Moveout approximation for P and SV waves in VTI and DTI media

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Outline

Introduction

Transverselly isotropic media

with vertical axis of symmetry (VTI)

Traveltime approximations for P and SV waves

Tests of the formulae

Dip-constrained transversely isotropic media (DTI)

Traveltime approximations for P and SV waves

Tests of the formulae

Conclusions

Introduction

Moveout approximations

- expansion of T^2 in terms of the squared offset

hyperbolic, non-hyperbolic, ...

- expansion of T^2 in terms of the deviations

of anisotropy from isotropy

VTI media

Unconverted P- or SV-wave

reflected from a plane reflector in a VTI layer

$$T^2(x) = (4H^2 + x^2)/v^2(\mathbf{n})$$

$T(x)$ - traveltime at the offset x

$v(\mathbf{n})$ - ray velocity

$\mathbf{n}$ - unit vector in the direction of the slowness vector

H - depth of the plane reflector

$\Rightarrow$ 2D problem in the (x_1, x_3) plane

VTI media

Normalized moveout formula

$$\bar{x} = x/2H \; , \qquad T_0 = 2H/V$$

$$T^2(\bar{x}) = V^2 T_0^2 \, (1 + \bar{x}^2)/v^2(\mathbf{n})$$

T_0 - two-way zero-offset traveltime

$\bar{x}$ - normalized offset

V - vertical velocity

P-wave ($V^2 = \alpha^2 = A_{33}$)

SV-wave ($V^2 = \beta^2 = A_{55}$)

$A_{\alpha\beta}$ - density-normalized elastic moduli in the Voigt notation

VTI media

Problem

Need to determine: $v^2(\mathbf{n})$

Available: $\mathbf{N}$, and for it, $c^2(\mathbf{N})$

$c(\mathbf{N})$ - phase velocity

$v(\mathbf{n})$ - ray velocity

$\mathbf{n}$ - unit vector in the direction of the slowness vector

$\mathbf{N}$ - unit vector in the direction of the ray-velocity vector $\mathbf{v}$

$\Rightarrow$ need to find the relation between $v^2(\mathbf{n})$ and $c^2(\mathbf{N})$

VTI media

Weak-anisotropy approximation

First-order approximation of $c^2(\mathbf{N})$

P wave $\quad c^2(\mathbf{N}) = \alpha^2[1 + 2(\delta_W - \epsilon_W)N_1^2 N_3^2 + 2\epsilon_W N_1^2]$

$$\epsilon_W = (A_{11} - \alpha^2)/2\alpha^2 \ , \quad \delta_W = (A_{13} + 2A_{55} - \alpha^2)/\alpha^2 \ , \quad \alpha^2 = A_{33}$$

SV wave $\quad c^2(\mathbf{N}) = \beta^2(1 + 2\sigma_W N_1^2 N_3^2)$

$$\sigma_W = r^{-2}(\epsilon_W - \delta_W) \ , \quad r = \beta/\alpha \ , \quad \beta^2 = A_{55}$$

VTI media

Specification of the ray direction

$\mathbf{N}$ - unit vector in the direction of the ray-velocity vector $\mathbf{v}(\mathbf{n})$

$$N_1 = \bar{x}/\sqrt{1+\bar{x}^2}\,, \qquad N_3 = 1/\sqrt{1+\bar{x}^2}$$

First-order relation between N and n

P wave: $$\mathbf{N}(\mathbf{n}) = \mathbf{n} + 2c^{-2}(\mathbf{n})B(\mathbf{n})\mathbf{e}(\mathbf{n})$$

$$B(\mathbf{n}) = \alpha^2 n_1 n_3[\delta_W - 2(\delta_W - \epsilon_W)n_1^2]$$

SV wave: $$\mathbf{N}(\mathbf{n}) = \mathbf{n} + 2c^{-2}(\mathbf{n})E(\mathbf{n})\mathbf{e}(\mathbf{n})$$

$$E(\mathbf{n}) = \beta^2 \sigma_W n_1 n_3 (n_3^2 - n_1^2)$$

$\mathbf{e}$ - unit vector, $\mathbf{e} \perp \mathbf{n}$ in the (x_1, x_3) plane

VTI media

First-order relations between $v^2(\mathbf{n})$ and $c^2(\mathbf{N})$

P wave: $$v^2(\mathbf{n}) = c^2(\mathbf{N}) - 4c^{-2}(\mathbf{N})B^2(\mathbf{N})$$

$$B(\mathbf{N}) = B(\mathbf{n}) = \alpha^2 n_1 n_3[\delta_W - 2(\delta_W - \epsilon_W)n_1^2]$$

SV wave: $$v^2(\mathbf{n}) = c^2(\mathbf{N}) - 4c^{-2}(\mathbf{N})E^2(\mathbf{N})$$

$$E(\mathbf{N}) = E(\mathbf{n}) = \beta^2 \sigma_W n_1 n_3 (n_3^2 - n_1^2)$$

Traveltime approximations for P waves

a) Assumption $\mathbf{n} = \mathbf{N} \Rightarrow v^2(\mathbf{n}) = c^2(\mathbf{N})$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3/P(\bar{x})$$

b) Assumption $\mathbf{n} \neq \mathbf{N} \Rightarrow v^2(\mathbf{n}) \neq c^2(\mathbf{N})$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3 P(\bar{x})/[P^2(\bar{x}) - Q^2(\bar{x})]$$

c) Assumption $\mathbf{n} \neq \mathbf{N}$ and second-order $v^2(\mathbf{N})$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3 P(\bar{x})/[P^2(\bar{x}) + aQ^2(\bar{x})]$$

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\delta_W \bar{x}^2 + 2\epsilon_W \bar{x}^4 \;, \qquad Q(\bar{x}) = 2\bar{x}[2\epsilon_W \bar{x}^2 + \delta_W(1-\bar{x}^2)]$$

$$a = (r^2 - 3/4)/(1-r^2)\;, \qquad r = \beta/\alpha$$

Traveltime approximations for SV waves

a) Assumption $\mathbf{n} = \mathbf{N} \Rightarrow v^2(\mathbf{n}) = c^2(\mathbf{N})$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3/P(\bar{x})$$

b) Assumption $\mathbf{n} \neq \mathbf{N} \Rightarrow v^2(\mathbf{n}) \neq c^2(\mathbf{N})$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3 P(\bar{x})/[P^2(\bar{x}) - Q^2(\bar{x})]$$

c) Assumption $\mathbf{n} \neq \mathbf{N}$ **and second-order** $v^2(\mathbf{N})$

$$T^2(\bar{x}) = T_0^2(1+\bar{x}^2)^3 P(\bar{x})/[P^2(\bar{x}) - Q^2(\bar{x}) - (1-r^2)^{-1}R^2(\bar{x})]$$

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\sigma_W \bar{x}^2 \ , \qquad Q(\bar{x}) = 2\sigma_W \bar{x}(1-\bar{x}^2)$$

$$R(\bar{x}) = r^{-1}\bar{x}[2\epsilon_W \bar{x}^2 + \delta_W(1-\bar{x}^2)] \ , \qquad r = \beta/\alpha$$

Traveltime approximations for P waves

Reference moveout formula

Long-spread moveout approximation (Tsvankin, 2001)

$$T^2(\bar{x}) = T_0^2[1 + R_\delta \bar{x}^2 - 2\eta R_\delta^2 \bar{x}^4/(1 + S R_\delta \bar{x}^2)]$$

$$R_\delta = (1 + 2\delta)^{-1} \ , \qquad S = R_\delta(1 + 2\epsilon) \ , \qquad \eta = R_\delta(\epsilon - \delta)$$

