Calculation of anisotropic-ray-theory S-wave rays in velocity model SC1_II with a split intersection singularity

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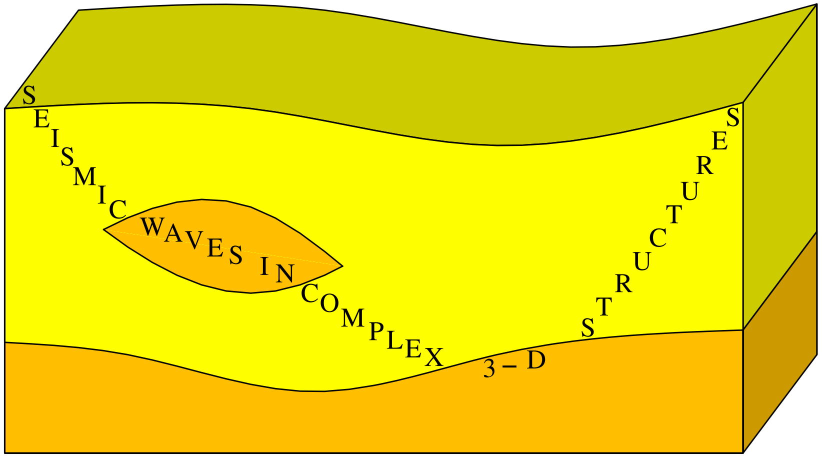

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SW3D Consortium meeting, Prague, 23-24.6.2014

Anisotropic-ray-theory S-wave rays in package CRT

Previous versions of the SW3D software package CRT were designed to trace the anisotropic–ray–theory P–wave rays and the anisotropic common S–wave rays using the average S–wave Hamiltonian function. We did not consider anisotropic–ray–theory S–wave rays for obvious problems with S–wave slowness surface singularities. Now we have added an optional possibility to calculate anisotropic–ray–theory S–wave rays to the package CRT version 7.10 for testing purposes.

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In a generally anisotropic medium, the S–wave slowness sheets of the slowness surface are usually mostly separated and intersect in at up to 32 point S–wave singularities. In this case, outside the point singularities, the anisotropic–ray–theory rays stay at the faster or slower S–wave slowness sheet, respectively. When approaching the point singularities, the limiting case again corresponds to staying at the faster or slower S–wave slowness sheet, respectively.

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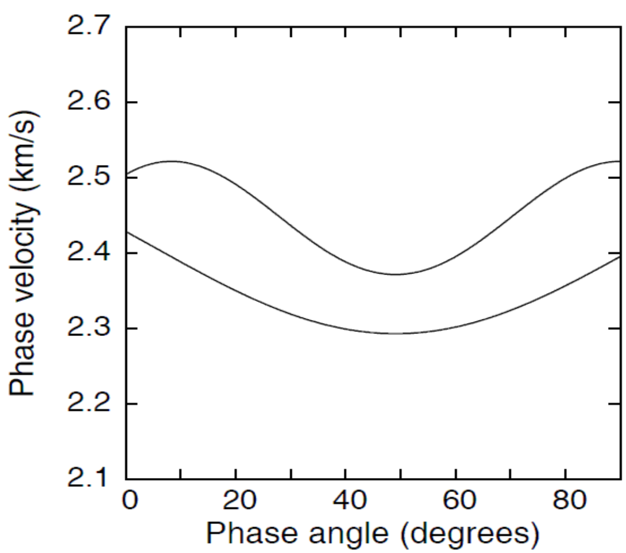
Optional tracing of anisotropic–ray–theory S–wave rays in package CRT is designed for general anisotropy. We thus a priori choose the faster S wave or the slower S wave. In each step of anisotropic–ray–theory S–wave ray tracing, the Christofel matrix is calculated together with its eigenvalues and corresponding eigenvectors. We then select the a priori given anisotropic–ray–theory S wave (the slower one or the faster one) for the calculation of the ray. If the S-wave eigenvalues are equal*, tracing of the ray is terminated.

(* first version of the code)

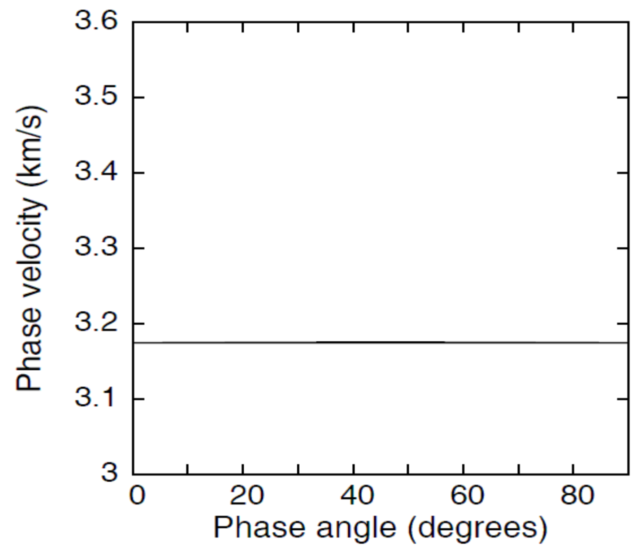
Model SC1_II

At the depth of 0 km, velocity model SC1 II is transversely isotropic with the tilted axis of symmetry. At this depth, the slowness surface contains an intersection singularity. At the depth of 1.5 km, velocity model SC1 II is very close to isotropic, but is slightly cubic and its symmetry axes coincide with the coordinate axes. This means that, at all depths except 0 km, velocity model SC1 II is very close to transversely isotropic, but is slightly tetragonal.

model SC2 (TI): Shearer & Chapman (1989)

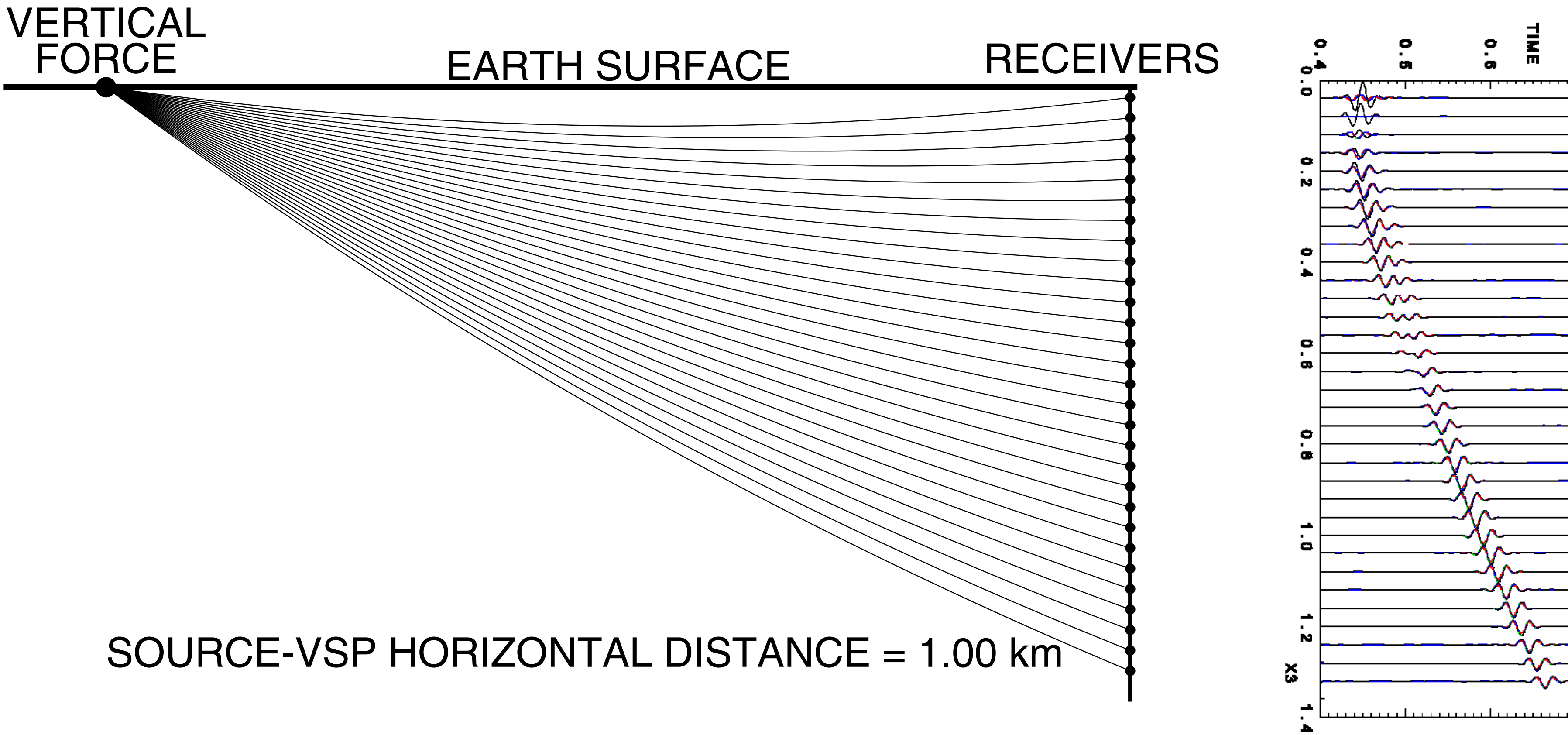


z=0 km

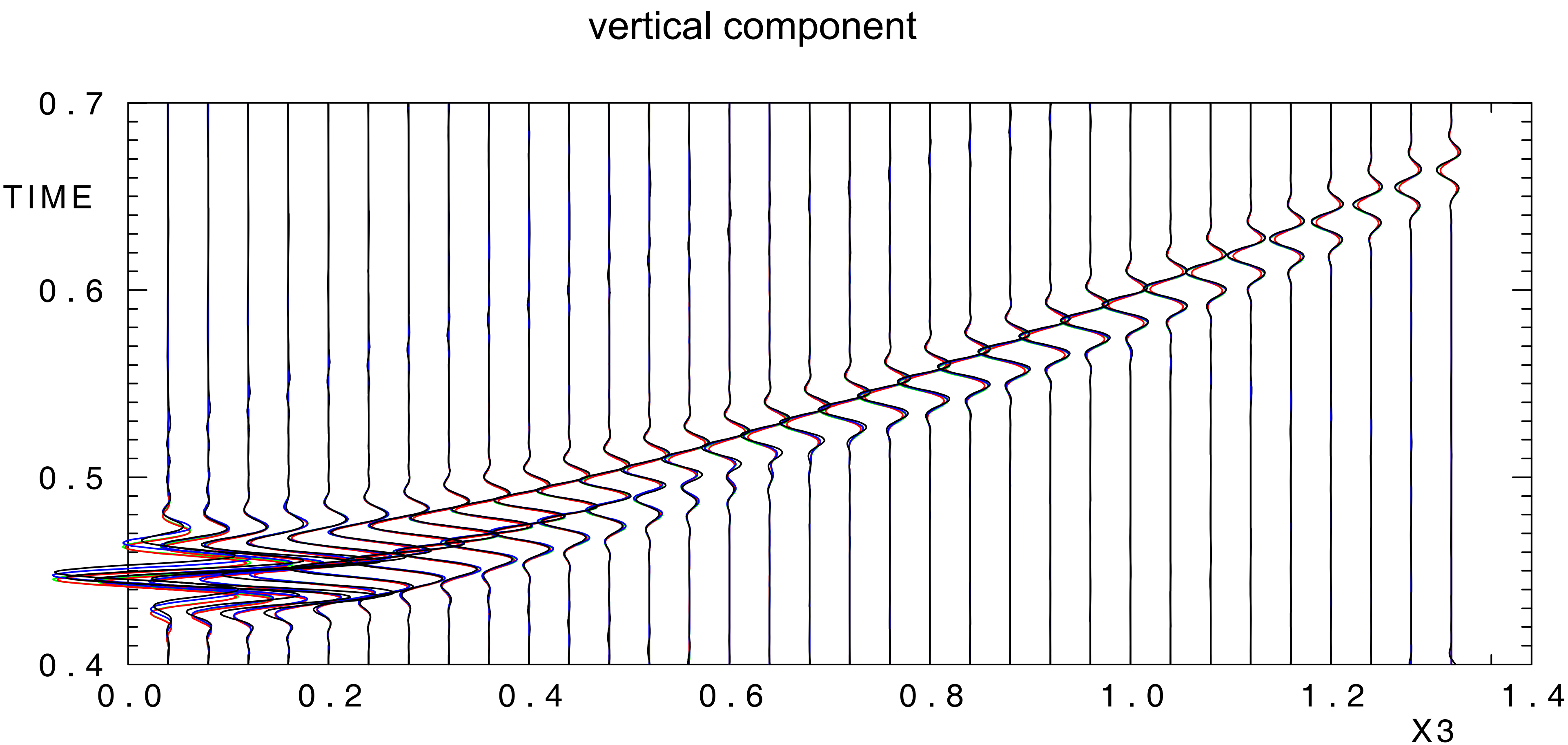


z=1.5 km

Synthetic seismograms in model SC1_II



Synthetic seismograms – various coupling ray theories



Bulant, Pšenčík, Farra & Tessmer (2011)

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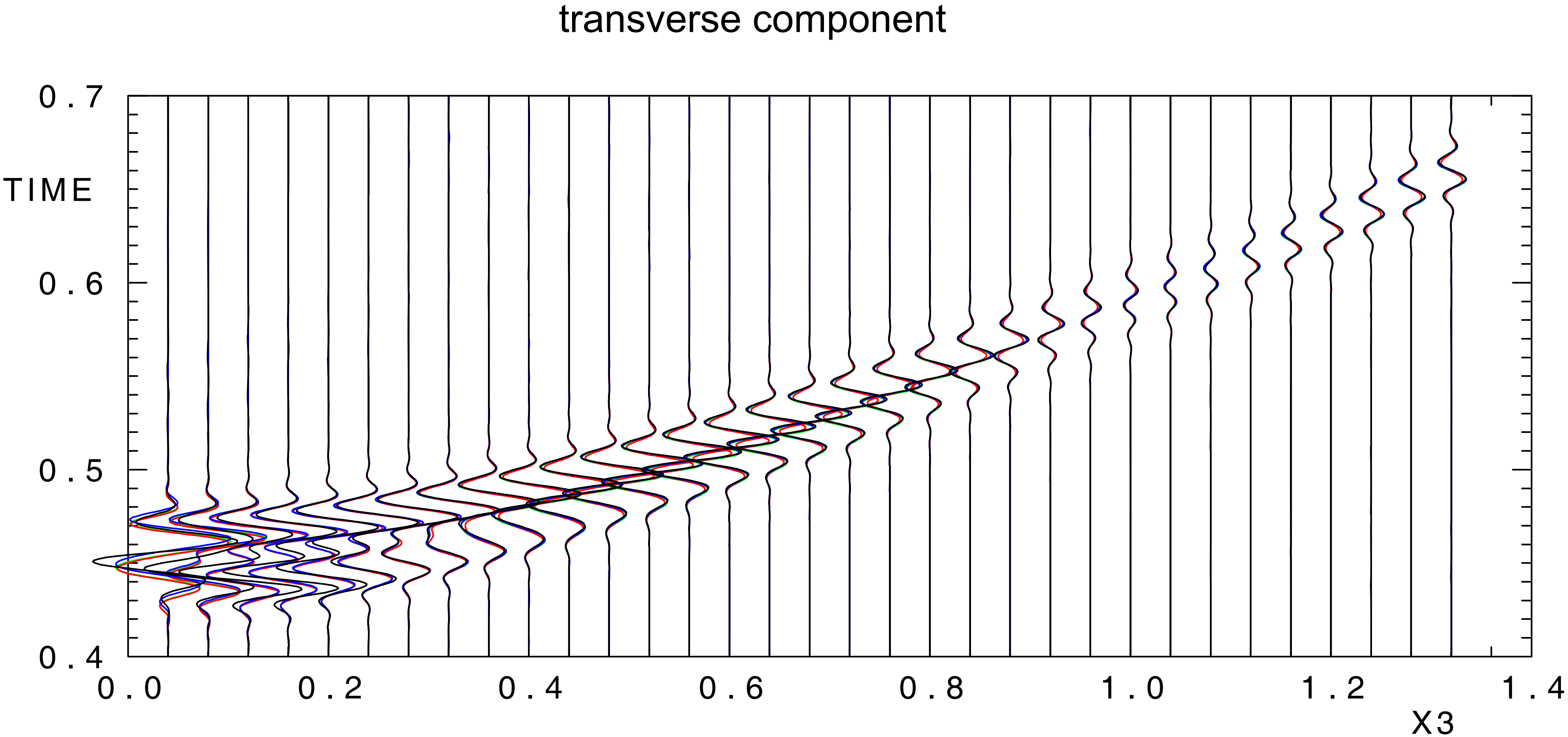

