Two-point paraxial traveltimes

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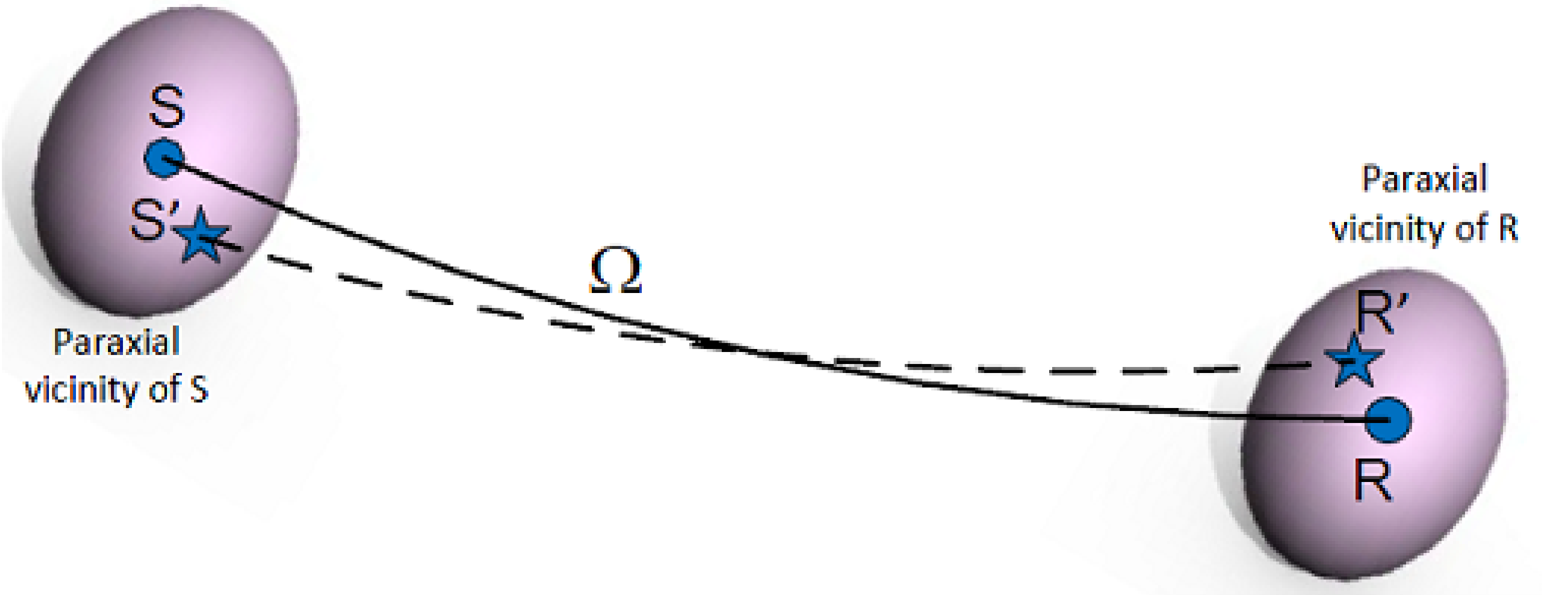
Conclusions

Introduction

Two-point traveltime $T(R, S)$ between S and R known

Two-point paraxial traveltime $T(R', S')$ between S' and R' to be determined



Introduction

Ray-centred coordinate system q_i

non-orthogonal, curvilinear, fixed origin

coordinate line $q_3 \Leftrightarrow$ the reference ray Ω

q_1, q_2 - Cartesian coordinates in the plane $\perp$ slowness vector $\mathbf{p}$

Wavefront orthonormal coordinate system y_i

local Cartesian, moving origin

y_i - Cartesian coordinates, $y_1 \equiv q_1, \; y_2 \equiv q_2, \; \mathbf{y_3} \parallel \mathbf{p}$