ϵ , δ - Thomsen's parameters

Traveltime approximations for SV waves

Reference moveout formula

Rational approximation (Stovas, 2010)

$$T^2(\bar{x}) = T_0^2[1 + R_\sigma \bar{x}^2 + AR_\sigma^2 \bar{x}^4/(1 + BR_\sigma \bar{x}^2)]$$

$$A = 2\sigma B \quad , \qquad B = R_\sigma^2(1 - r^2 + 2\delta)/(1 - r^2)$$

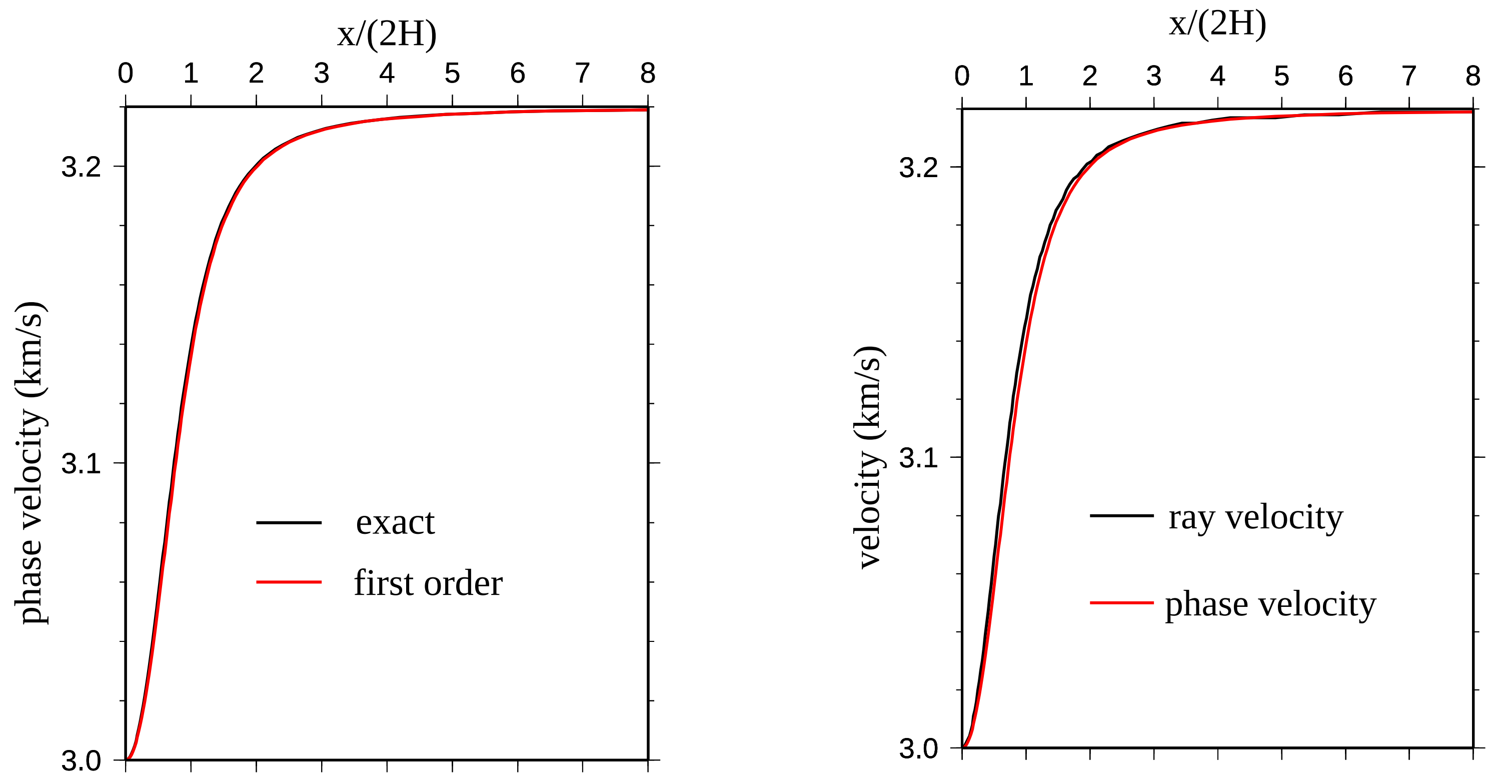
$$R_\sigma = (1 + 2\sigma)^{-1} \quad , \qquad \sigma = r^{-2}(\epsilon - \delta) \quad , \qquad r = \beta/\alpha$$

ϵ , δ - Thomsen's parameters

Tests of the formulae

P wave, limestone (anisotropy $\sim$ 8%)

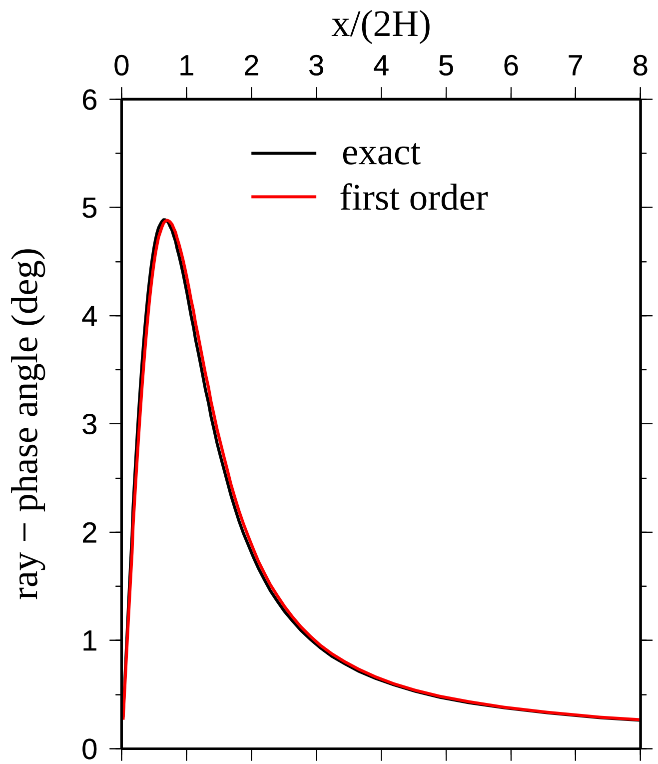
α=3.0 km/s, β=1.707 km/s, ϵ_W=0.076, δ_W=0.133

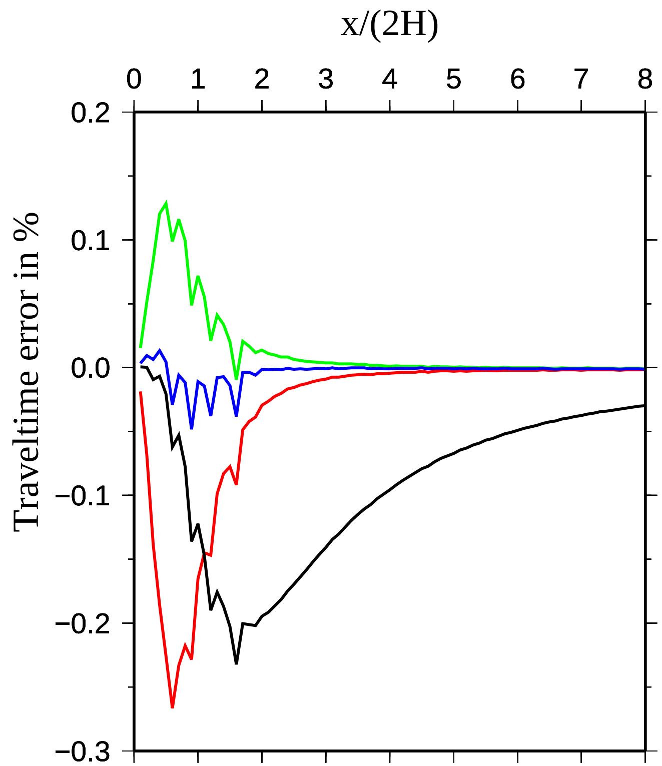


Tests of the formulae

P wave, limestone (anisotropy $\sim$ 8%)

1st ord., N = n, **1st ord., N $\neq$ n**, **2nd ord.**, long-spread approx.