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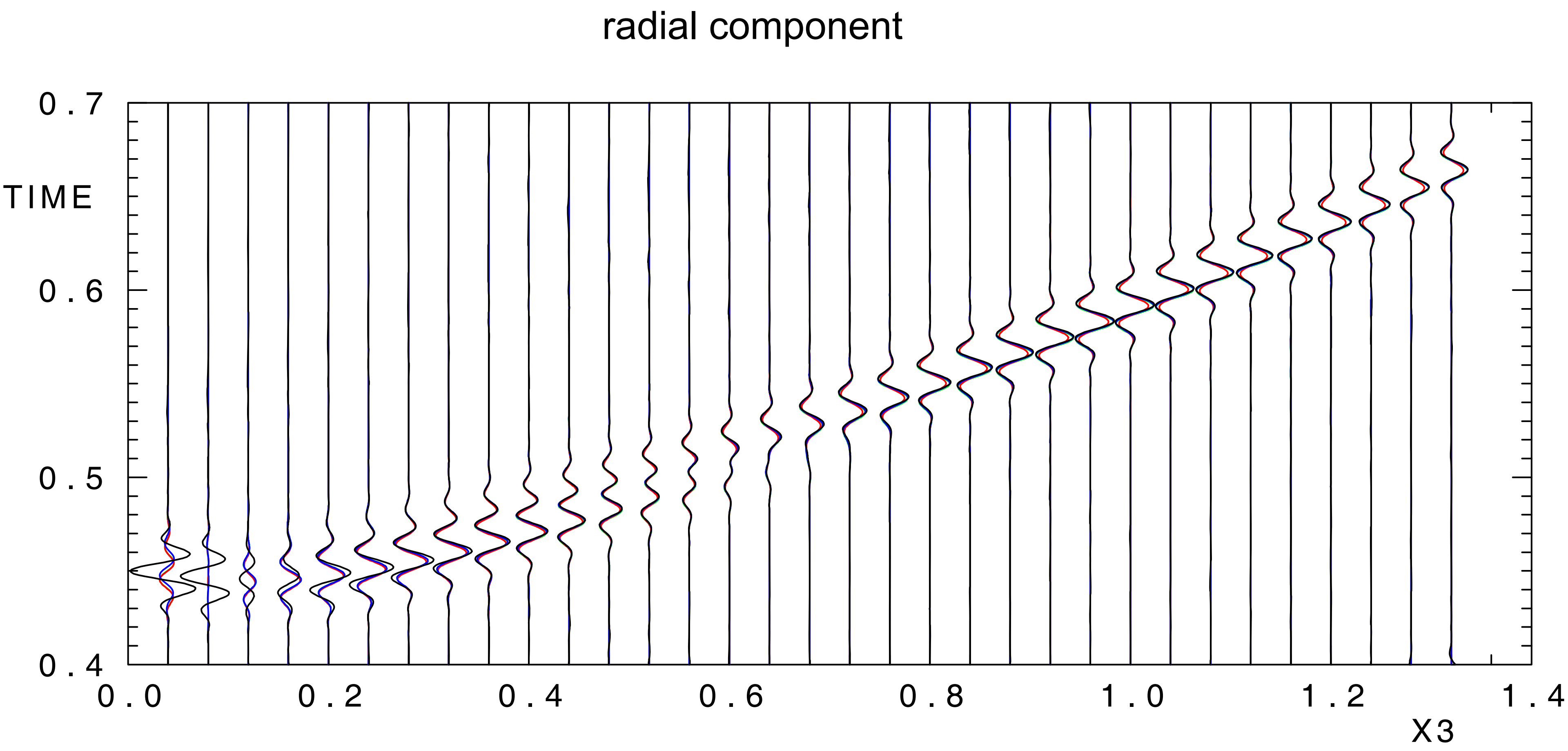

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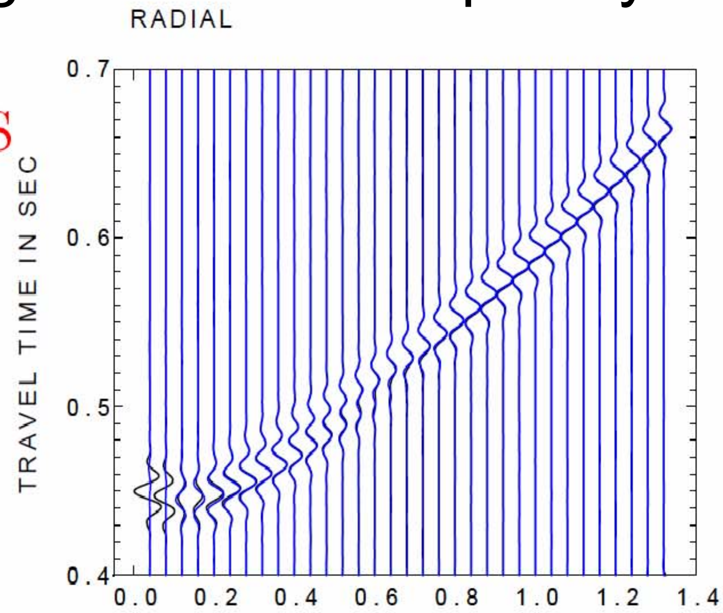
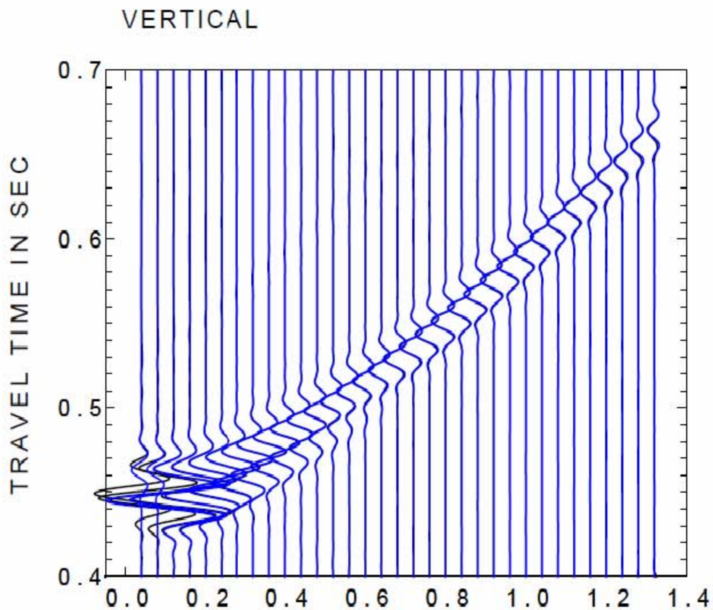
Synthetic seismograms – anisotropic ray theory by ANRAY

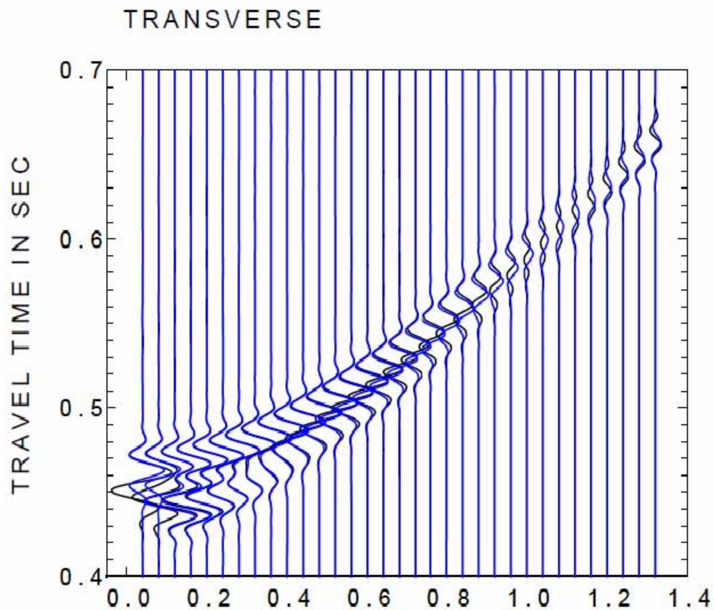
Comparisons

SC2: FM RT



Pšenčík, Farra & Tessmer (2011)

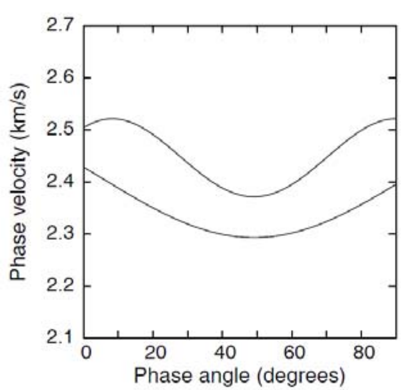




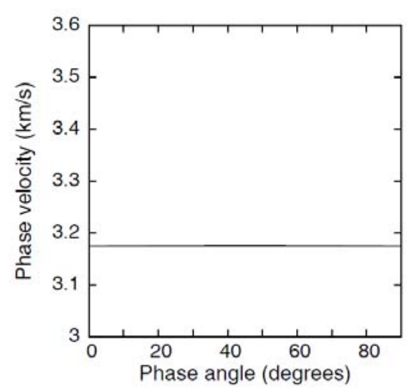
Synthetic seismograms in model SC1_II

Pšenčík, Farra & Tessmer (2011) used velocity model SC1_II to compare synthetic seismograms calculated by different ray theories with the seismograms calculated by Fourier pseudospectral method. They obtained considerable differences between the seismograms for shallow receivers and concluded that the coupling ray theory along common anisotropic S–wave rays failed because the slowness vectors corresponding to the two anisotropic–ray–theory rays have too different orientation and it is impossible to approximate them by the slowness vector of the common anisotropic S–wave ray.

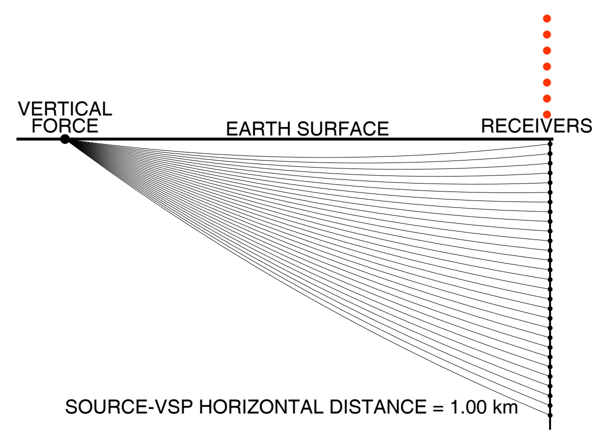
In order to check the above mentioned hypothesis, we tried to trace the anisotropic-ray-theory rays in the model SC1_II. Because the phase-velocity section shown by Pšenčík, Farra & Tessmer (2011) indicated possible existence of the intersection singularity above the original surface of the model, we decided to extend the model in the vertical direction, and we added 13 new receivers above the original vertical receiver profile.