Two-point paraxial traveltime formula DRT in ray-centred coordinates

$$T(R', S') = T(R, S) + \delta x_i^R p_i(R) - \delta x_i^S p_i(S)$$

$$-\delta x_i^S A_{ij}(R, S)\delta x_j^R + \tfrac{1}{2}\delta x_i^R M_{ij}(R, S)\delta x_j^R + \tfrac{1}{2}\delta x_i^S N_{ij}(R, S)\delta x_j^S$$

$$\delta x_i^R = x_i(R') - x_i(R)\ , \qquad \delta x_i^S = x_i(S') - x_i(S)$$

$$A_{ij}(R, S) = f_{Mi}^S(\mathbf{Q}_2^{(q)-1})_{MN} f_{Nj}^R$$

$$M_{ij}(R, S) = f_{Mi}^R(\mathbf{P}_2^{(q)}\mathbf{Q}_2^{(q)-1})_{MN} f_{Nj}^R + (p_i\eta_j + p_j\eta_i - p_i p_j \mathcal{U}_k \eta_k)_R$$

$$N_{ij}(R, S) = f_{Mi}^S(\mathbf{Q}_2^{(q)-1}\mathbf{Q}_1^{(q)})_{MN} f_{Nj}^S - (p_i\eta_j + p_j\eta_i - p_i p_j \mathcal{U}_k \eta_k)_S$$

Two-point paraxial traveltime formula DRT in ray-centred coordinates

x_i - Cartesian coordinates

Quantities obtained from tracing the reference ray Ω

p_i - components of the slowness vector $\mathbf{p}$

$\boldsymbol{\mathcal{U}} = d\mathbf{x}/d\tau$ - ray-velocity vector

$\boldsymbol{\eta} = d\mathbf{p}/d\tau$ - $\boldsymbol{\eta}$-vector

Two-point paraxial traveltime formula DRT in ray-centred coordinates

Vectors $\mathbf{f}_I$: $(\mathbf{f}_1, \mathbf{f}_2) \perp \Omega$

not necessarily unit and mutually perpendicular

$$\mathbf{f}_1 = \mathcal{C}^{-1}(\mathbf{e}_2 \times \boldsymbol{\mathcal{U}}) \ , \qquad \mathbf{f}_2 = \mathcal{C}^{-1}(\boldsymbol{\mathcal{U}} \times \mathbf{e}_1)$$

$\mathcal{C}$ - phase velocity

Vectors $\mathbf{e}_I$: $(\mathbf{e}_1, \mathbf{e}_2) \perp \mathbf{p}$

unit and mutually perpendicular

$$\mathrm{d}\mathbf{e}_I/\mathrm{d}\tau = -(\mathbf{e}_I \cdot \boldsymbol{\eta})\mathbf{p}/(\mathbf{p} \cdot \mathbf{p})$$

Two-point paraxial traveltime formula
DRT in ray-centred coordinates

$\mathbf{Q}_1^{(q)}$, $\mathbf{Q}_2^{(q)}$, $\mathbf{P}_2^{(q)}$ - 2×2 submatrices

of the 4×4 ray propagator matrix $\mathbf{\Pi}^{(q)}$

$$\mathbf{\Pi}^{(q)}(R,S) = \mathbf{\Pi}^{(q)}(\tau,\tau_0) = \begin{pmatrix} \mathbf{Q}_1^{(q)}(R,S) & \mathbf{Q}_2^{(q)}(R,S) \\ \mathbf{P}_1^{(q)}(R,S) & \mathbf{P}_2^{(q)}(R,S) \end{pmatrix}$$

$\mathbf{\Pi}^{(q)}$ - solution of the dynamic ray tracing in ray-centred coordinates

$$\mathrm{d}\mathbf{\Pi}^{(q)}(\tau,\tau_0)/\mathrm{d}\tau = \mathbf{S}(\tau)\mathbf{\Pi}^{(q)}(\tau,\tau_0)$$

$\mathbf{S}$ - 4×4 DRT system matrix

Initial conditions: $\mathbf{\Pi}^{(q)}(\tau_0,\tau_0) = \mathbf{I}$; $\mathbf{I}$ - 4×4 identity matrix