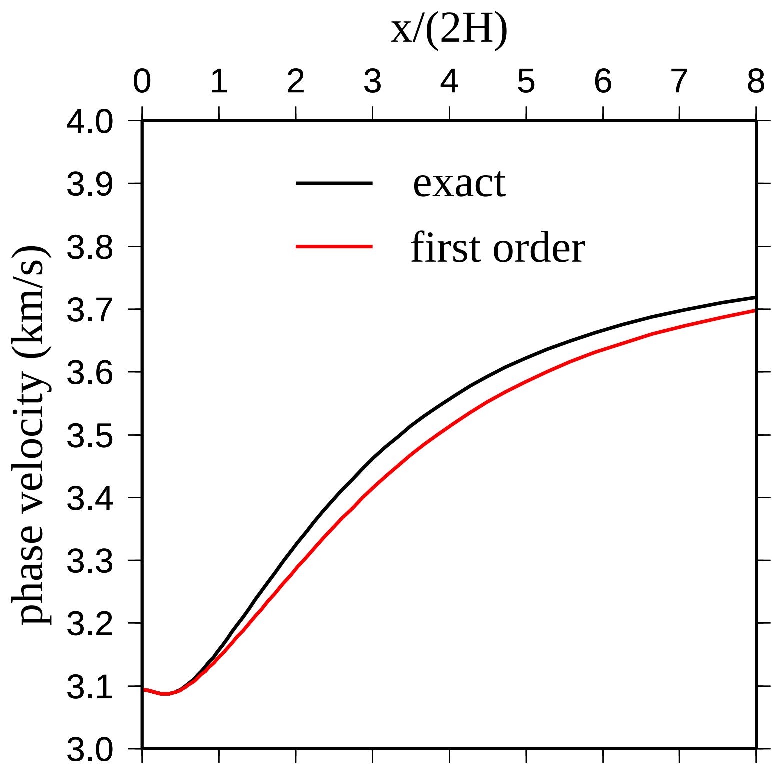


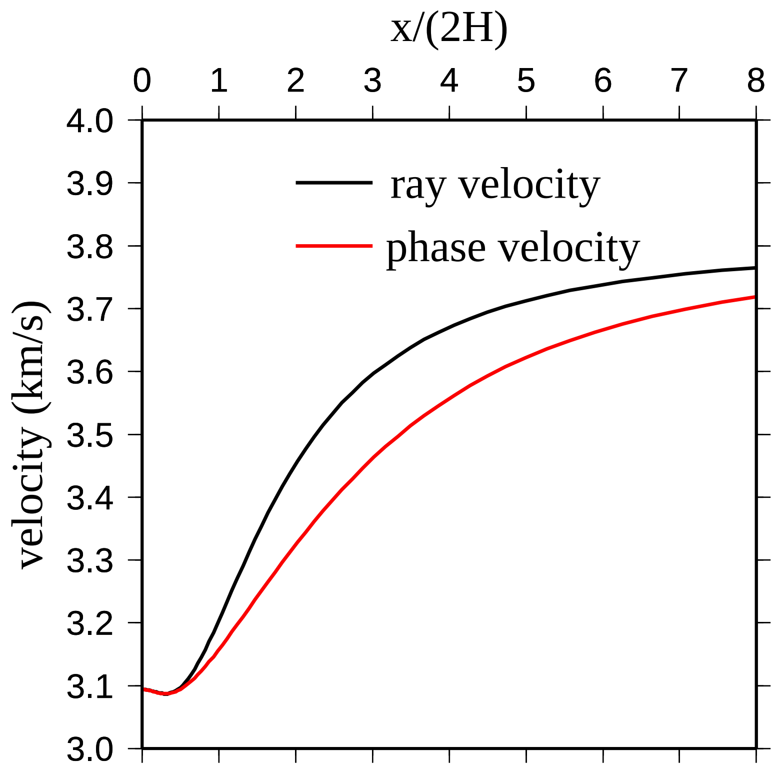


Tests of the formulae

P wave, Greenhorn shale (anisotropy $\sim$ 26%)

α=3.094 km/s, β=1.51 km/s, ϵ_W=0.256, δ_W=-0.0523





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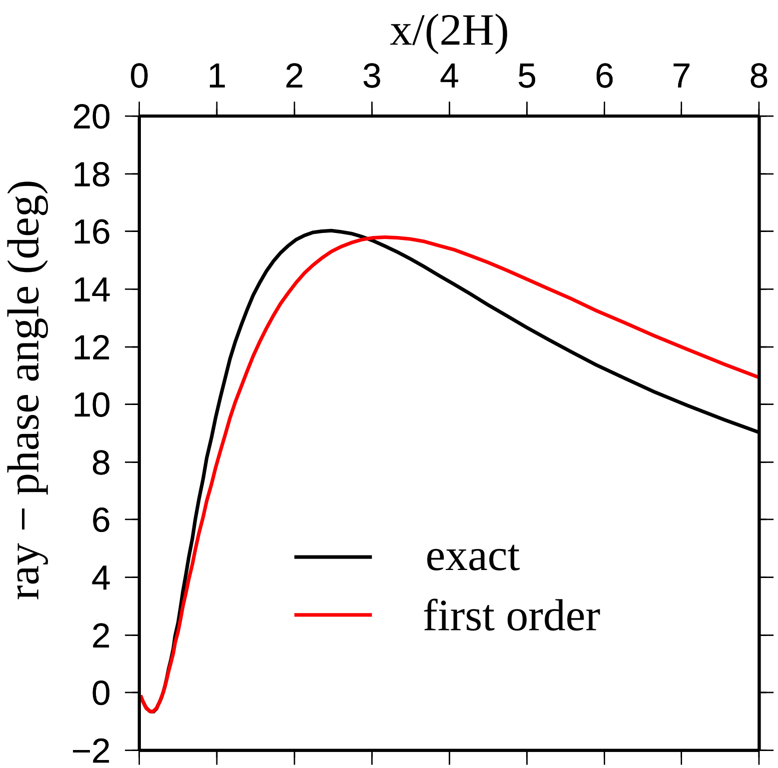
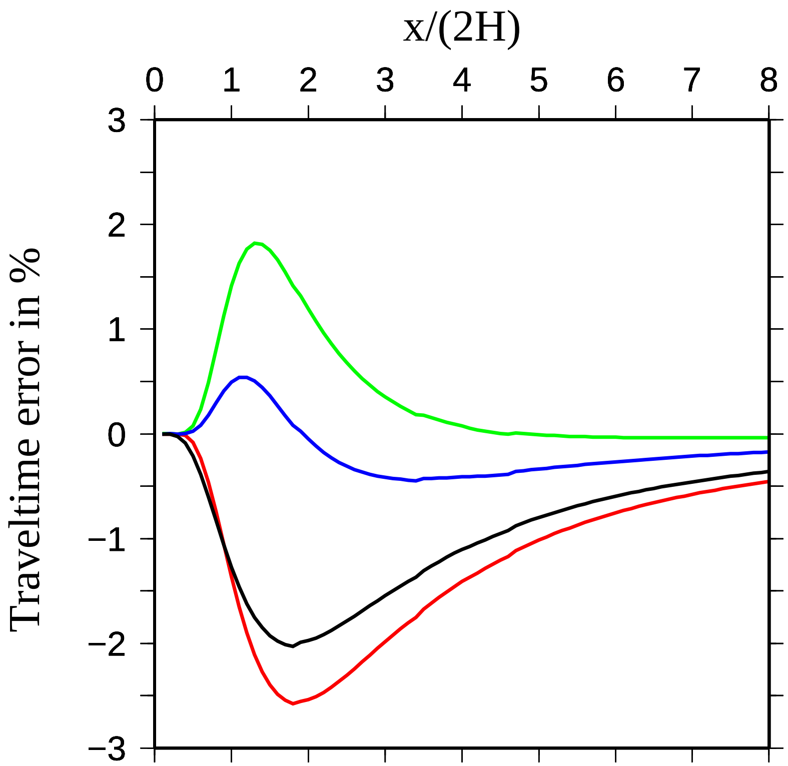
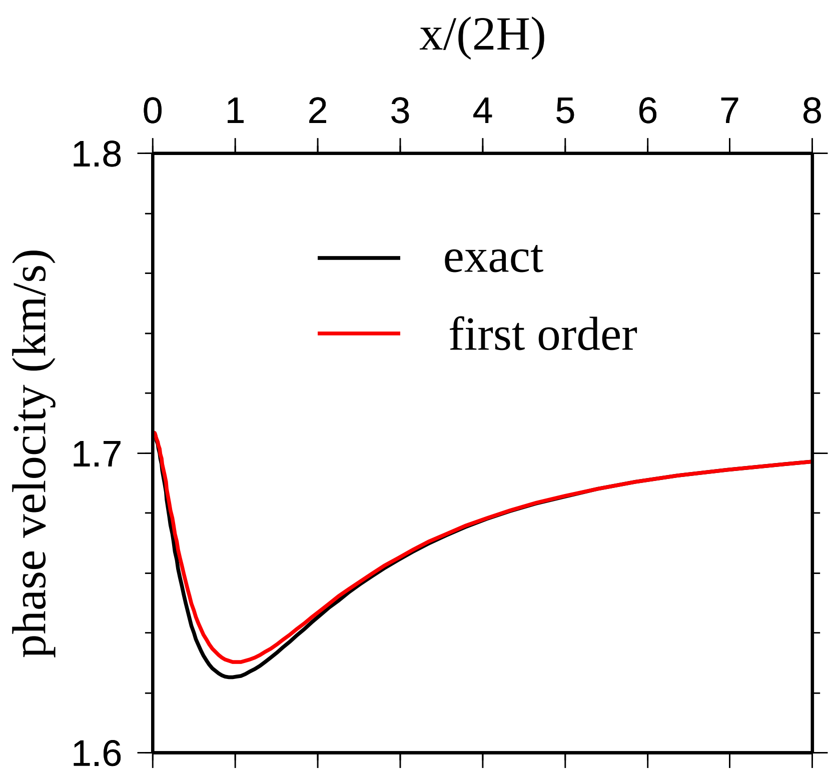