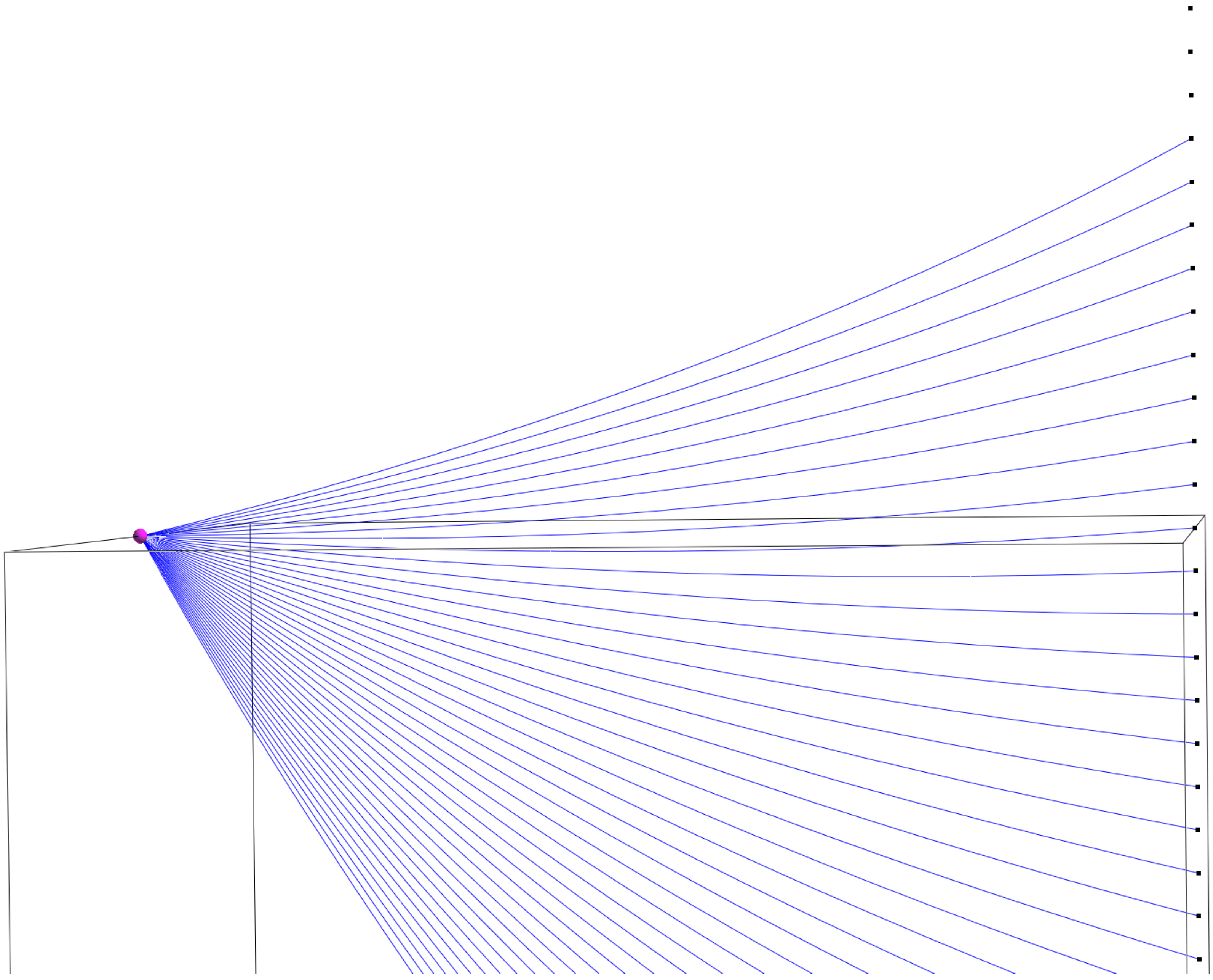
z=0 km



z=1.5 km



Common anisotropic S-wave rays traced by package CRT



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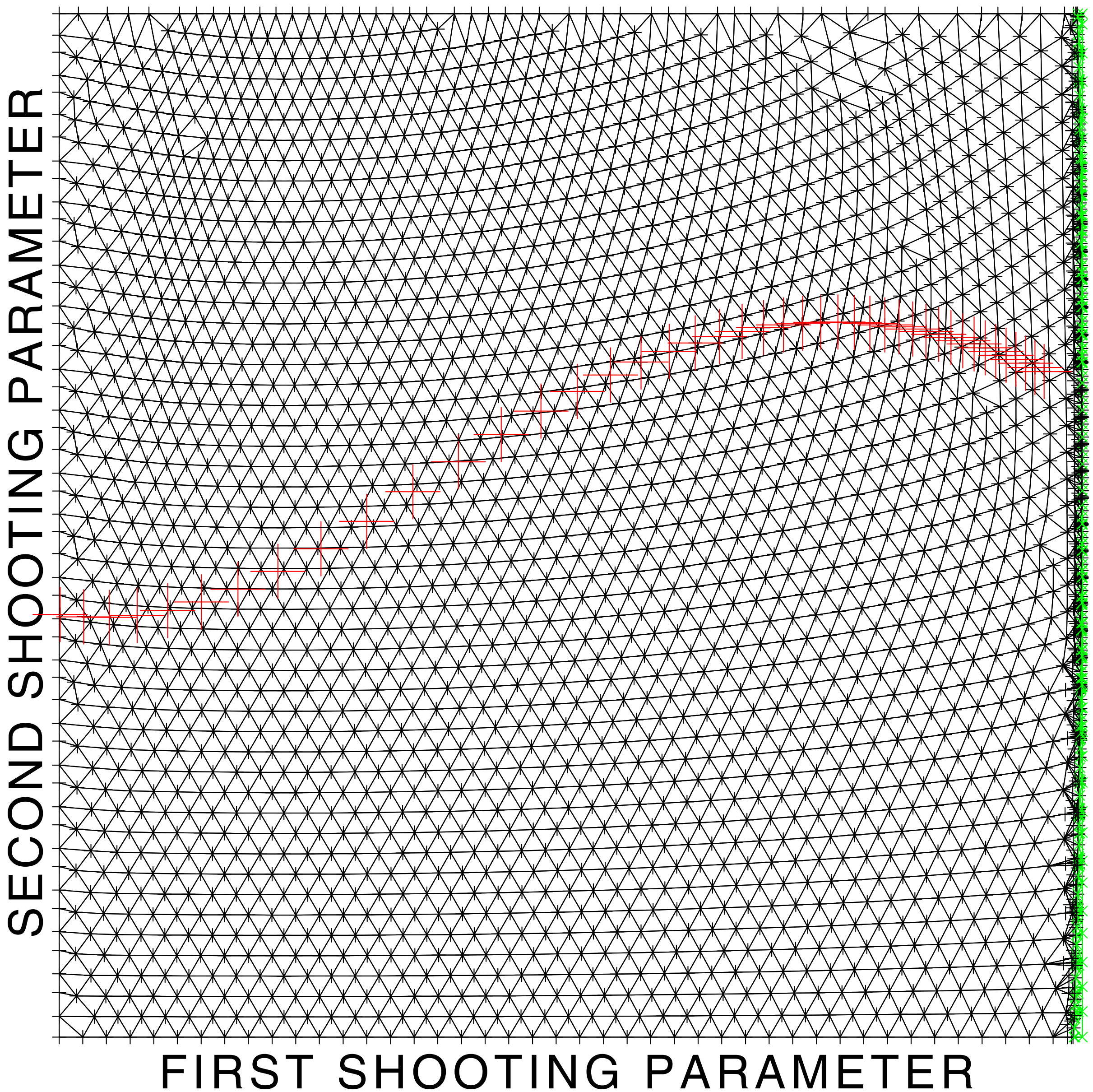
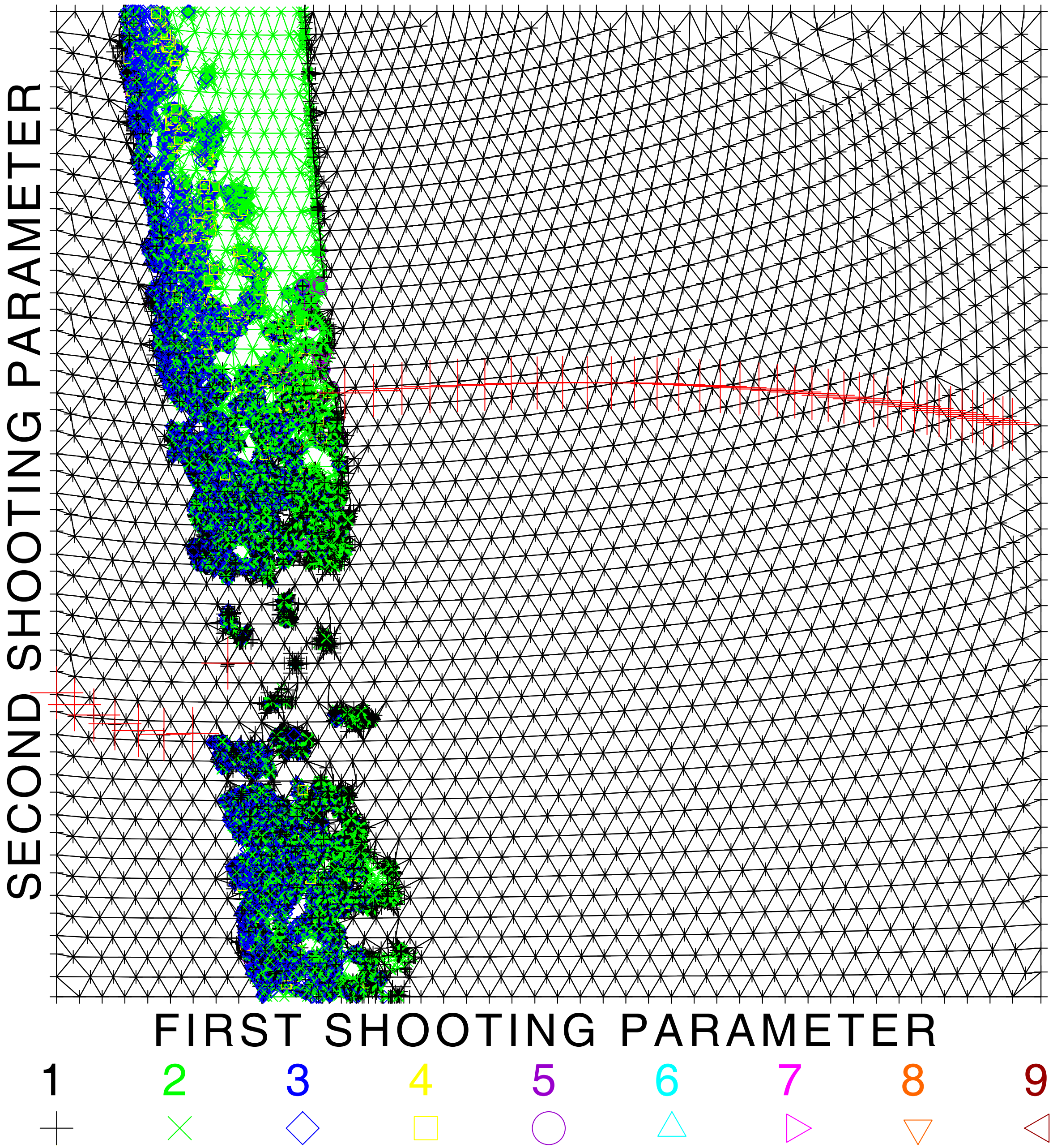


Anisotropic-ray-theory S-wave rays traced by package CRT in model SC1_II

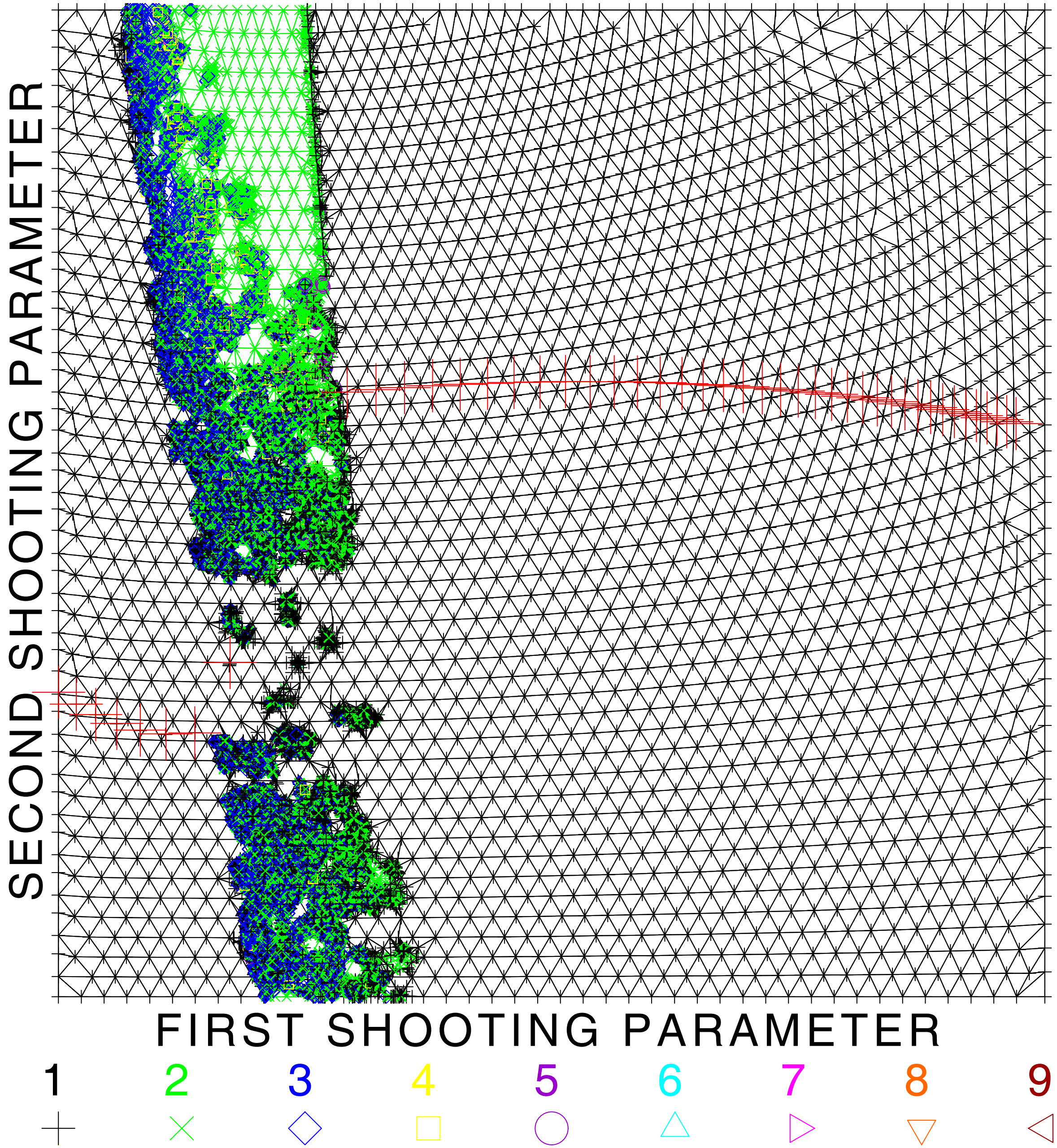
PART 1: TWO-POINT RAY TRACING

Anisotropic-ray-theory S1 (slower) wave rays traced by package CRT



Anisotropic-ray-theory S1 (slower) wave rays traced by package CRT

- 9 ray histories
- KMAH index up to 10



Anisotropic-ray-theory S-wave rays at split intersection singularity in model SC1_II

At the depth of 0 km, velocity model SC1 II is transversely isotropic with the tilted axis of symmetry. At this depth, the slowness surface contains an intersection singularity. At the depth of 1.5 km, velocity model SC1 II is very close to isotropic, but is slightly cubic and its symmetry axes coincide with the coordinate axes. This means that, at all depths except 0 km, velocity model SC1 II is very close to transversely isotropic, but is slightly tetragonal.

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Whereas the (exactly) transversely isotropic medium contains the intersection singularity through which the rays pass without rotation of the eigenvectors of the Christoffel matrix, in the slightly tetragonal medium, the intersection singularity is split, the slower S–wave slowness sheet separates from the faster S–wave slowness sheet, forming smooth but very sharp edges on both sheets – “split intersection singularity”.

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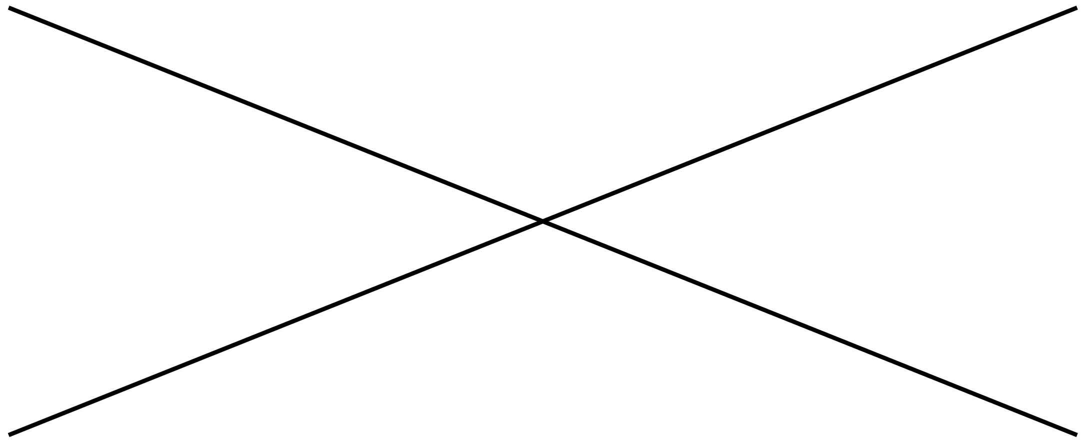
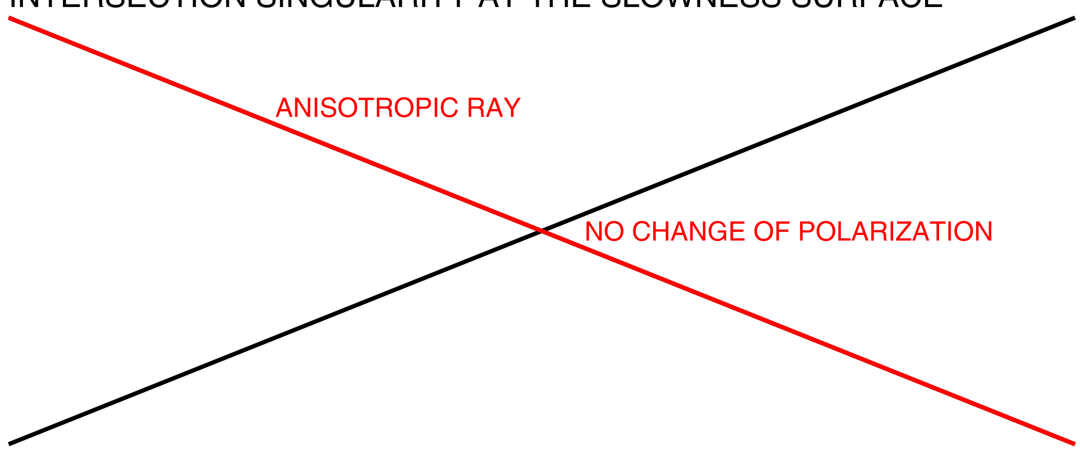
When the slowness vector of a ray smoothly pass through a split intersection singularity, the ray–velocity vector rapidly changes its direction and creates a sharp bend on the ray. This sharp bend is connected with a rapid rotation of the eigenvectors of the Christoffel matrix by 90 degree.

