Paraxial Super-Gaussian beams

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Amplitude cross–section:

\[ \exp(-ax^N), \quad N = 4, 6, 8, \ldots \]

Plot of Gaussian function \( \exp(-x^2) \) and supergauss functions \( \exp(-x^4), \exp(-x^6), \exp(-x^8), \exp(-x^{10}) \) and \( \exp(-x^{12}) \).
Matrix

\[ Q_{ia} = \frac{\partial x_i}{\partial \gamma_a} \]

of geometrical spreading is real-valued.

Travel-time derivatives (Klimeš, 2002):

\[ \tau_{ij...n} = T_{ab...f} Q_{ai}^{-1} Q_{bj}^{-1} \cdots Q_{fn}^{-1} , \]

\[ T_{ab...f}(\gamma) = T_{ab...f}(\gamma^0) + \int_{\gamma^0}^{\gamma} d\gamma K_{ij...n} Q_{ia} Q_{jb} \cdots Q_{nf} . \]

The integration kernels \( K_{ij...n} \) corresponding to the \( N^{th} \)-order derivatives of travel time are real-valued:

\[ \text{Im}[T_{ab...f}(\gamma)] = \text{Im}[T_{ab...f}(\gamma^0)] . \]

The lowest–order (\( N^{th} \)-order) paraxial approximation of the imaginary part of the travel time is constant along all paraxial rays.
The Super–Gaussian beams are thus equivalent to the zero–order ray–
theory wavefield with the real–valued travel time and with the initial
Super–Gaussian amplitude profile, without the diffracted wavefield which
could result from the representation theorem.

The diffraction of a beam is thus satisfactorily included in the case of
paraxial Gaussian beams, but not in the case of paraxial Super–Gaussian
beams.
References (online at “http://sw3d.cz”)


Acknowledgements

The research has been supported:

by the Grant Agency of the Czech Republic under contract P210/10/0736,

by the Ministry of Education of the Czech Republic within research project MSM0021620860,

and by the consortium “Seismic Waves in Complex 3-D Structures”

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