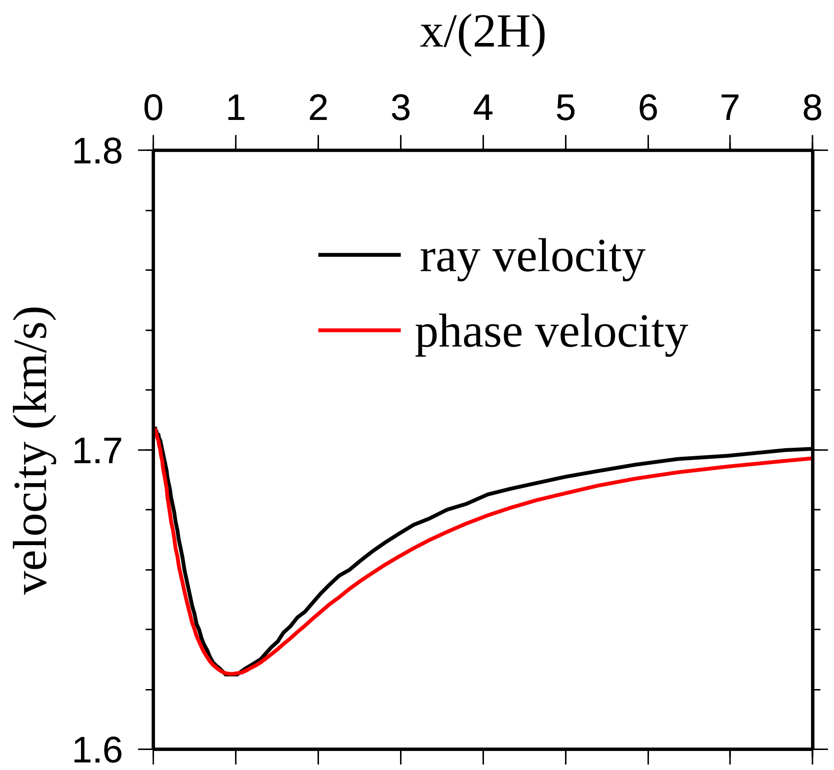


Tests of the formulae

SV wave, limestone (anisotropy $\sim$ 5%)

α=3.0 km/s, β=1.707 km/s, ϵ_W=0.076, δ_W=0.133

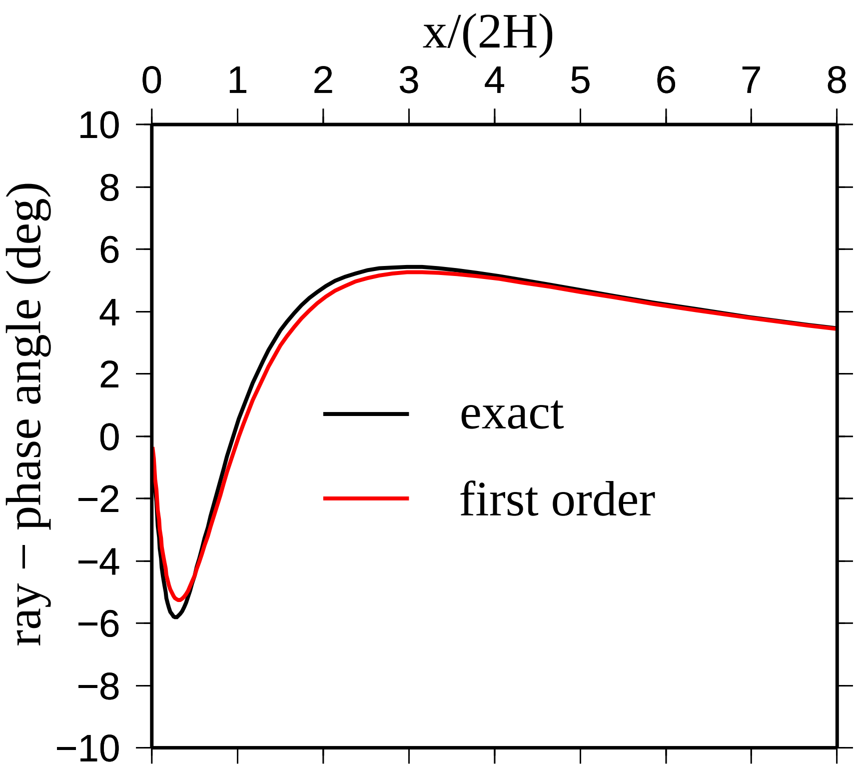


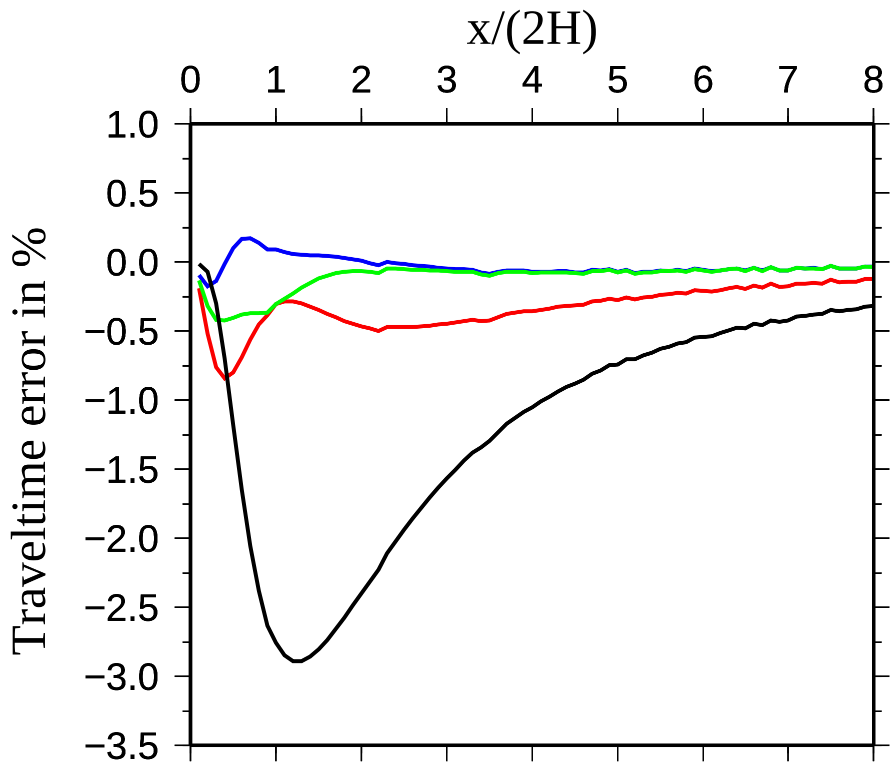


Tests of the formulae

SV wave, limestone (anisotropy $\sim$ 5%)

1st ord., $\mathbf{N = n}$, **1st ord., $\mathbf{N \neq n}$**, **2nd ord.**, rational approx.