Anisotropic-ray-theory S-wave rays at intersection singularity

INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE

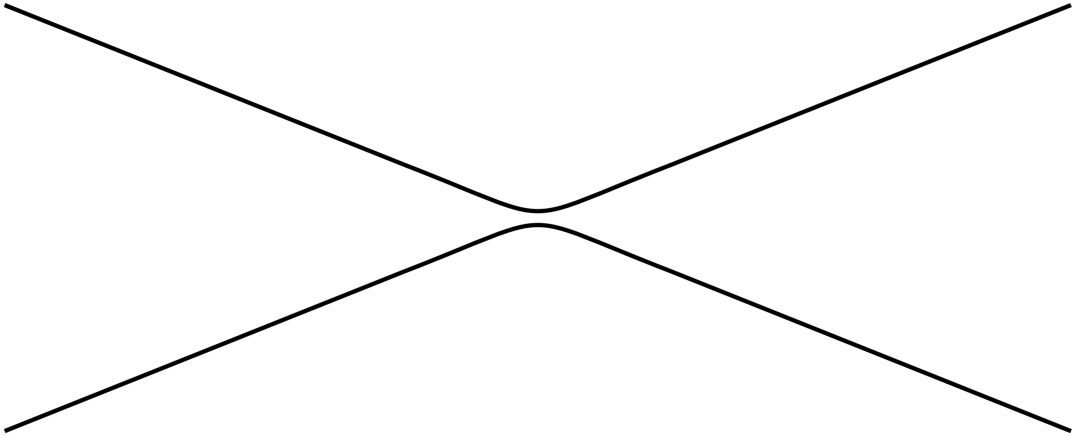


INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE



Anisotropic-ray-theory S-wave rays at split intersection singularity

SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE



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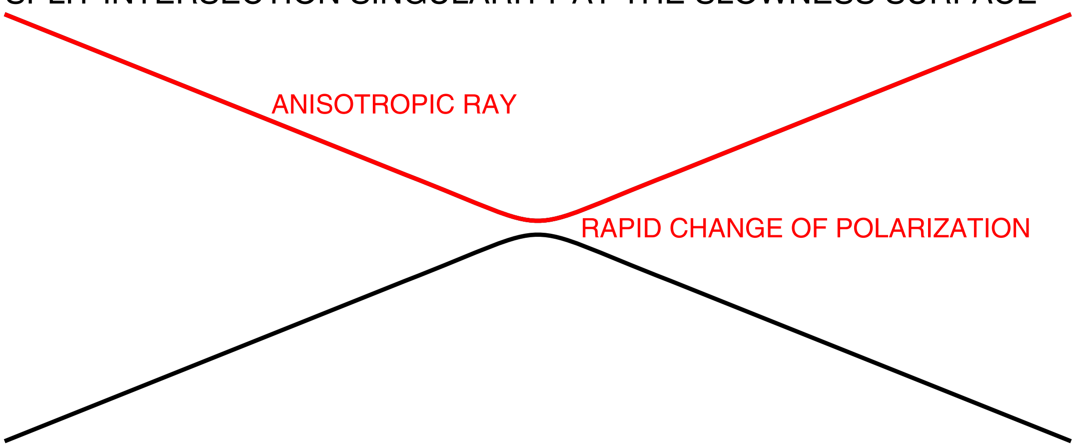
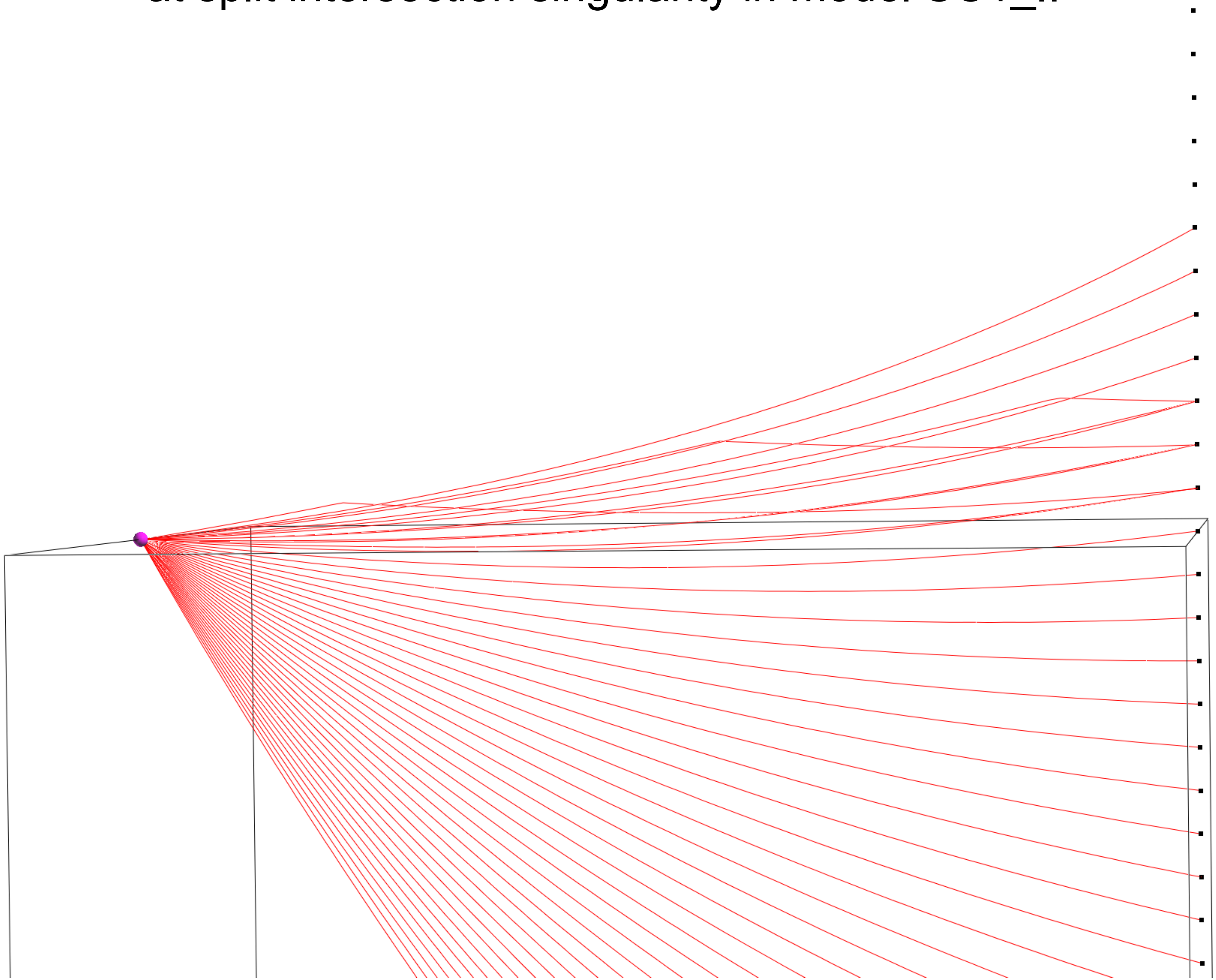
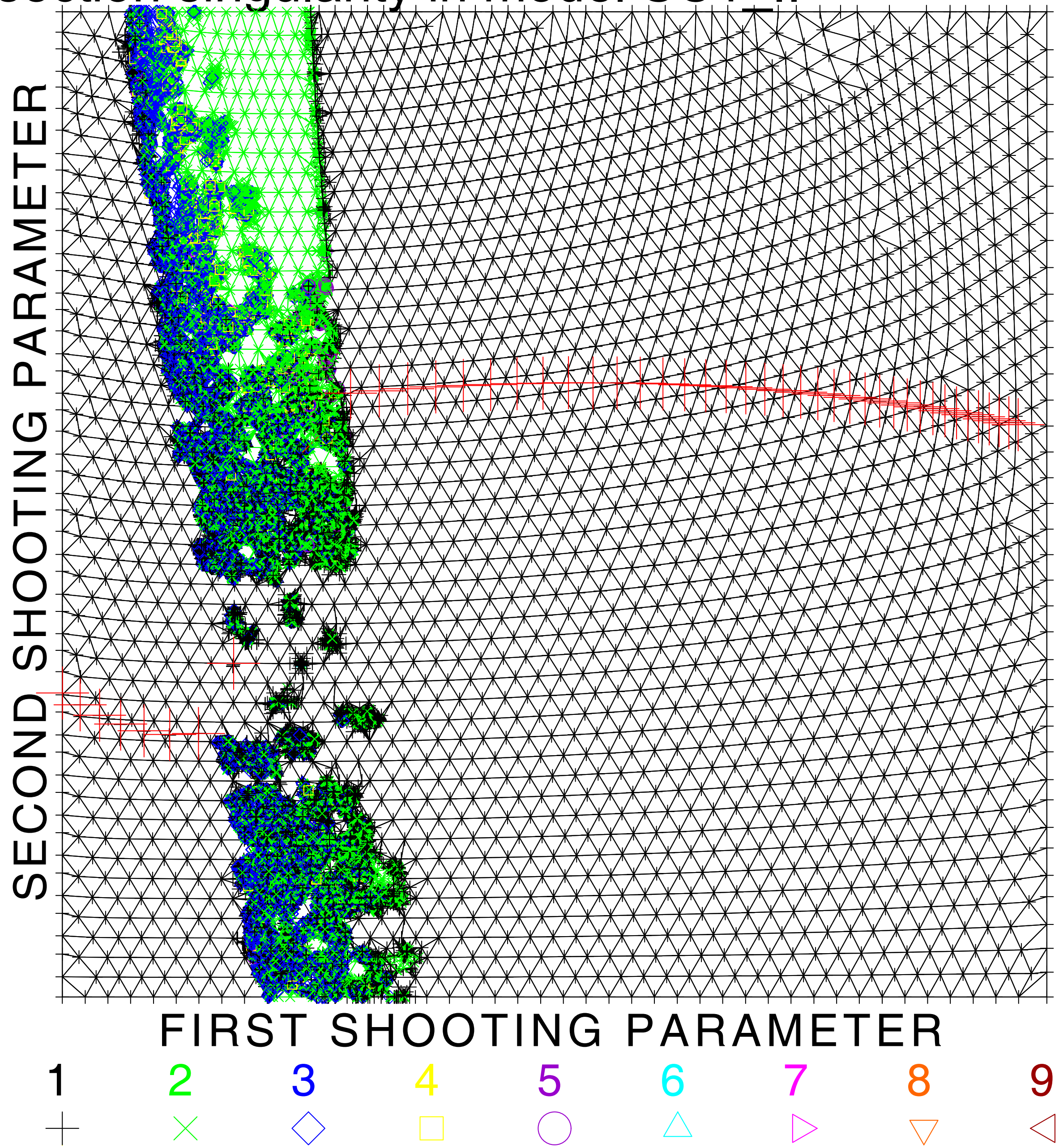


Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II



Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II

- 9 ray histories
- KMAH index up to 10
- the belt with “strange” rays corresponds to the rays bent at the split intersection singularity



Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II

The rays of the selected anisotropic–ray–theory S wave can safely be traced by solving Hamilton's equations of rays. Unfortunately, the equations of geodesic deviation (paraxial ray equations, dynamic ray tracing equations) contain the second–order derivatives of the Hamiltonian function with respect to the slowness vector. Expressions for these derivatives contain the difference of the S–wave eigenvalues of the Christofel matrix in the denominator.

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If the difference of the S–wave eigenvalues of the Christofel matrix is smaller than the rounding error, the second–order derivatives of the Hamiltonian function with respect to the slowness vector become random and, in consequence, the matrix of geometrical spreading becomes random, too.

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If we wish the rays with a reasonably defined matrix of geometrical spreading and a reasonably defined phase shift due to caustics, we have to terminate tracing of a ray if the relative difference between the S-wave eigenvalues of the Christoffel matrix is smaller then the prescribed limit which we named DSWAVE. The maximum angular numerical error of the eigenvectors of the Christoffel matrix in radians is then roughly equal to the ratio of the relative rounding error to parameter DSWAVE.

Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II

Initial-value rays traced with “reasonable” value of DSWAVE

- 5 ray histories

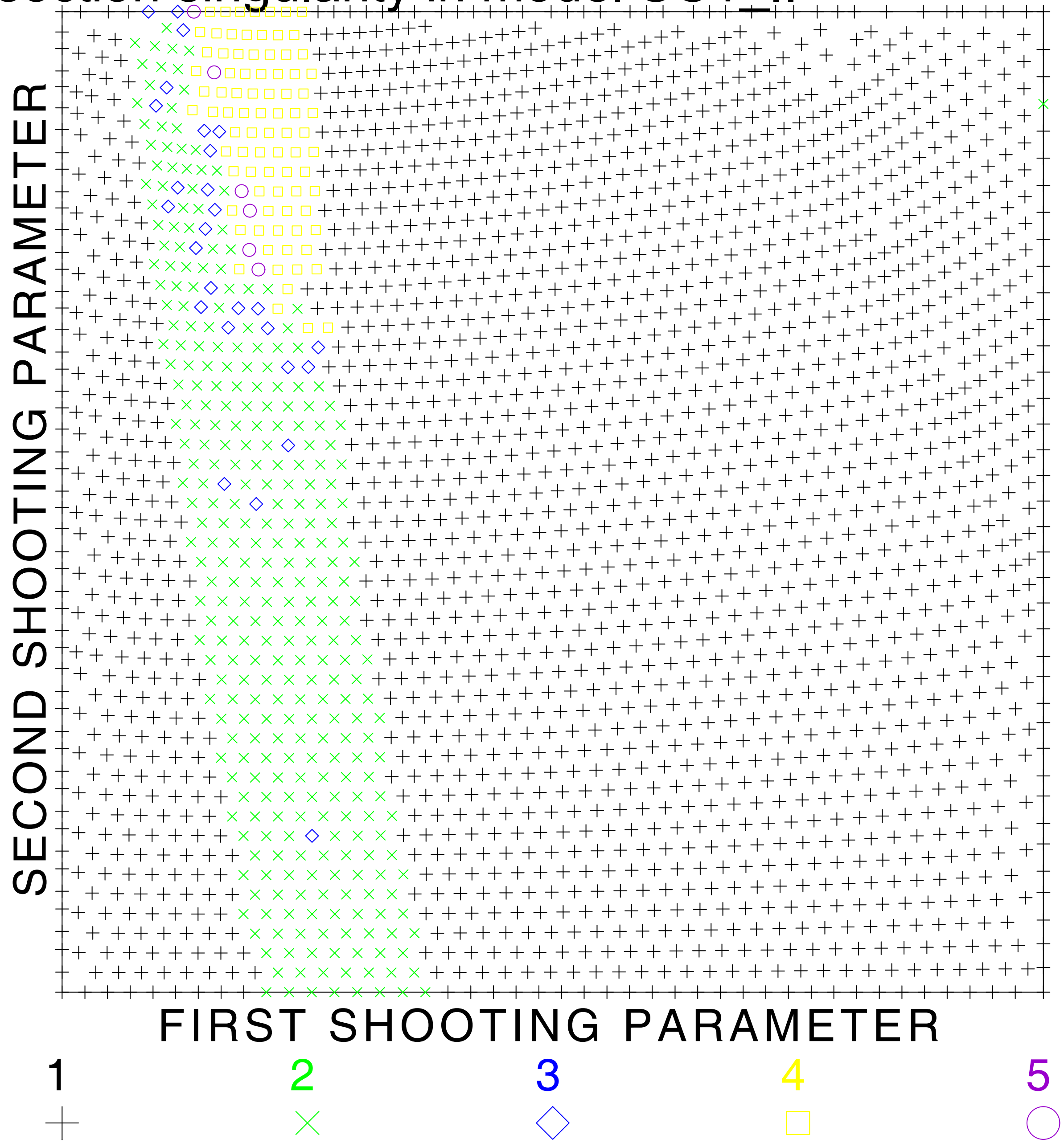
- rays of histories 2 and 3 terminated due to DSWAVE, KMAH index 0 and 1

- rays of histories 4 and 5 not terminated, KMAH index 1 and 2

⇒ DSWAVE too small

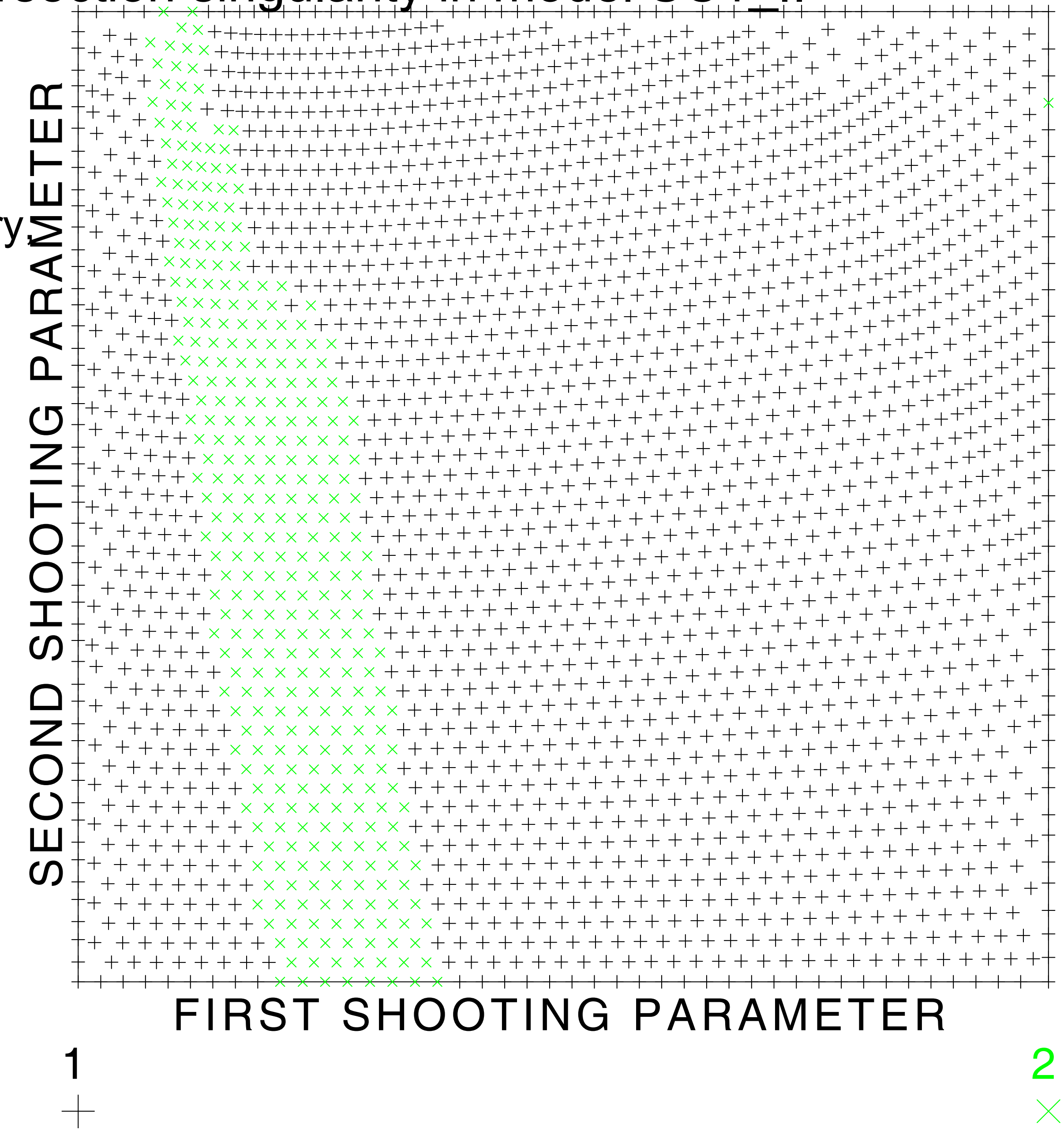
X

the belt too wide, all two-point rays bent at the singularity lost



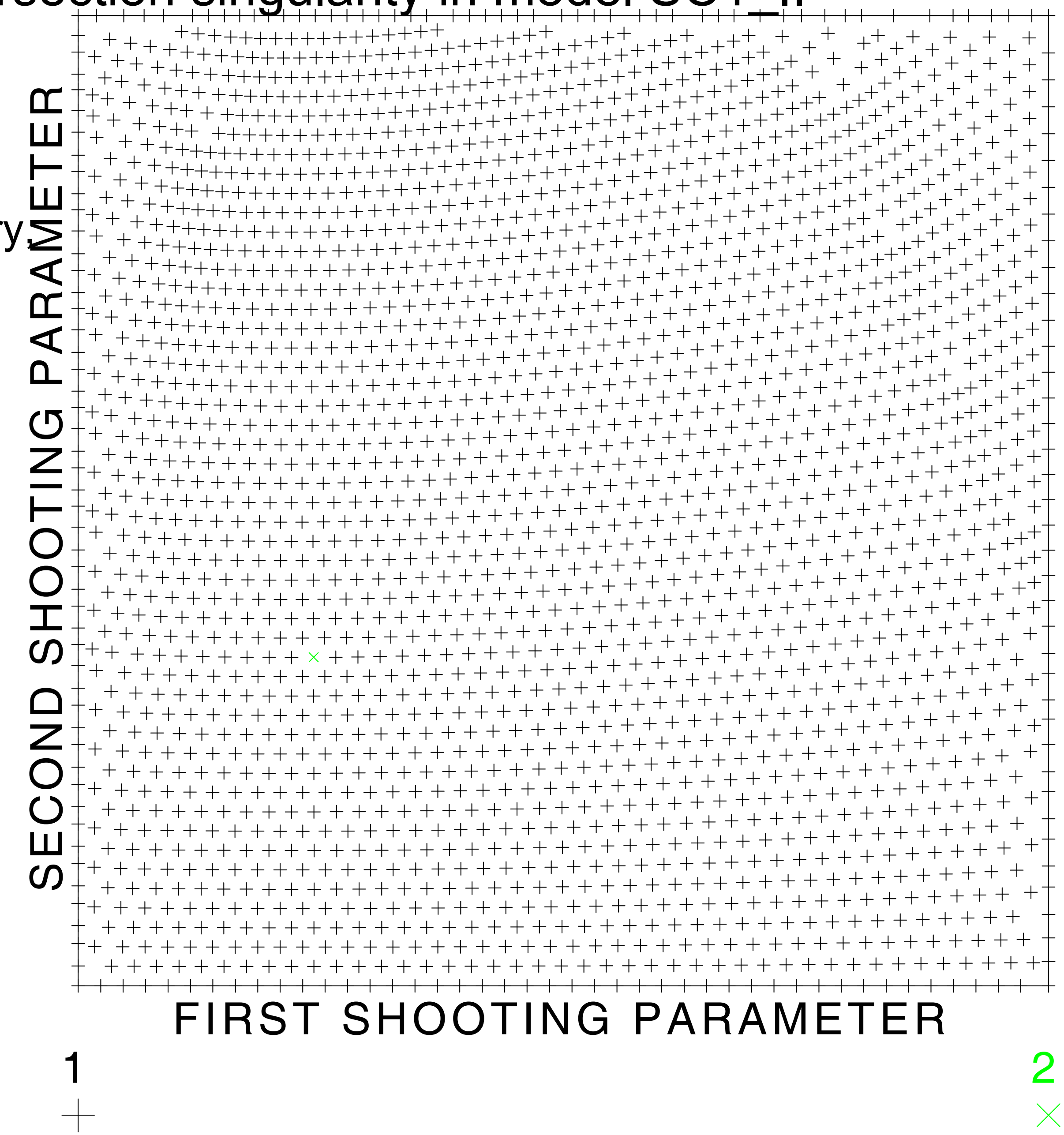
Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II

Initial-value rays traced with KMAH index excluded from the ray history, "reasonable" value of DSWAVE



Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II

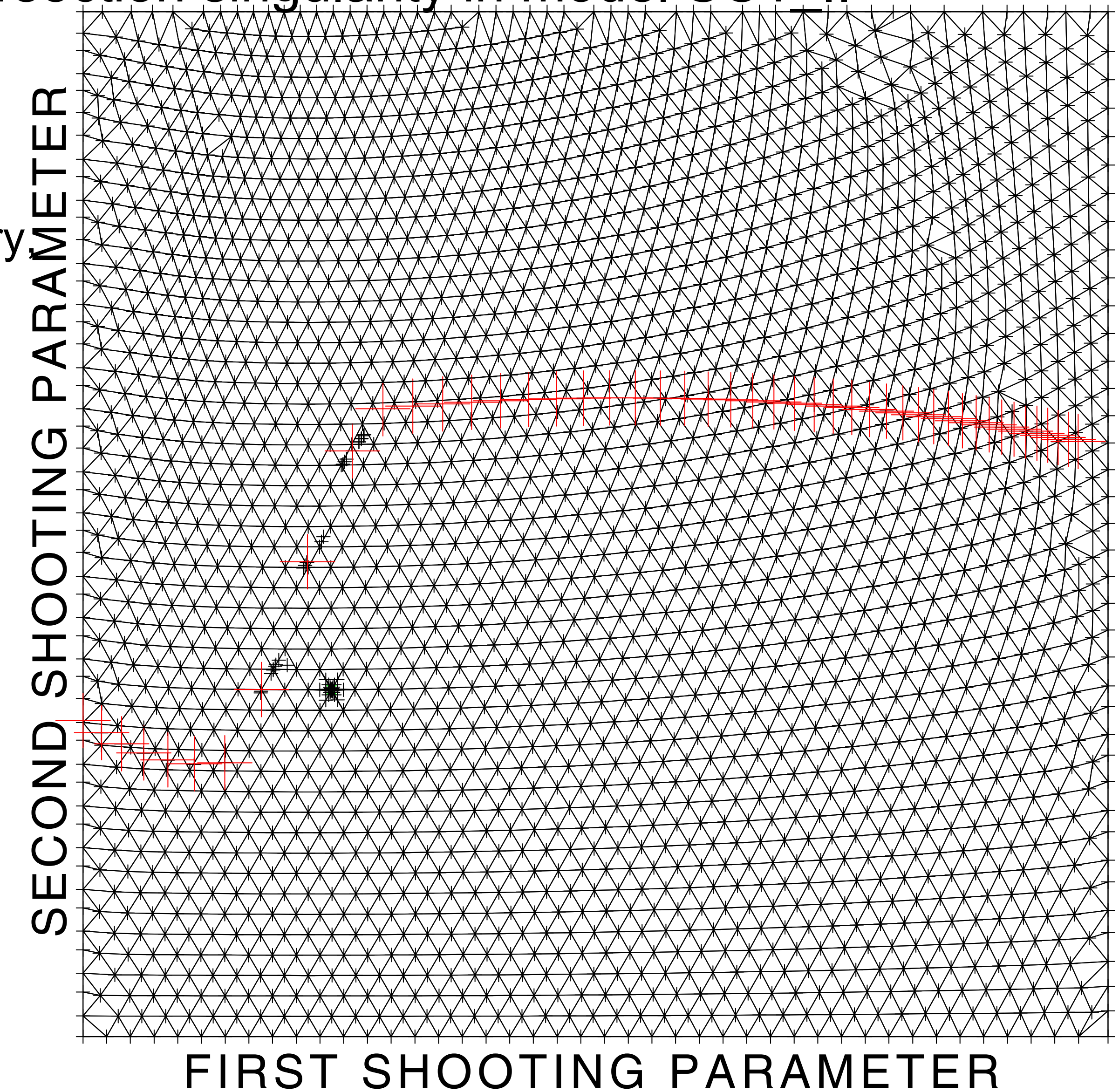
Initial-value rays
traced with KMAH index
excluded from the ray history,
minimum DSWAVE



Anisotropic-ray-theory S1-wave rays at split intersection singularity in model SC1_II

Two-point rays
traced with KMAH index
excluded from the ray history,
minimum DSWAVE

