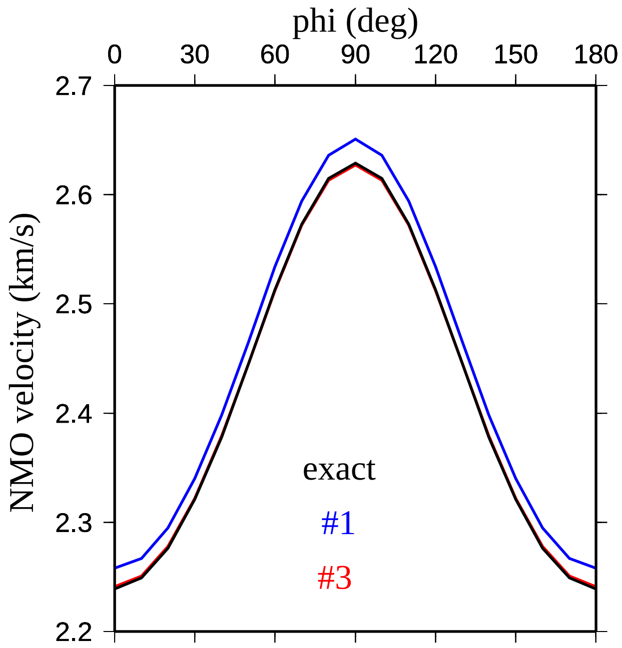


Tests of the formulae

ORT model, anisotropy $\sim$ 25%

α_0=2.437 km/s, β_0=1.414 km/s, ϵ_x=0.258, ϵ_y=0.328

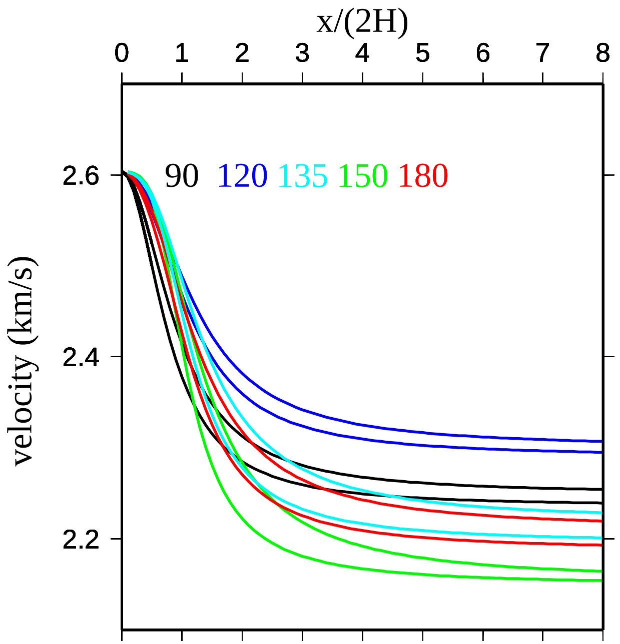
δ_x=0.077, δ_y=–0.083, δ_z=0.340

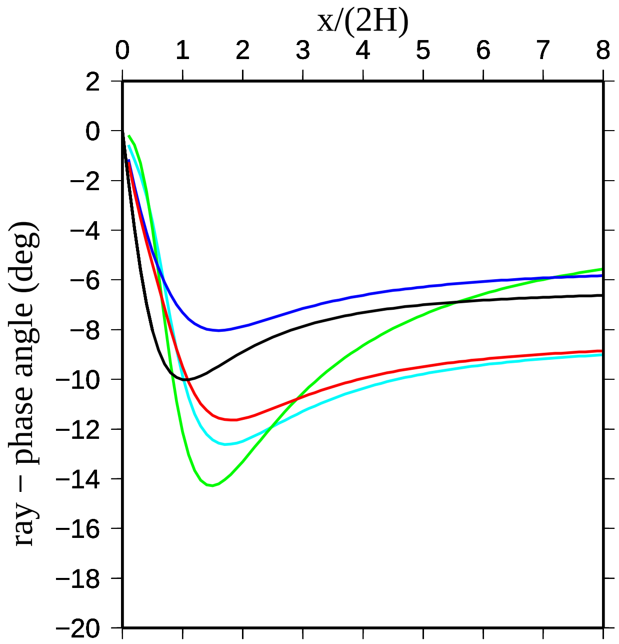


Tests of the formulae

MONO model, anisotropy ∼ 15%

α_0=2.604 km/s, β_0=1.566 km/s, ϵ_x=–0.135, ϵ_y=–0.124, δ_x=–0.128, δ_y=–0.057, δ_z=–0.241, ϵ_{16}=0.057, ϵ_{26}=–0.043, χ_z=–0.071