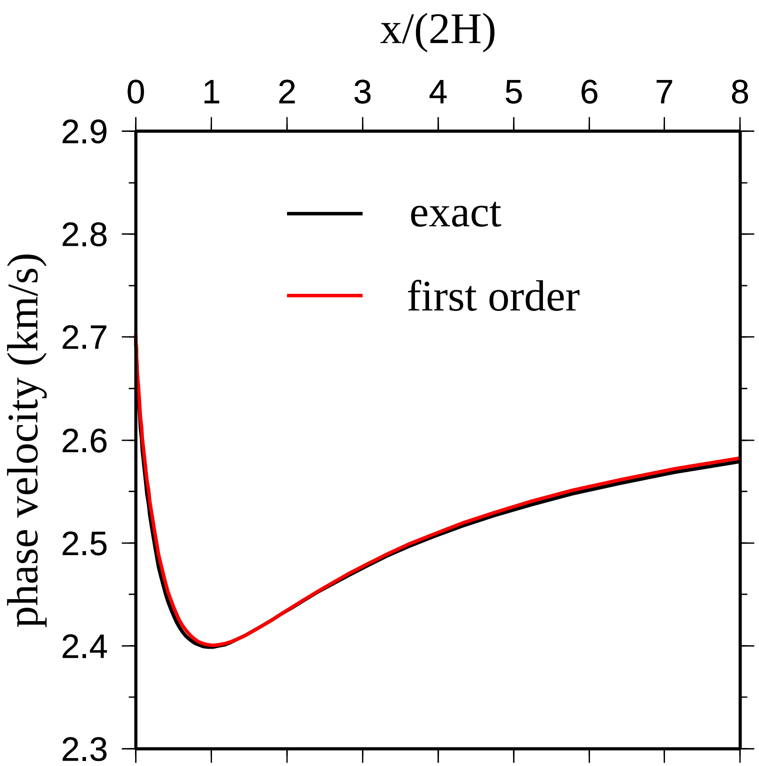


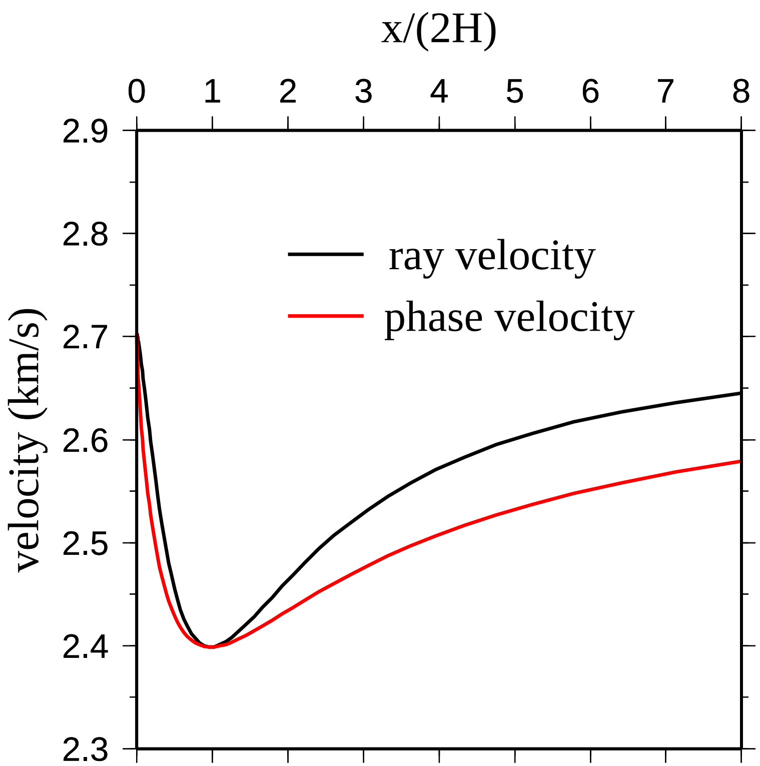


Tests of the formulae

SV wave, Mesaverde mudshale (anisotropy $\sim$ 12%)

α=4.53 km/s, β=2.703 km/s, ϵ_W=0.034, δ_W=0.184





Tests of the formulae

SV wave, Mesaverde mudshale (anisotropy $\sim$ 12%)

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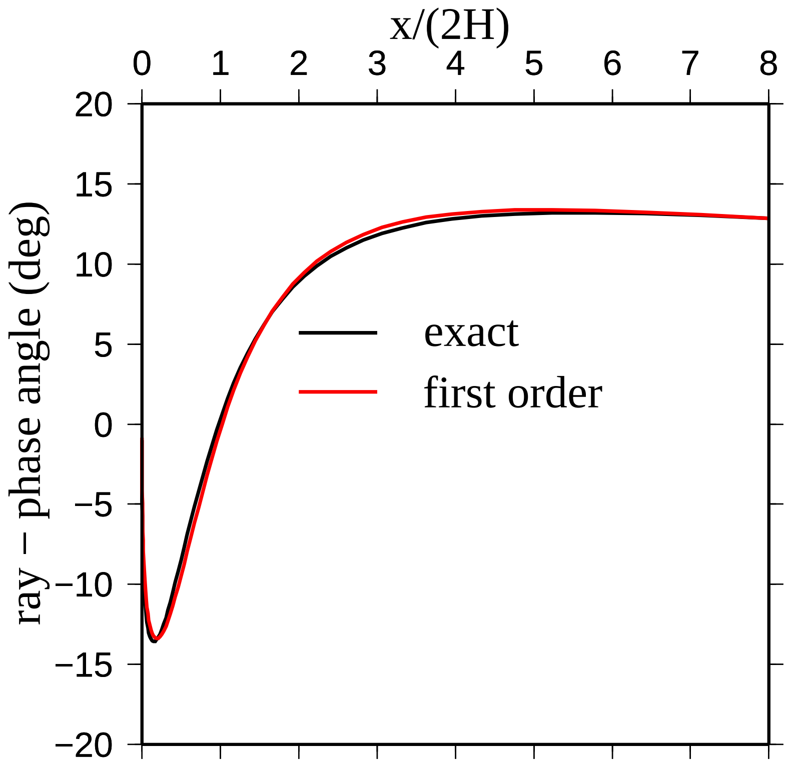
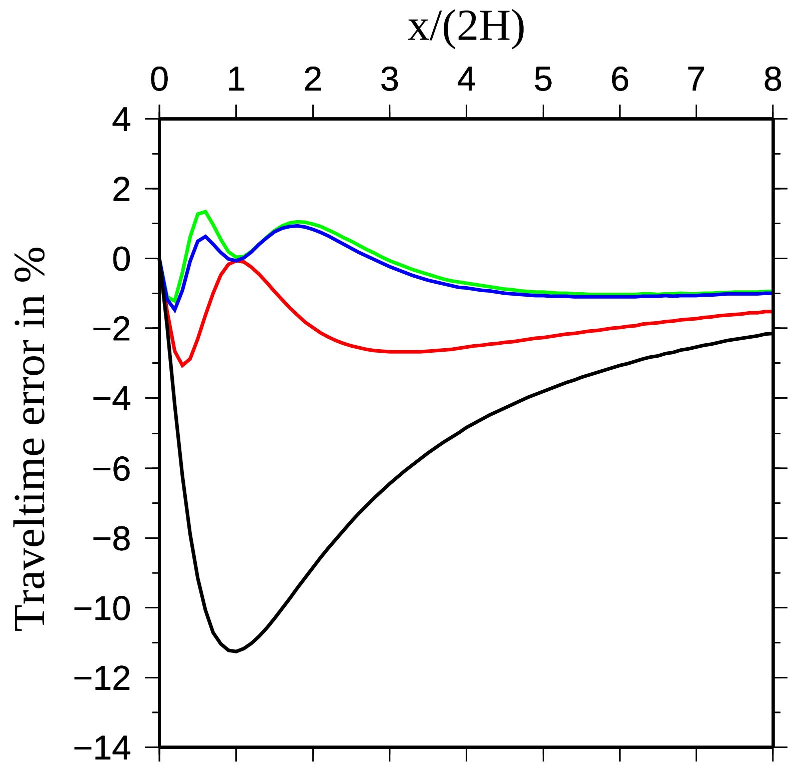
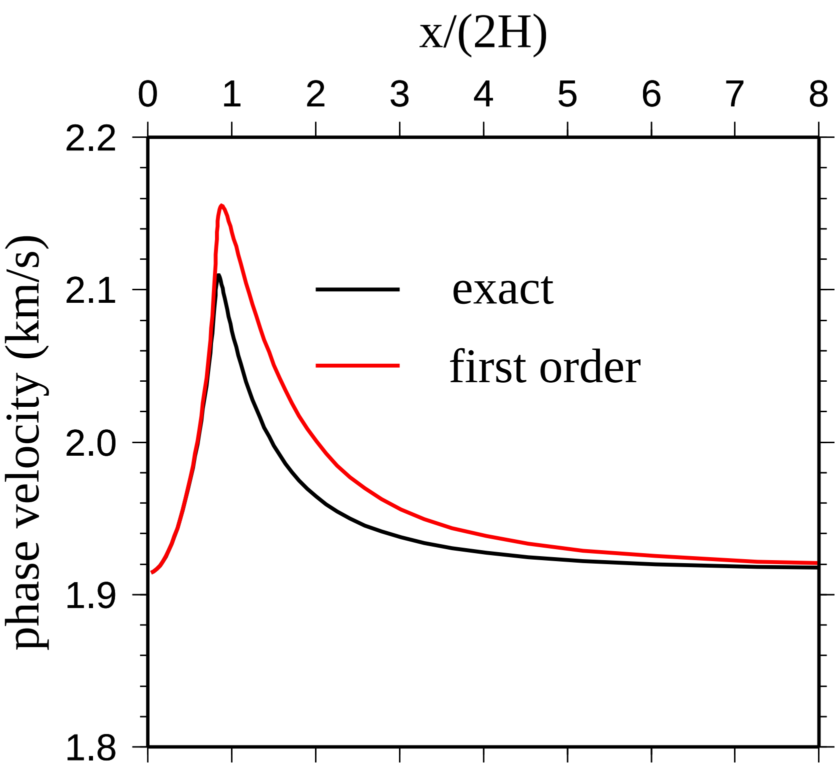