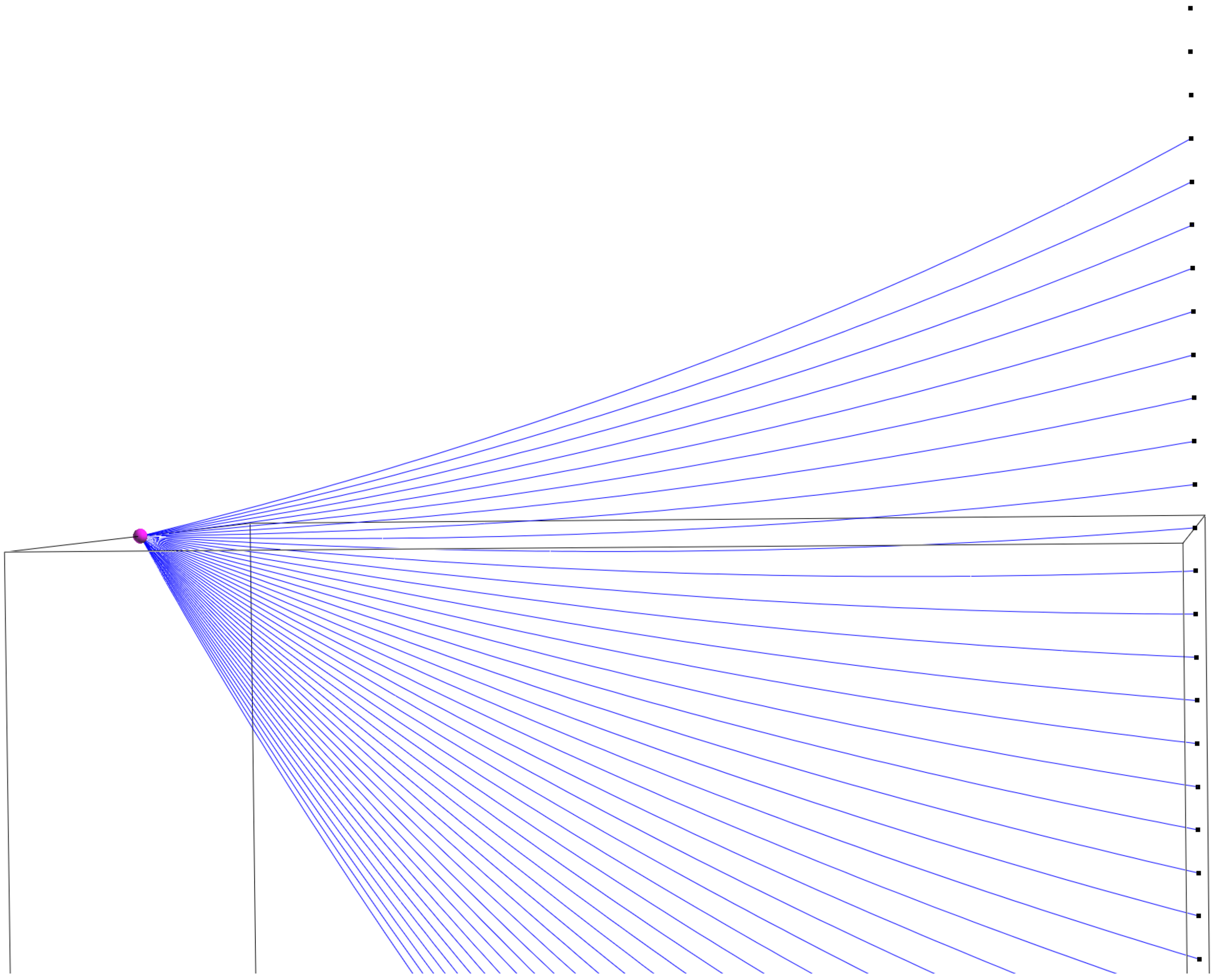
notice the problems with
finding two-point rays
due to incorrect matrix
of geometrical spreading



Anisotropic-ray-theory S-wave rays traced by package CRT in model SC1_II

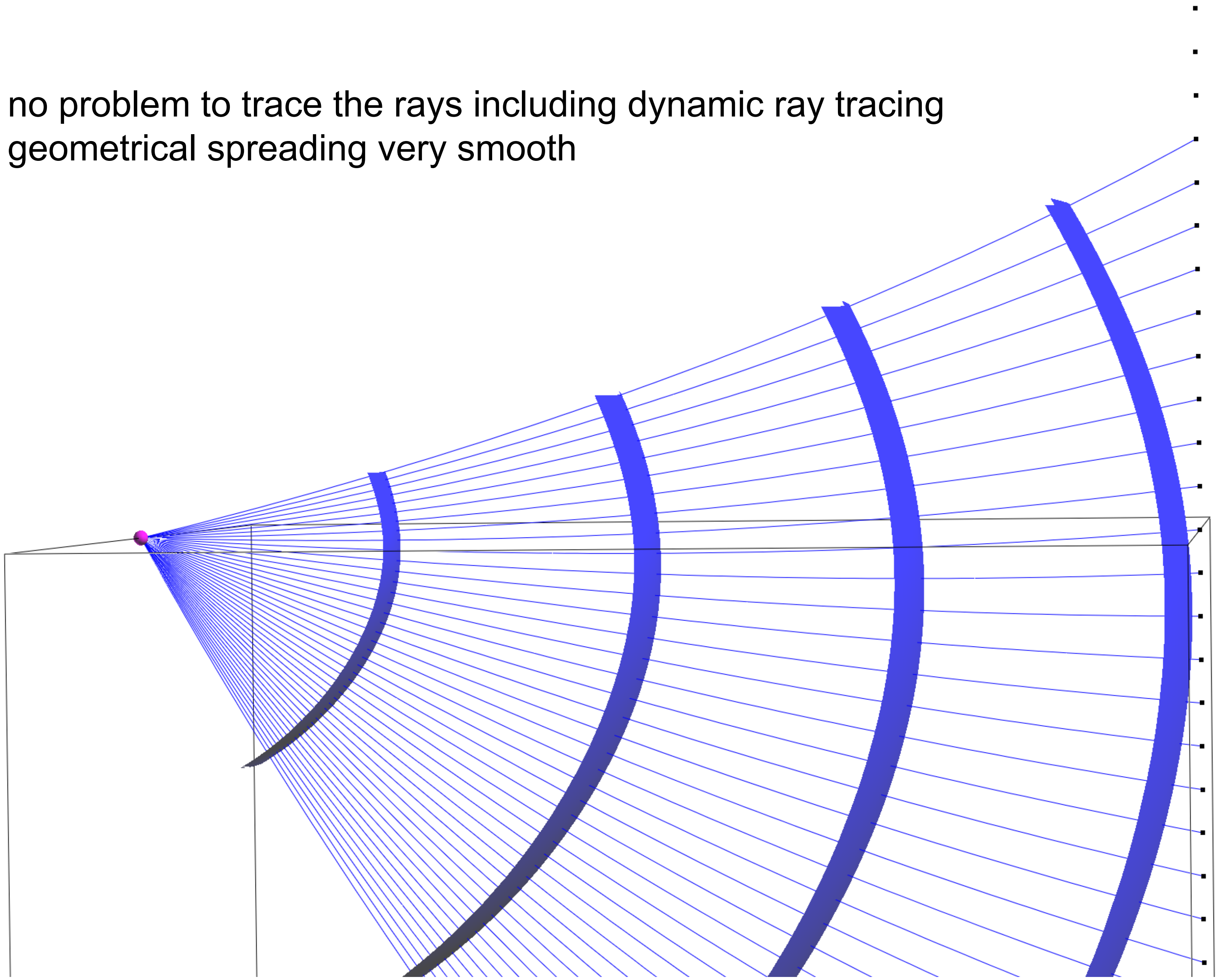
PART 2: RESULTING TWO-POINT RAYS

Common anisotropic S-wave rays

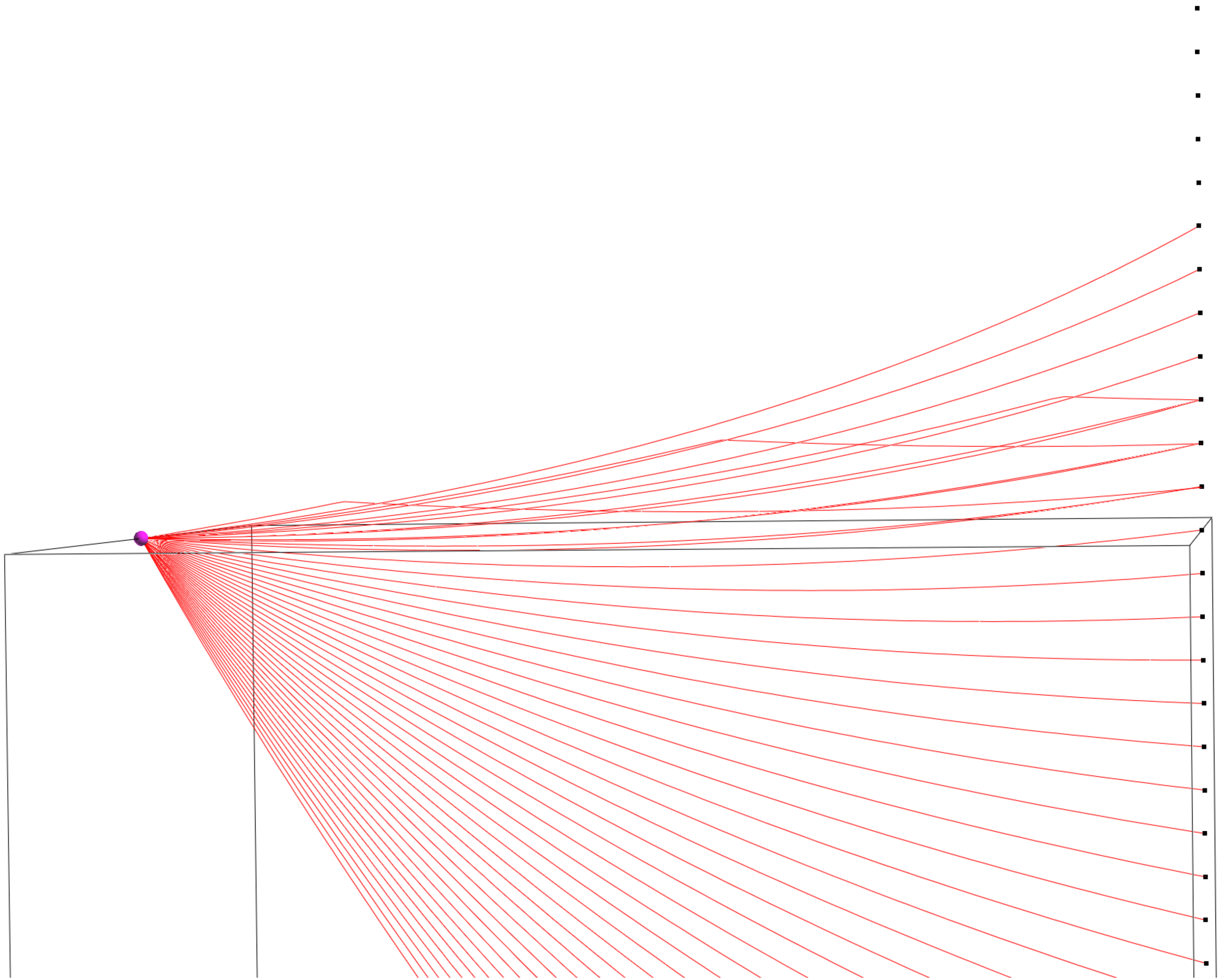


Common anisotropic S-wave rays

- no problem to trace the rays including dynamic ray tracing
- geometrical spreading very smooth