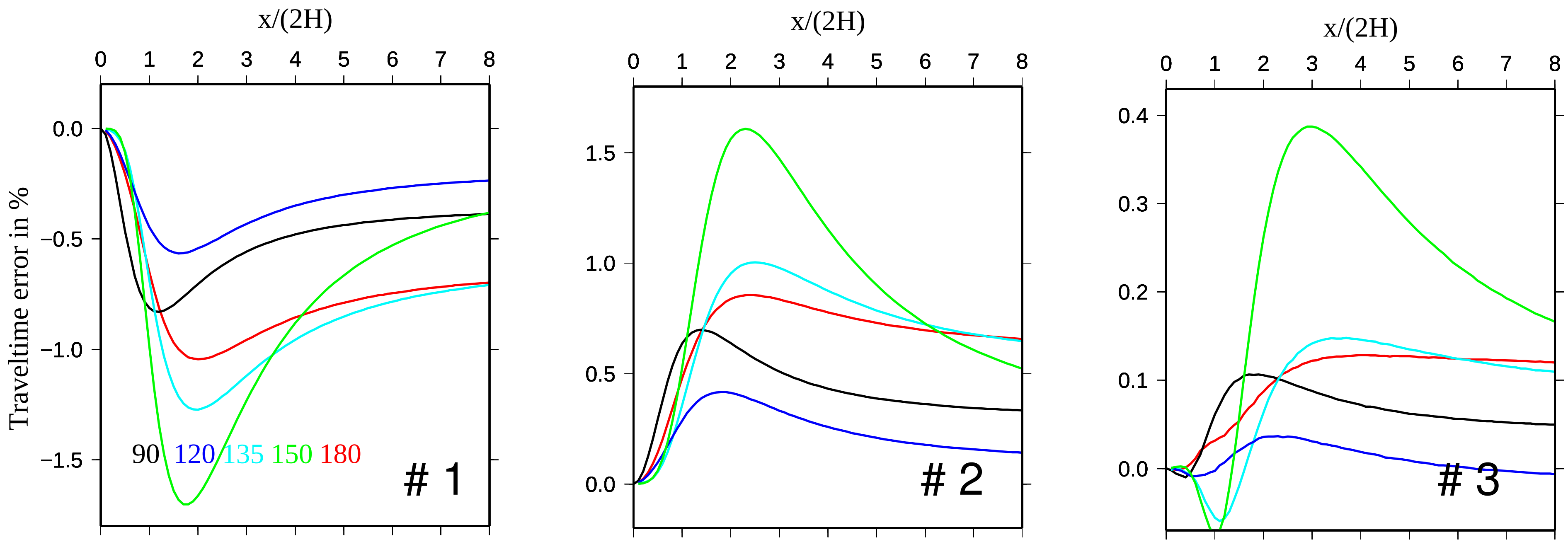




Tests of the formulae

MONO model, anisotropy ~ 15%

α_0=2.604 km/s, β_0=1.566 km/s, ϵ_x=–0.135, ϵ_y=–0.124, δ_x=–0.128, δ_y=–0.057, δ_z=–0.241, ϵ_{16}=0.057, ϵ_{26}=–0.043, χ_z=–0.071



Tests of the formulae

MONO model, anisotropy $\sim$ 15%

α_0=2.604 km/s, β_0=1.566 km/s, ϵ_x=–0.135, ϵ_y=–0.124, δ_x=–0.128, δ_y=–0.057, δ_z=–0.241, ϵ_{16}=0.057, ϵ_{26}=–0.043, χ_z=–0.071