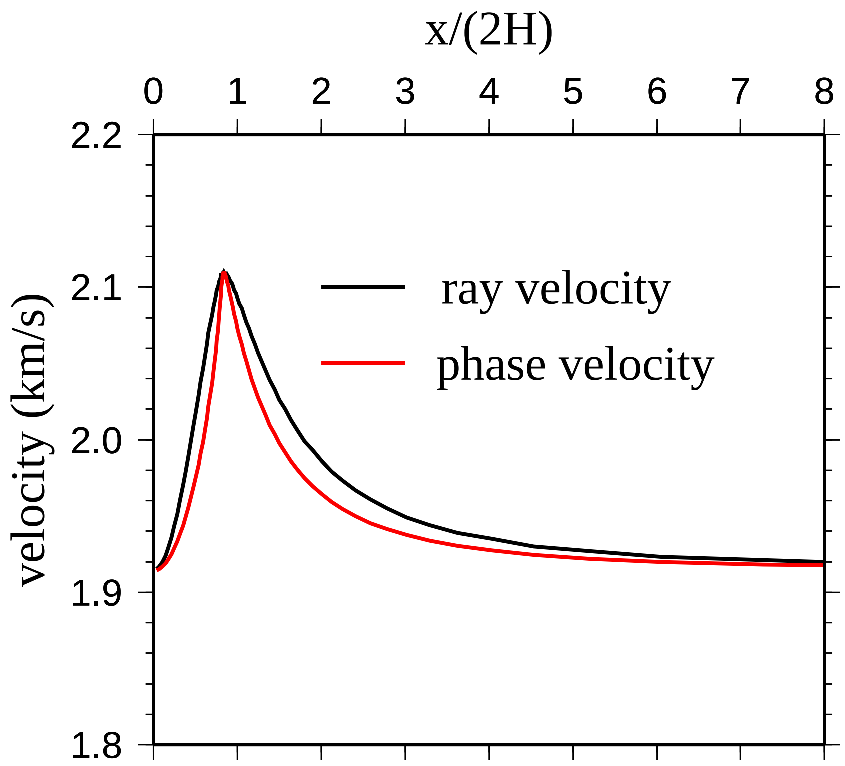


Tests of the formulae

SV wave, hard shale (anisotropy $\sim$ 12%)

α=3.0 km/s, β=1.914 km/s, ϵ_W=0.252, δ_W=0.034





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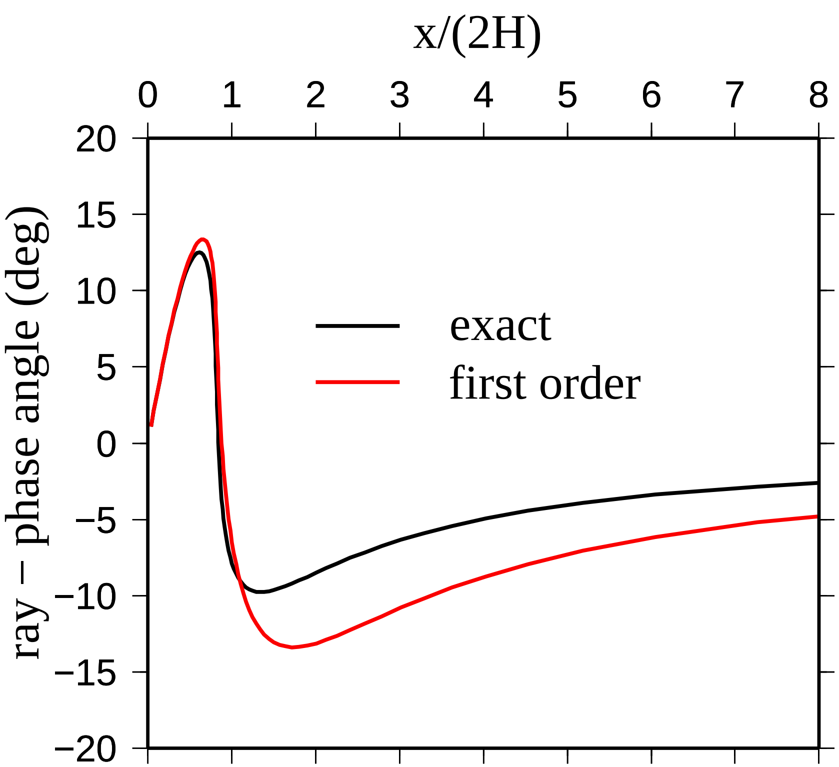
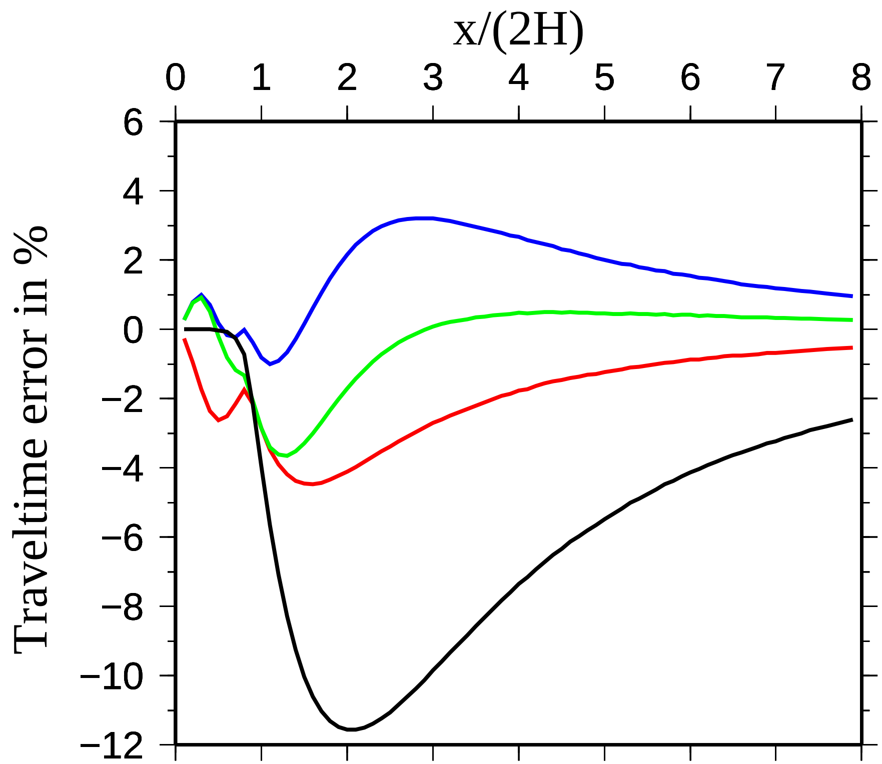
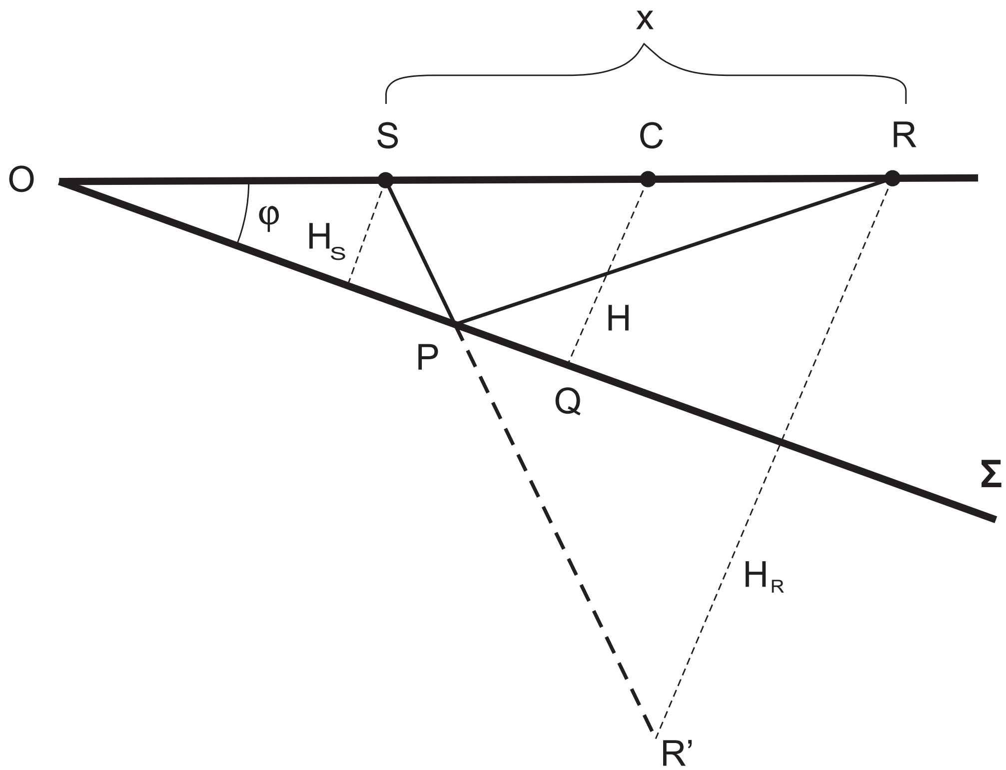




