Approximating the complex-valued Green-tensor amplitude by a real-valued Green-tensor amplitude

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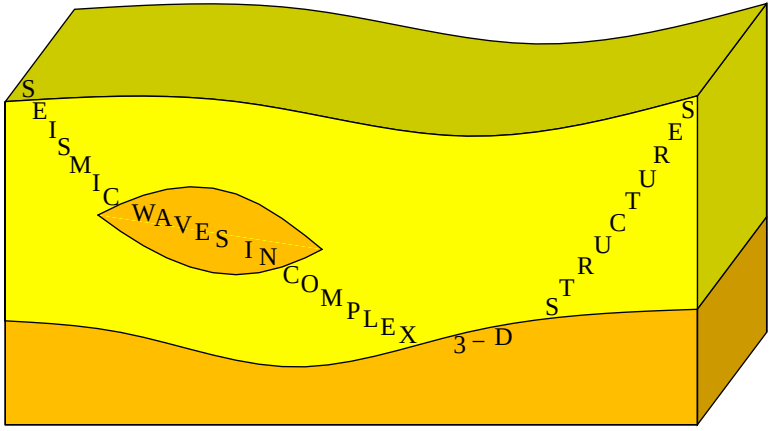

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Introduction

The tensorial amplitude of the S-wave Green tensor is often complex-valued, especially if the wavefield is recorded at the Earth surface (Červený, 2001).

The method of Šílený & Milev (2008), Šílený et al. (2009) and Horálek & Šílený (2013) for determining the real-valued seismic moment tensor from the maximum real-valued vectorial amplitude picked in the polarization diagram requires the approximation of the complex-valued 3×3 Green-tensor amplitude by a real-valued Green-tensor amplitude with a phase shift.

This approximation has been proposed by Klimeš (2014).

Approximation of the complex-valued Green-tensor amplitude

Given complex-valued Green-tensor amplitude $\mathbf{G}^{\mathrm{C}}$ maps seismic force $\mathbf{f}$ onto the complex-valued displacement $\mathbf{u}^{\mathrm{C}}$:

$$\mathbf{u}^{\mathrm{C}} = \mathbf{G}^{\mathrm{C}}\,\mathbf{f} \quad . \tag{1}$$

Approximate real-valued Green-tensor amplitude $\mathbf{G}^{\mathrm{R}}$ with phase shift factor $\exp(\mathrm{i}\varphi)$, map seismic force $\mathbf{f}$ onto the approximate complex-valued displacement $\mathbf{u}^{\mathrm{R}}$:

$$\mathbf{u}^{\mathrm{R}} = \mathbf{G}^{\mathrm{R}}\,\mathbf{f}\,\exp(\mathrm{i}\varphi) \quad . \tag{2}$$

The square of the wavefield error of this approximation is

$$\delta^2 = (\mathbf{u}^{\mathrm{C}} - \mathbf{u}^{\mathrm{R}})^{+}(\mathbf{u}^{\mathrm{C}} - \mathbf{u}^{\mathrm{R}}) \quad , \tag{3}$$

where $^{+}$ denotes the Hermitian adjoint.

We minimize the average $\langle\delta^2\rangle$ of the square δ^2 of wavefield error over all spatial directions of unit seismic force $\mathbf{f}$.

Real-valued Green-tensor amplitude with phase shift factor

Optimum approximate real-valued Green-tensor amplitude:

$$\mathbf{G}^{\mathrm{R}} = \mathrm{Re}(\mathbf{G}^{\mathrm{C}})\cos(\varphi) + \mathrm{Im}(\mathbf{G}^{\mathrm{C}})\sin(\varphi) \quad . \tag{10}$$

Optimum phase shift:

$$\varphi = \tfrac{1}{2}\arg\Big\{\mathrm{Tr}\big[\mathrm{Re}(\mathbf{G}^{\mathrm{C}})^{\mathrm{T}}\mathrm{Re}(\mathbf{G}^{\mathrm{C}}) - \mathrm{Im}(\mathbf{G}^{\mathrm{C}})^{\mathrm{T}}\mathrm{Im}(\mathbf{G}^{\mathrm{C}})\big] + \mathrm{i}\,2\,\mathrm{Tr}\big[\mathrm{Re}(\mathbf{G}^{\mathrm{C}})^{\mathrm{T}}\mathrm{Im}(\mathbf{G}^{\mathrm{C}})\big]\Big\} \quad . \tag{13}$$

Relative wavefield error

The relative root mean square error of the approximation of the complex-valued Green-tensor amplitude by the real-valued Green-tensor amplitude is

$$\rho = \sqrt{\langle \delta^2 \rangle \,/\, \langle |\mathbf{u}^{\mathrm{C}}|^2 \rangle} \quad , \tag{17}$$

where

$$\langle \delta^2 \rangle = \tfrac{1}{3} \,\mathrm{Tr}\big[(\mathbf{G}^{\mathrm{C}})^{+}\mathbf{G}^{\mathrm{C}} - (\mathbf{G}^{\mathrm{R}})^{\mathrm{T}}\mathbf{G}^{\mathrm{R}}\big] \tag{14}$$

is the average square of wavefield error, and

$$\langle |\mathbf{u}^{\mathrm{C}}|^2 \rangle = \tfrac{1}{3} \,\mathrm{Tr}\big[\mathrm{Re}(\mathbf{G}^{\mathrm{C}})^{\mathrm{T}}\mathrm{Re}(\mathbf{G}^{\mathrm{C}})\big] + \mathrm{Tr}\big[\mathrm{Im}(\mathbf{G}^{\mathrm{C}})^{\mathrm{T}}\mathrm{Im}(\mathbf{G}^{\mathrm{C}})\big] \tag{18}$$

is the average square of the norm of the complex-valued displacement over all spatial directions of unit seismic force $\mathbf{f}$.

If relative error (17) is acceptable, the real-valued Green-tensor amplitude can be used in the inversion according to Šílený & Milev (2008), Šílený et al. (2009), and Horálek & Šílený (2013). If relative error (17) is not acceptable, the corresponding data from the receiver should not be used in that inversion.

References

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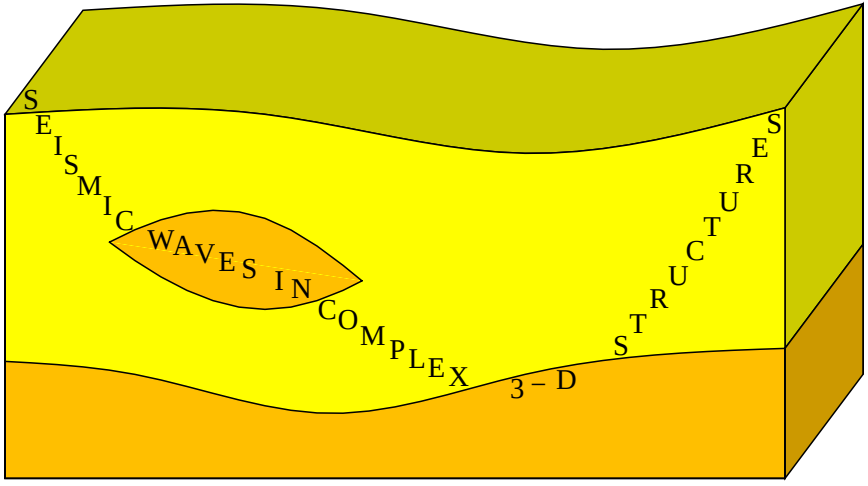

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