Anisotropic-ray-theory S1 (slower) wave rays



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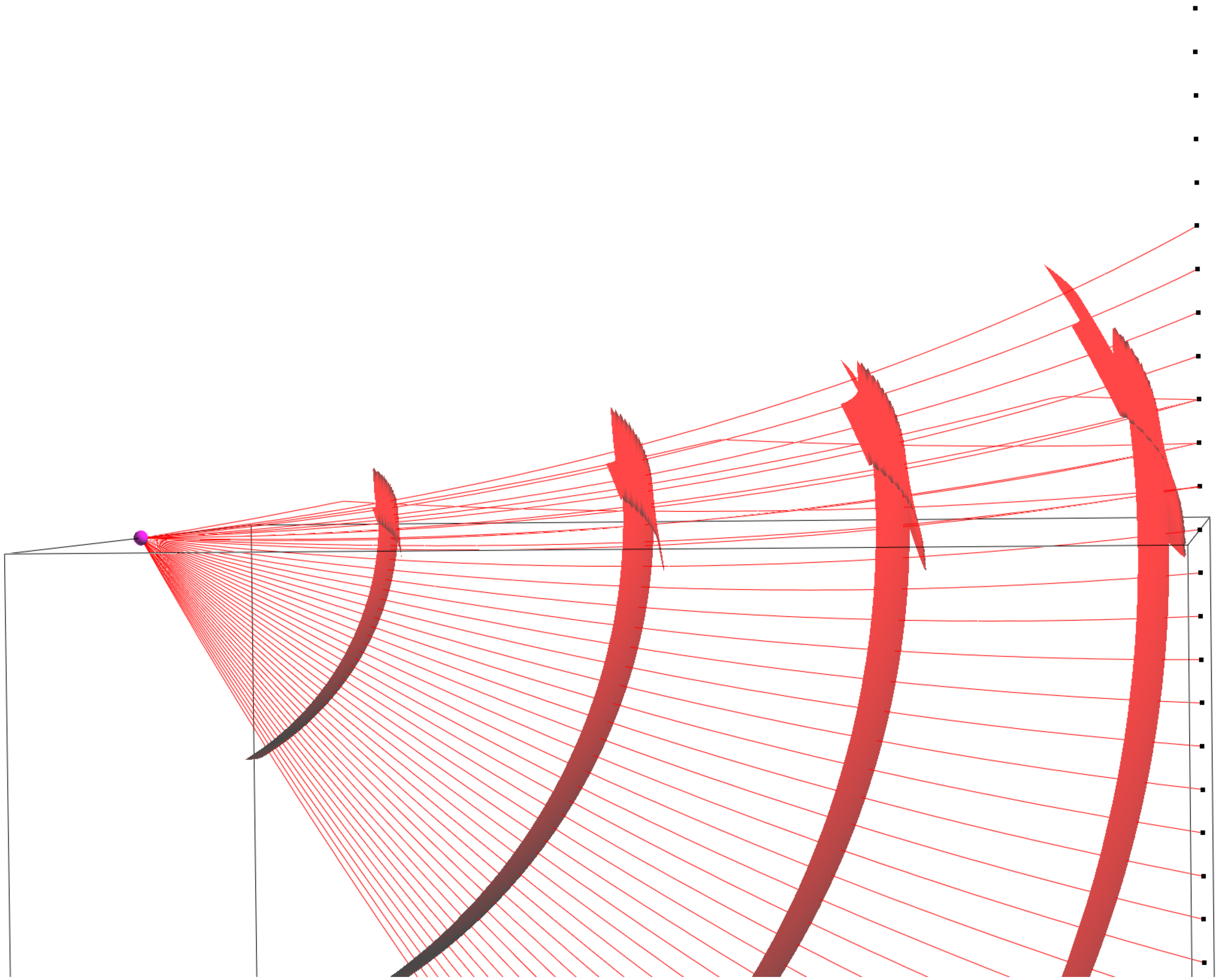
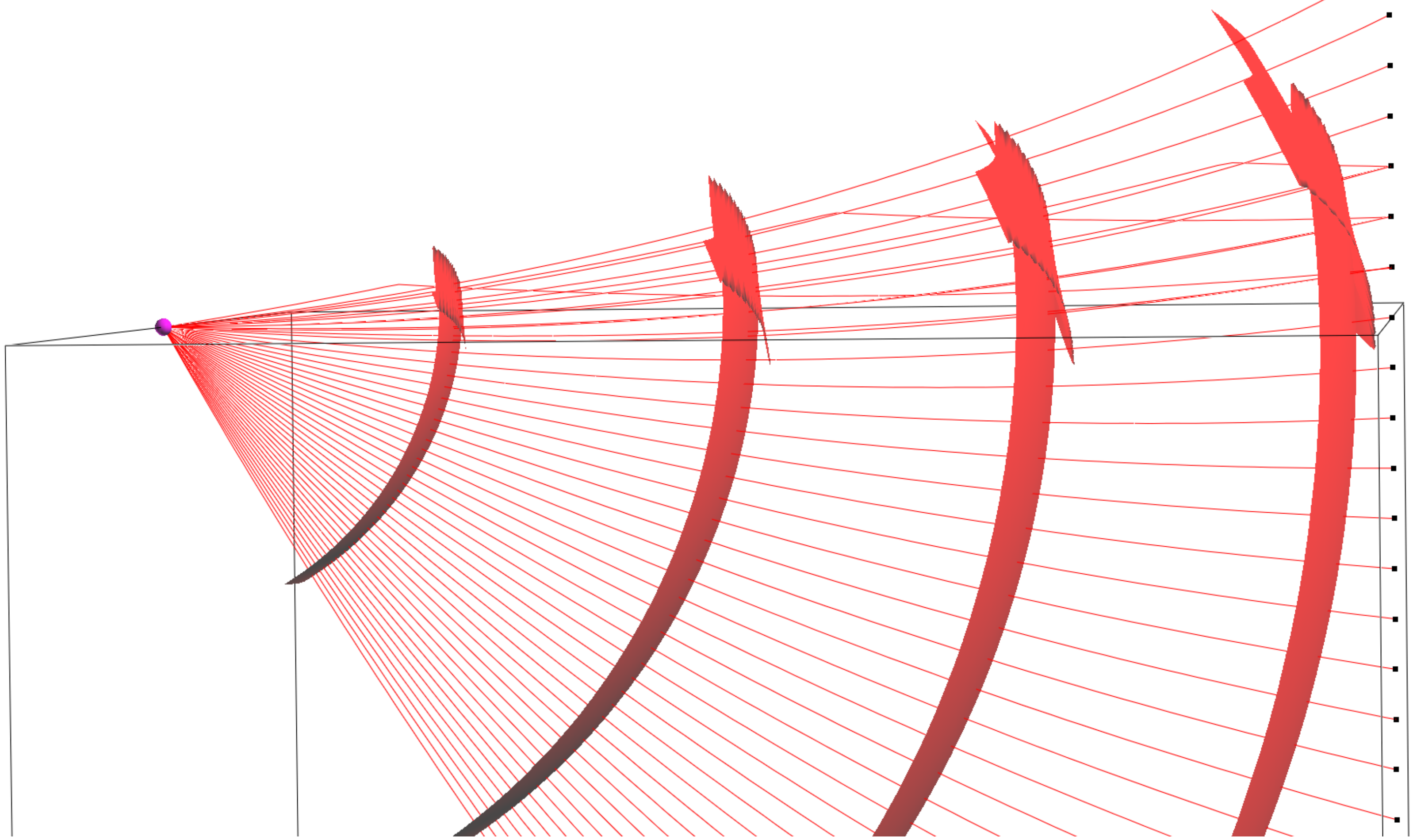
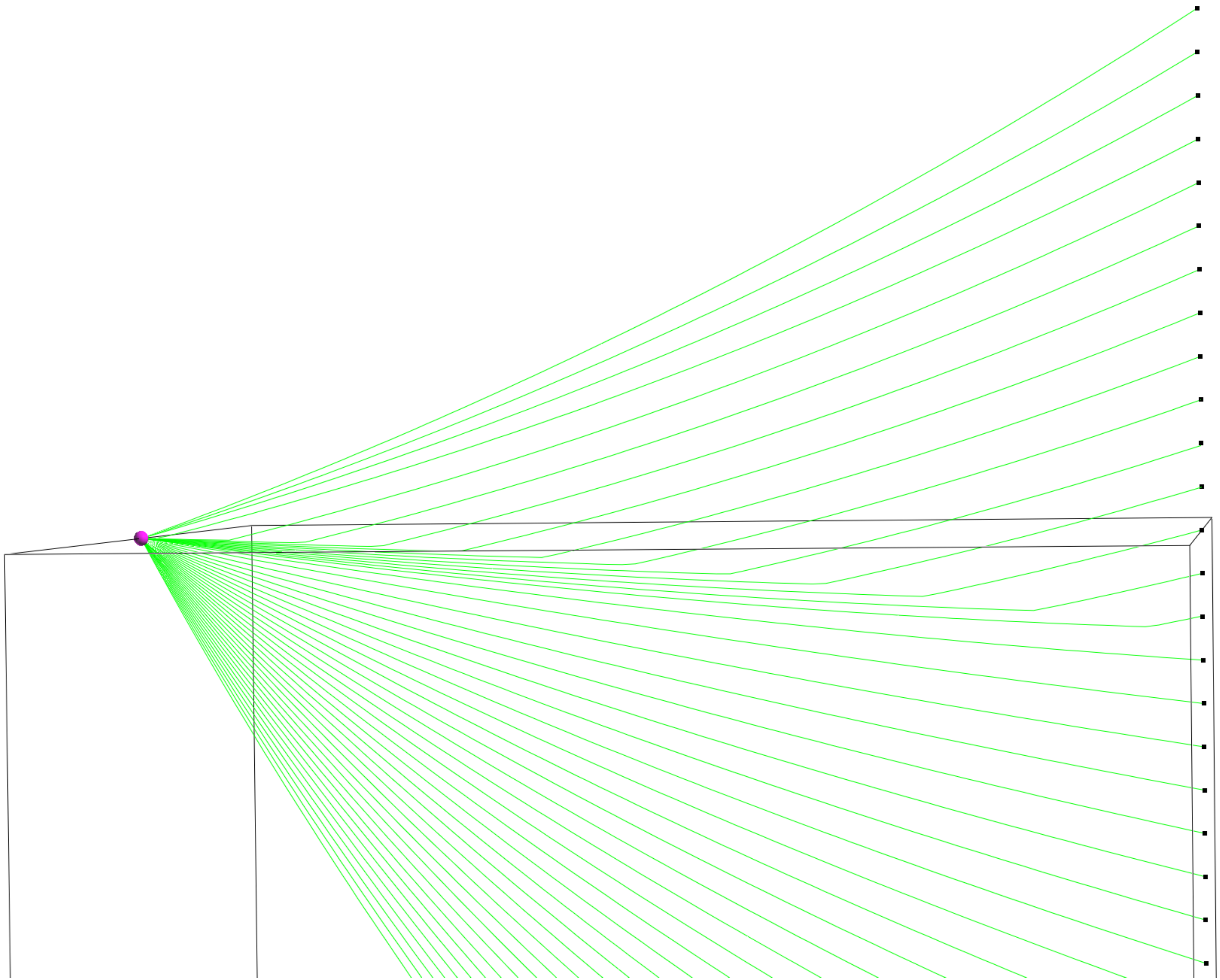


Anisotropic-ray-theory S1 (slower) wave rays

- problems to find two-point rays
- dynamic ray tracing fails
- the rays sharply bent at the split intersection singularity form a triplication



Anisotropic-ray-theory S2 (faster) wave rays



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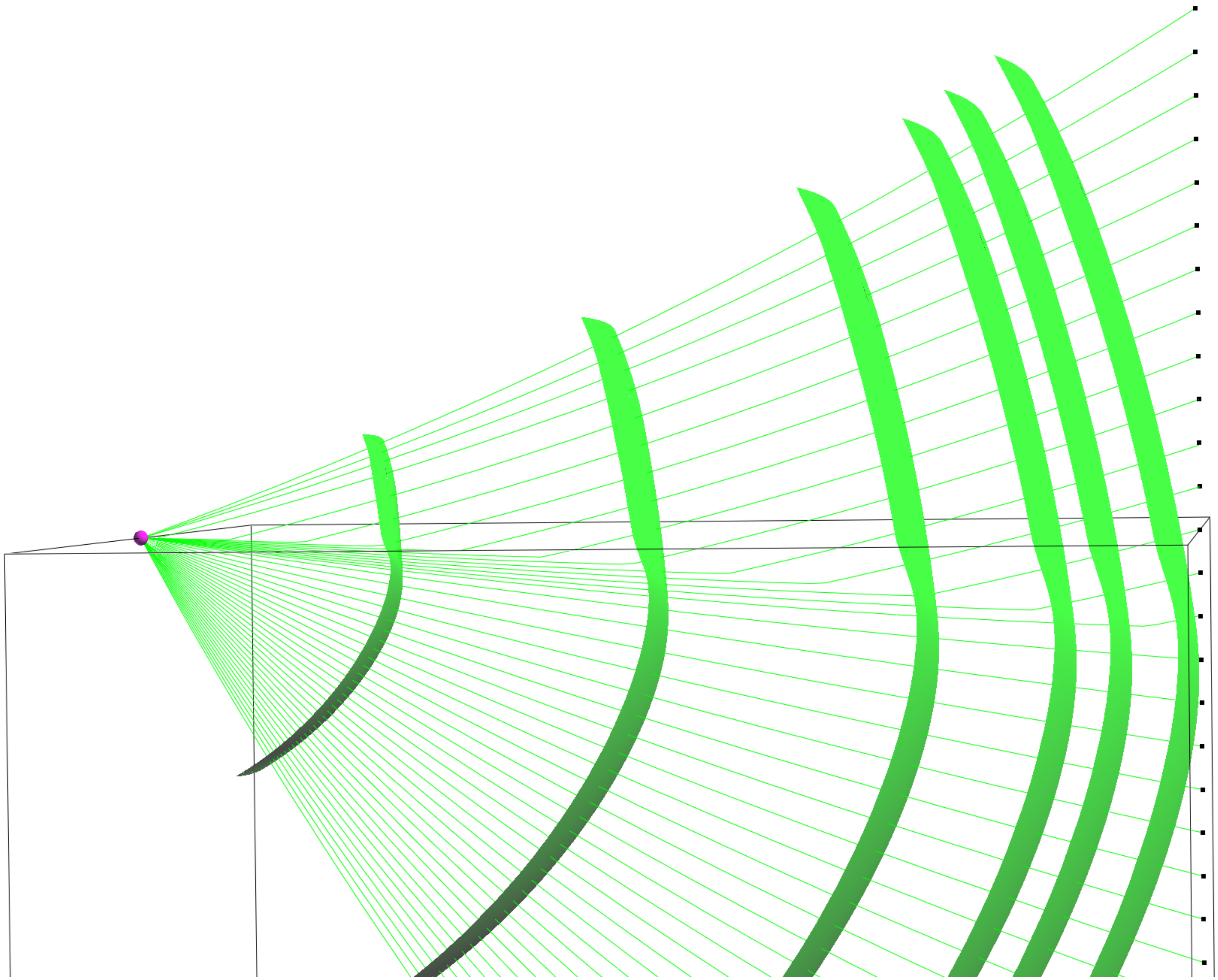
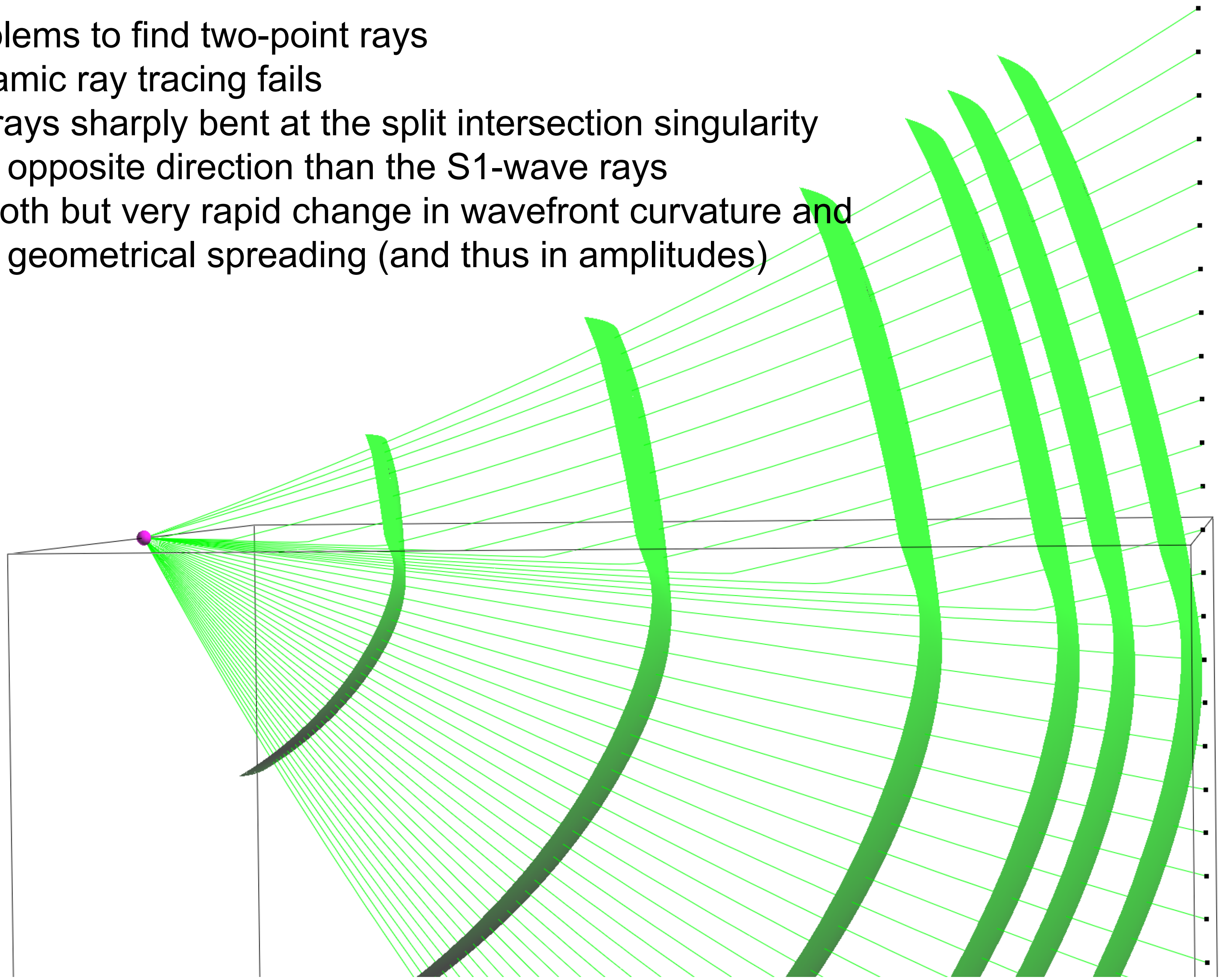
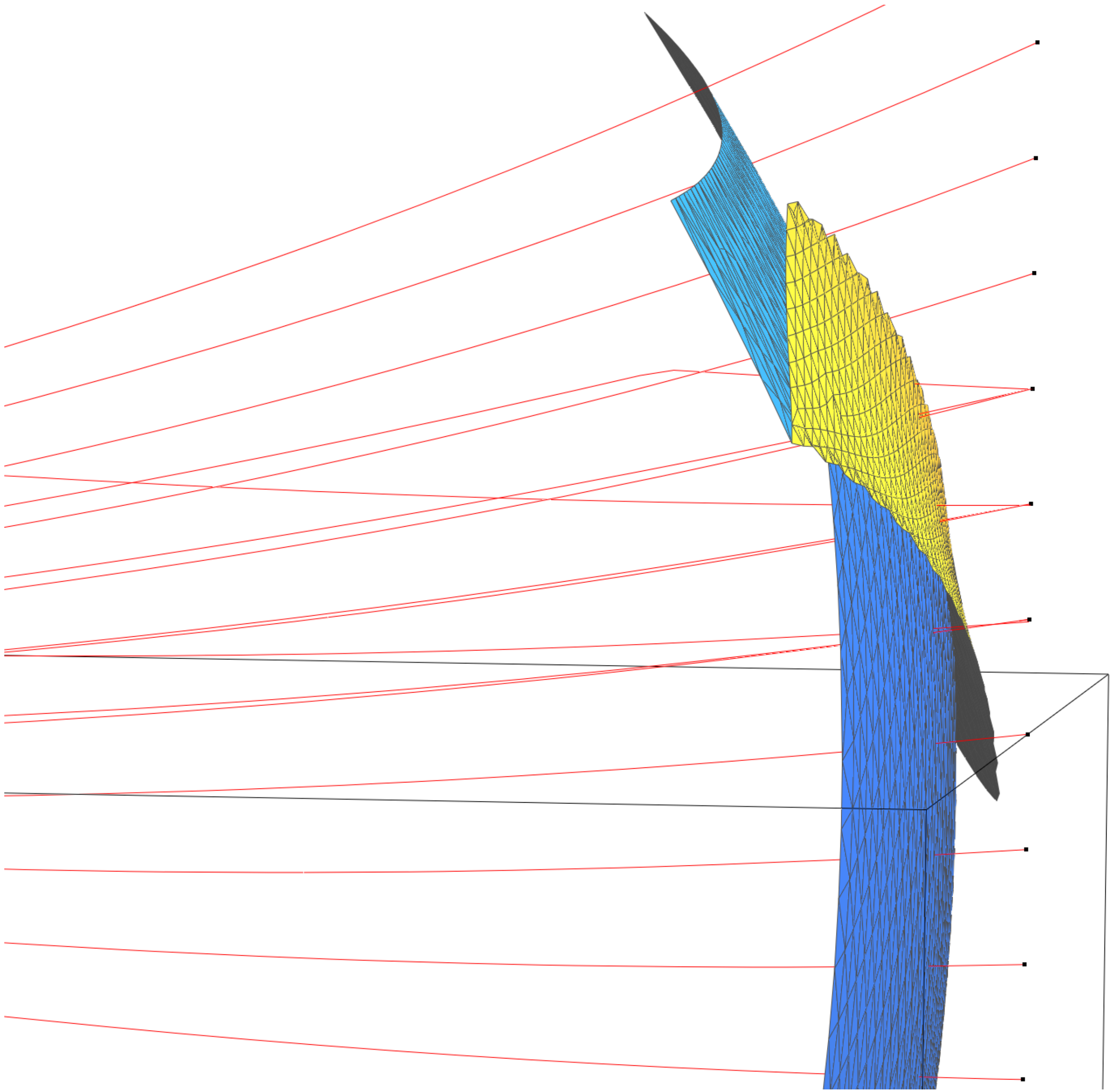