DTI media

Ray (SPR) of an unconverted P- or SV-wave

reflected from a dipping plane reflector Σ in a DTI layer



DTI media

Σ_a - plane defined by the source-receiver line and the normal to the reflector

$$T^2(x) = (4H^2 + x^2 \cos^2 \varphi)/v^2(\mathbf{n})$$

$T(x)$ - traveltime at the offset x

$v(\mathbf{n})$ - ray velocity

φ - apparent dip

$\mathbf{n}$ - unit vector in the direction of the slowness vector

H - orthogonal distance of the common midpoint to the reflector Σ

$\Rightarrow$ 2D problem in the Σ_a plane

DTI media

Normalized moveout formula

$$\bar{x} = x/2H \ , \qquad T_0 = 2H/V$$

$$T^2(\bar{x}) = V^2 T_0^2 \, (1 + \bar{x}^2 \cos^2 \varphi)/v^2(\mathbf{n})$$

T_0 - two-way zero-offset traveltime at the common midpoint

$\bar{x}$ - normalized offset

V - phase velocity along the symmetry axis

P-wave ($V^2 = \alpha^2 = A_{33}$)

SV-wave ($V^2 = \beta^2 = A_{55}$)

$A_{\alpha\beta}$ - density-normalized elastic moduli in *local coordinates*

DTI media

Transformation of VTI formulae into DTI formulae

$\bar{x} \Rightarrow \bar{x}\cos\varphi$

Finite offsets: $\bar{x} < 1/\sin\varphi$

Transformation from the plane Σ_a to 3D

$\bar{x}$, H and φ determined from a 3D specification

of the SR line, the reflector Σ and actual dip

(SR line cannot be chosen perpendicular to Σ)

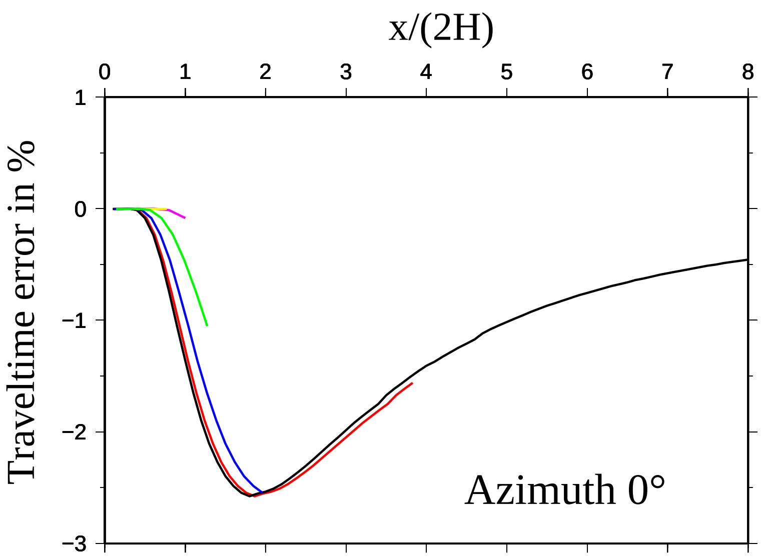
Tests of the formulae

P wave, Greenhorn shale (anisotropy $\sim$ 26%)

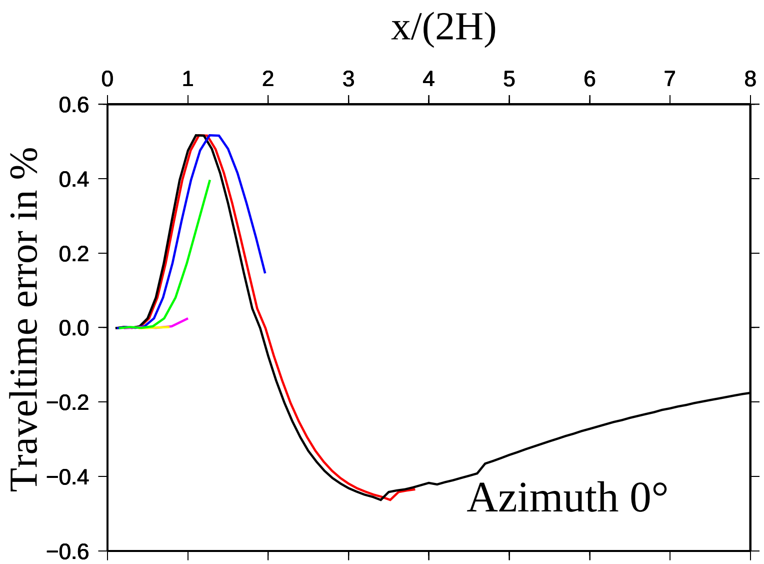
α=3.094 km/s, β=1.51 km/s, ϵ_W=0.256, δ_W=-0.0523

Actual dip: 0^o, **15^o**, **30^o**, **45^o**, **60^o**, **75^o**

First-order (assumption $\mathbf{N} = \mathbf{n}$)



Second-order



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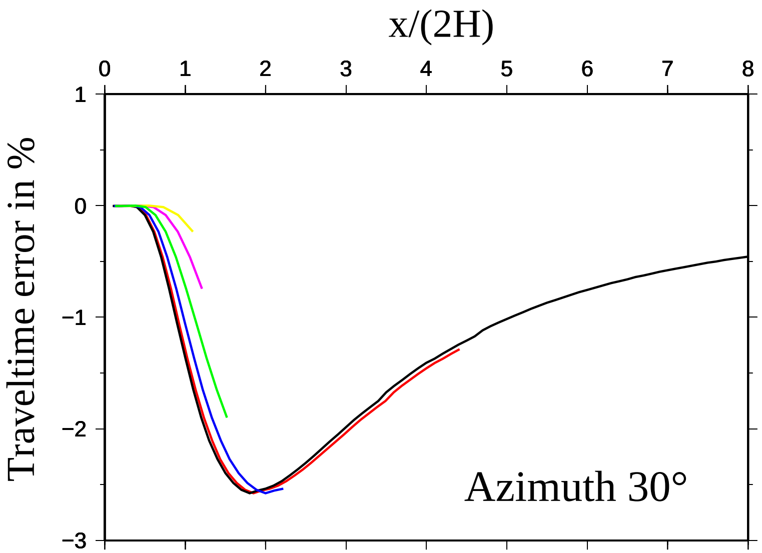
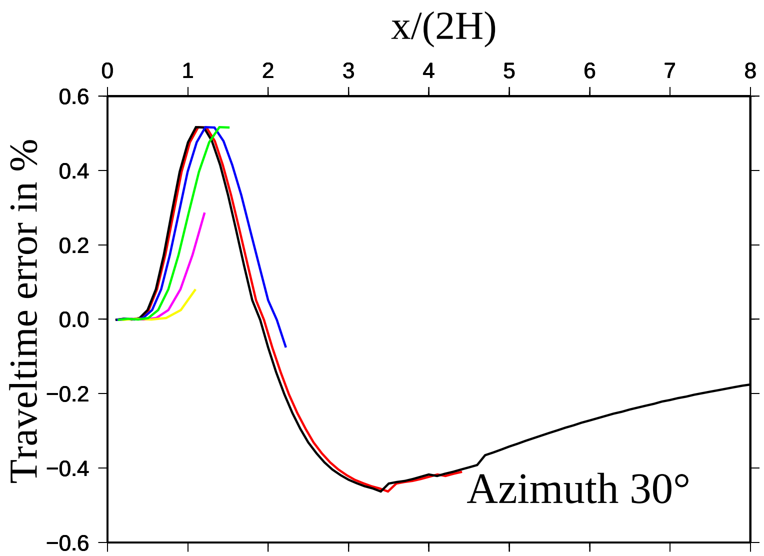