Two-point paraxial traveltime formula DRT in wavefront orthonormal coordinates

$$T(R', S') = T(R, S) + y_3(R')\mathcal{C}^{-1}(R) - y_3(S')\mathcal{C}^{-1}(S)$$

$$-y_i(S')A^{(y)}_{ij}(R, S)y_j(R') + \tfrac{1}{2}y_i(R')M^{(y)}_{ij}(R, S)y_j(R') + \tfrac{1}{2}y_i(S')N^{(y)}_{ij}(R, S)y_j(S')$$

$$A^{(y)}_{IJ}(R, S) = (\mathbf{Q}^{(y)-1}_2)_{IJ} = (\mathbf{Q}^{(q)-1}_2)_{IJ}$$

$$M^{(y)}_{IJ}(R, S) = (\mathbf{P}^{(y)}_2\mathbf{Q}^{(q)-1}_2)_{IJ} = (\mathbf{P}^{(q)}_2\mathbf{Q}^{(q)-1}_2)_{IJ}$$

$$N^{(y)}_{IJ}(R, S) = (\mathbf{Q}^{(y)-1}_2\mathbf{Q}^{(q)}_1)_{IJ} = (\mathbf{Q}^{(q)-1}_2\mathbf{Q}^{(q)}_1)_{IJ}$$

$$\mathbf{\Pi}^{(y)}(R, S) = \mathbf{\Pi}^{(q)}(R, S)$$

Two-point paraxial traveltime formula DRT in wavefront orthonormal coordinates

3×3 matrices $\mathbf{M}^{(y)}$ and $\mathbf{N}^{(y)}$ are symmetric

3×3 matrix $\mathbf{A}^{(y)}$ is non-symmetric

3rd columns and 3rd lines depend

on RT quantities $\mathcal{C}$, $\mathcal{U}_I^{(y)}$, $\eta_I^{(y)}$ at S and R

on DRT quantities $\mathbf{Q}_1^{(y)}(R,S)$, $\mathbf{Q}_2^{(y)}(R,S)$ and $\mathbf{P}_2^{(y)}(R,S)$

complete expressions given in the Report

Two-point paraxial traveltime formula

Two-point paraxial traveltime formulae using DRT

in ray-centred and wavefront orthonormal coordinates

are equivalent

Conclusions

- single ray approach
- S' and R' arbitrarily chosen in paraxial vicinities of S and R
- knowledge of the 4×4 ray propagator matrix sufficient
- the 4×4 ray propagator matrix $\mathbf{\Pi}^{(q)}(R, S)$ in ray-centred coordinates or $\mathbf{\Pi}^{(y)}(R, S)$ in wavefront orthonormal coordinates can be used
- $\mathbf{\Pi}^{(q)}(R, S) = \mathbf{\Pi}^{(y)}(R, S)$
- efficient, no problems with redundant DRT solutions
- formulae for $T(R', S')$ in both coordinates equivalent
- formulae for $T^2(R', S')$ available
- paraxial slowness vectors at R' and S'

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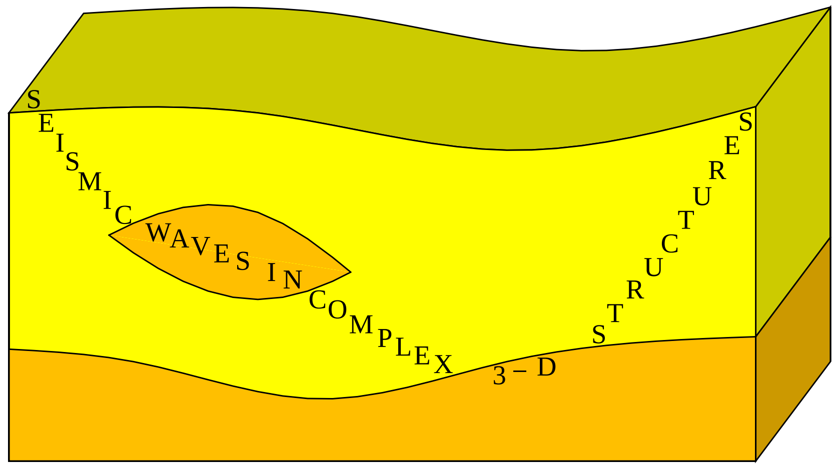

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