Anisotropic-ray-theory S2 (faster) wave rays

- problems to find two-point rays
- dynamic ray tracing fails
- the rays sharply bent at the split intersection singularity in the opposite direction than the S1-wave rays
- smooth but very rapid change in wavefront curvature and in the geometrical spreading (and thus in amplitudes)



Anisotropic-ray-theory S1 (slower) wave rays



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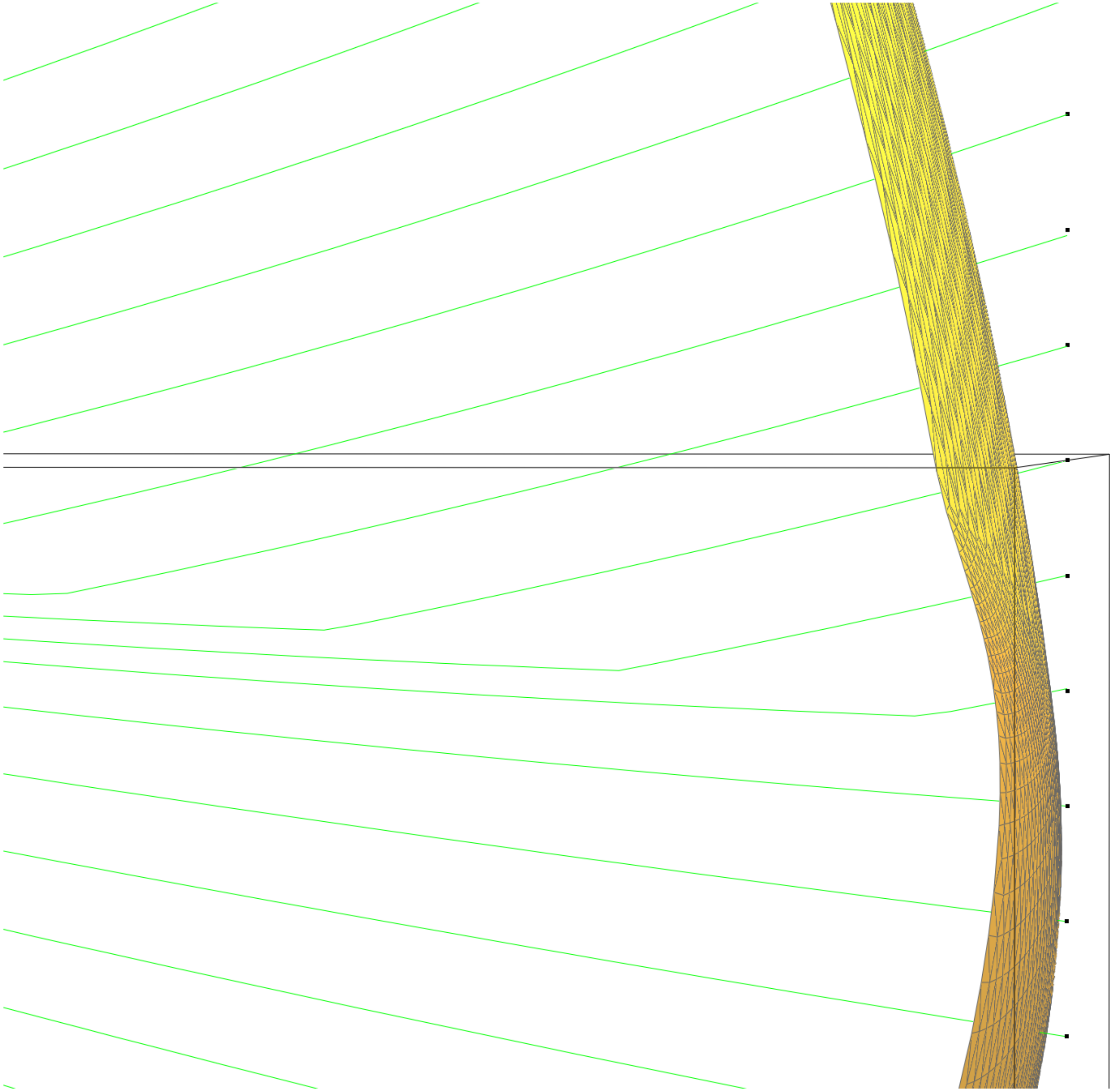
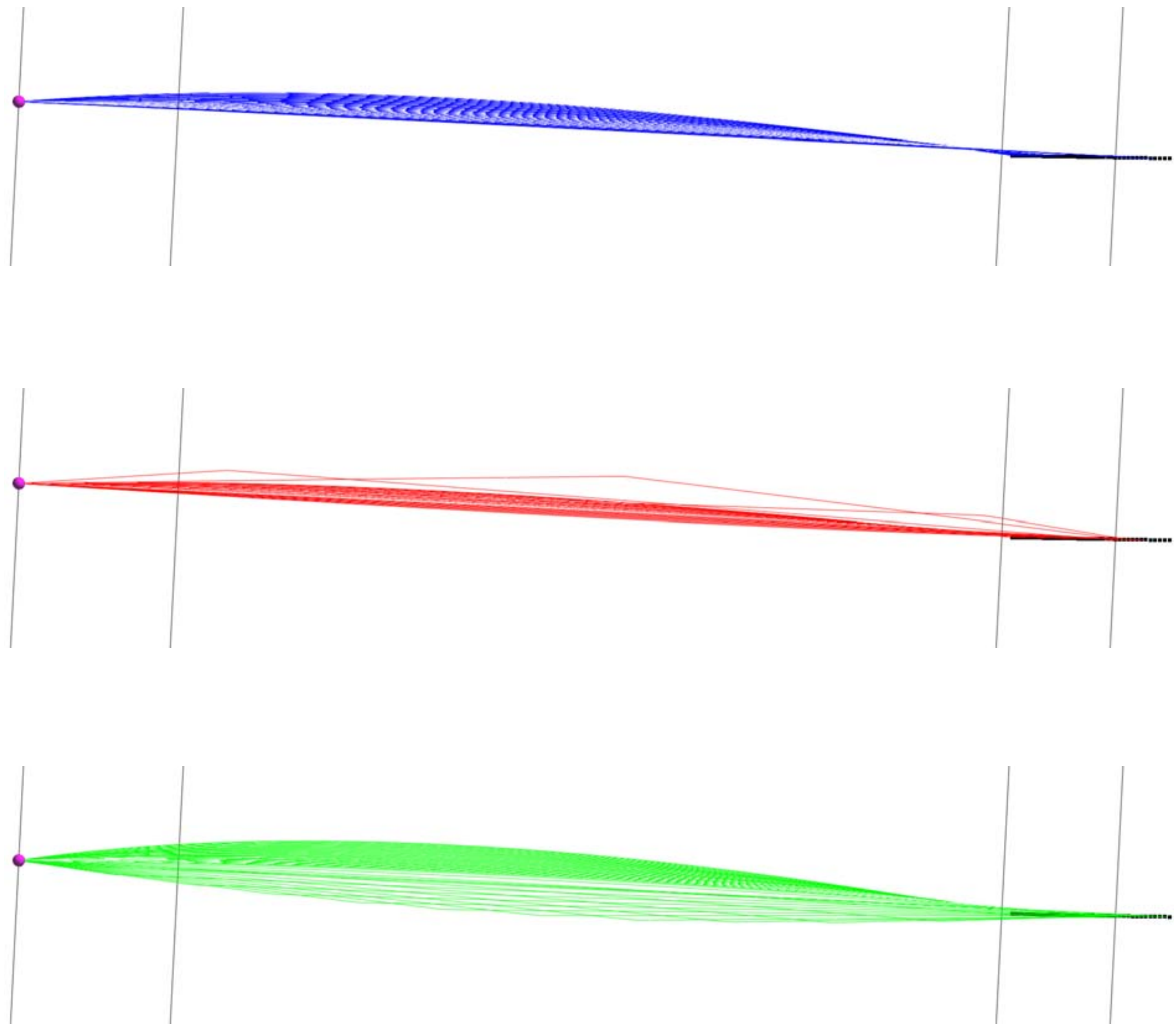
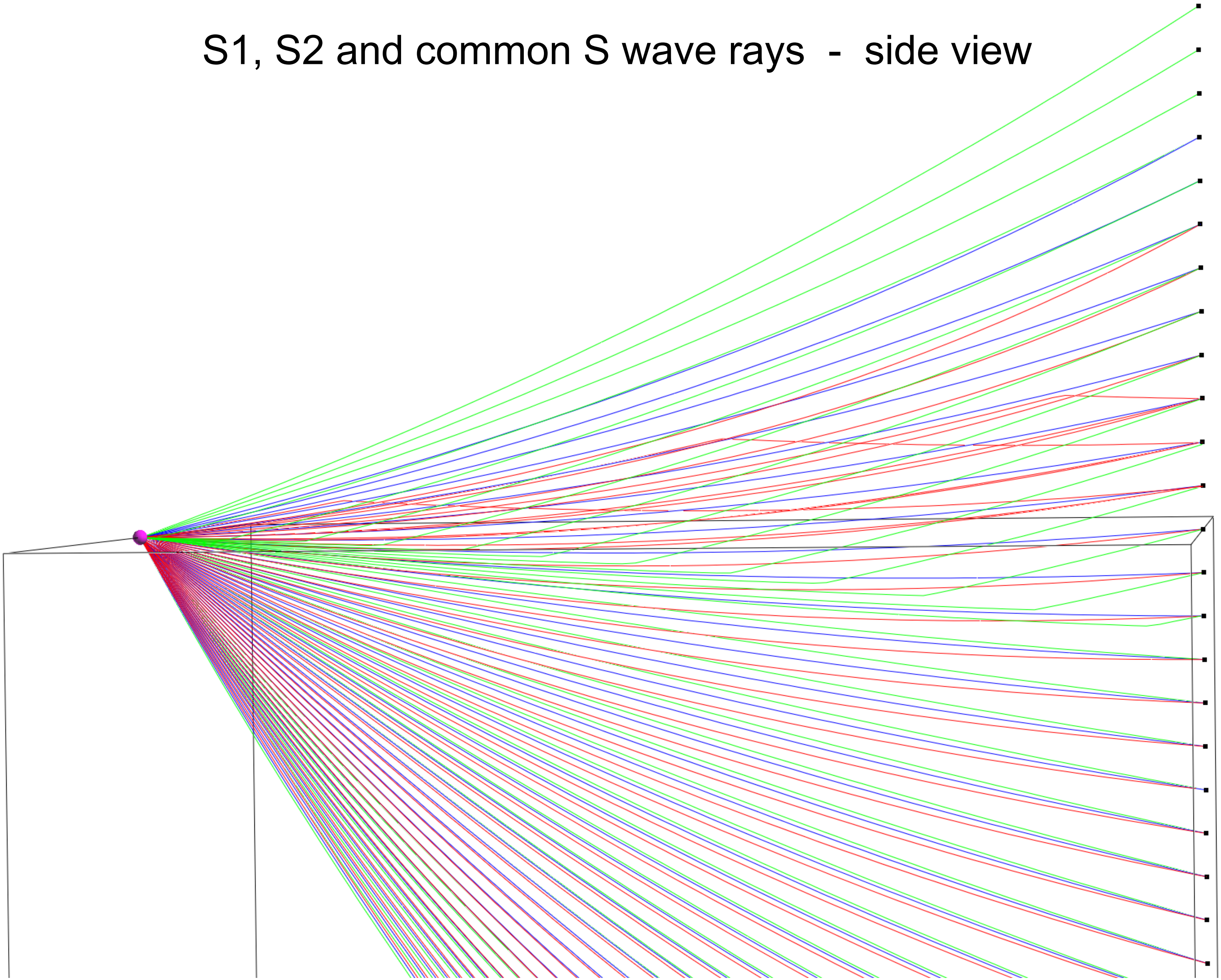


S1, S2 and common S wave rays - top view



S1, S2 and common S wave rays - side view



Conclusions

In velocity models with split intersection singularity at the slowness surface, the S–wave anisotropic–ray–theory rays do not describe the actual paths of wave propagation and do not represent reasonable reference rays for the coupling ray theory. In this case, the common anisotropic S–wave rays represent much better reference rays for the coupling ray theory.

Acknowledgments

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