Second-order



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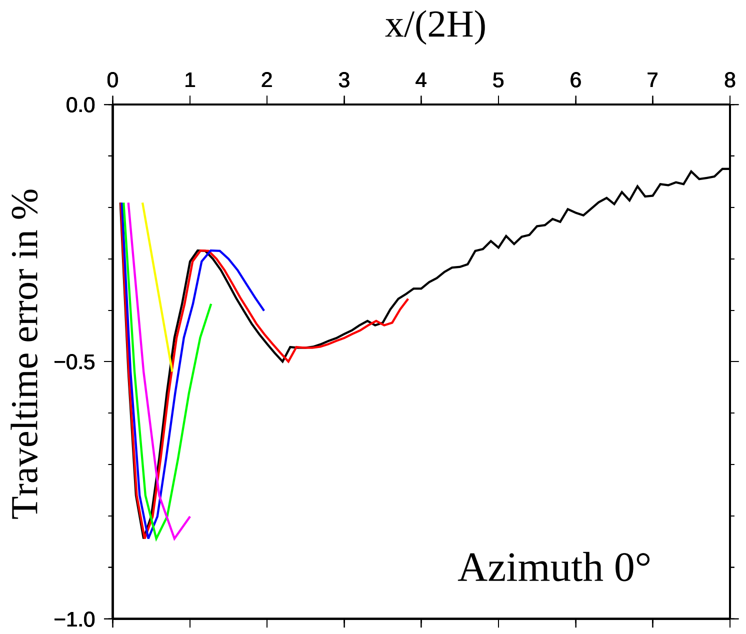
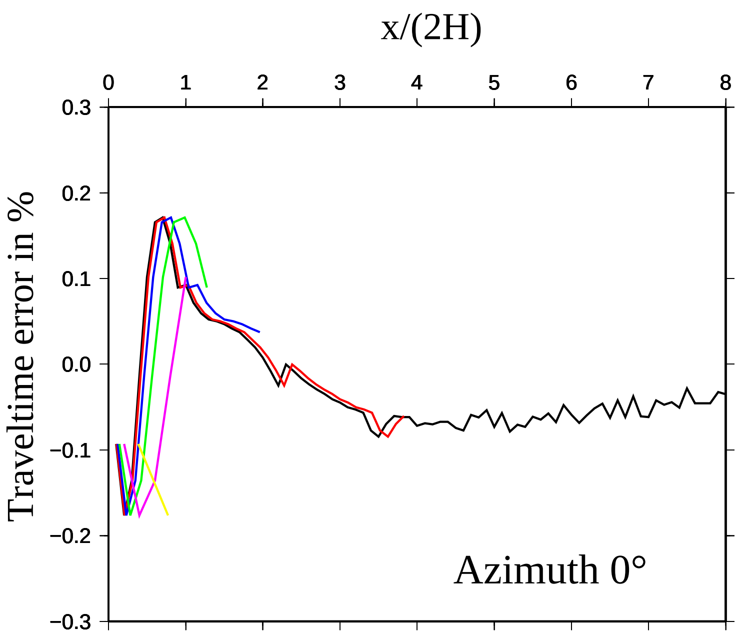


Second-order



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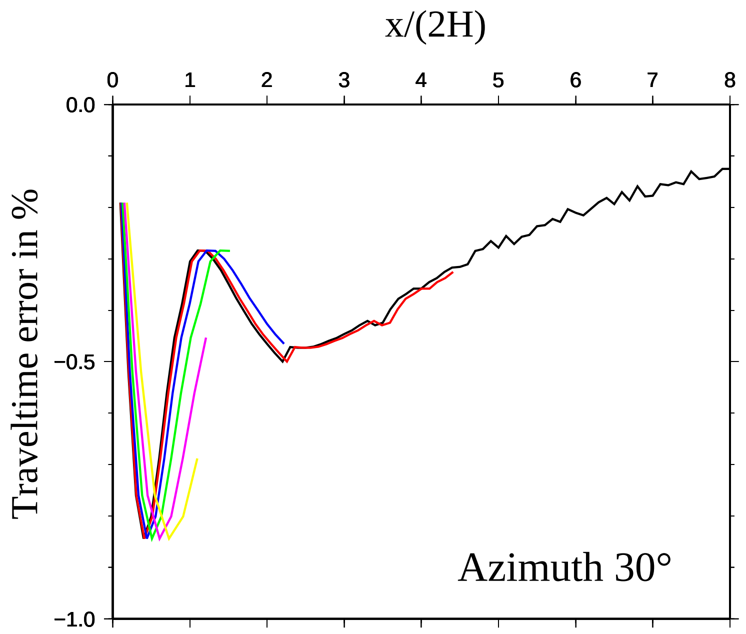
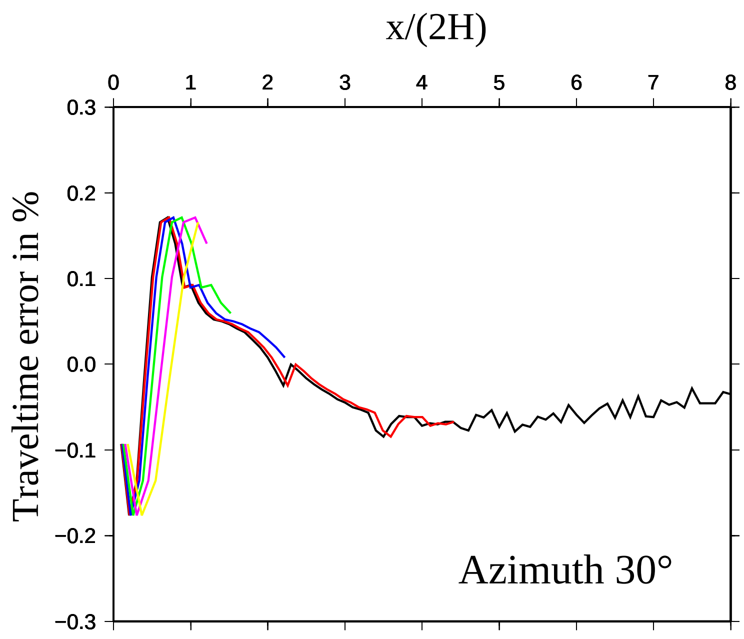


Second-order



Conclusions

- based on WA approximation
- no non-physical assumptions
- relatively simple formulae
- inaccuracies for large deviations of $\mathbf{n}$ and $\mathbf{N}$
- for small and large offsets accurate
- second-order formulae very accurate
- P waves: dependence on H, α, ϵ_W, δ_W (2nd-order also on r)
- SV waves: dependence on H, β, σ_W (2nd-order also on ϵ_W and r)
- dependence of relative traveltime errors on dip in DTI media small
- byproduct: simple expressions for NMO velocities

Generalizations

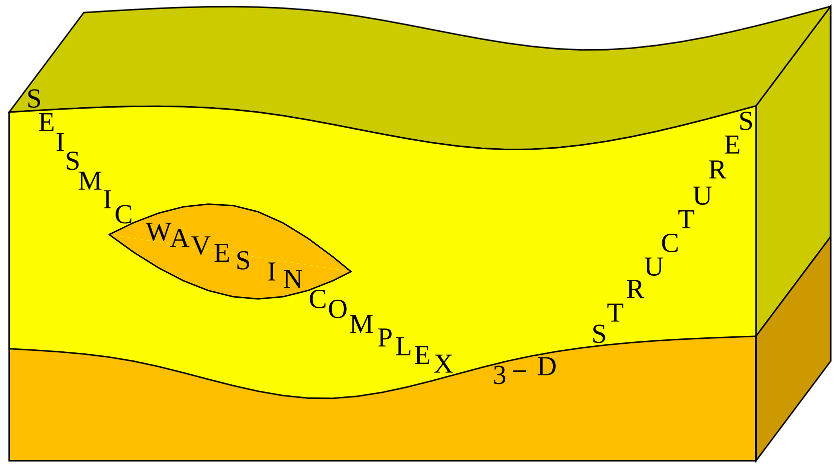
Straightforward

- TI media with symmetry axis parallel to Σ
  and reflected ray in the symmetry plane
- orthorhombic media with Σ and reflected ray,
  each in a symmetry plane

Possible

- monoclinic media with Σ in the symmetry plane
- TI media with axis of symmetry neither perpendicular
  nor parallel to Σ
- converted waves
- anisotropic media of lower symmetry

Acknowledgements



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