Single-frequency approximation of the coupling ray theory

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Idea

Anisotropic-ray-theory Green tensor:

\[ G_{ij}^{\text{ART}}(\mathbf{x}, \mathbf{x}_0, \omega) = \sum_{K=1}^{2} A^{(K)}(\mathbf{x}, \mathbf{x}_0) g_i^{(K)}(\mathbf{x}) g_j^{(K)}(\mathbf{x}_0) \exp[i\omega\tau^{(K)}(\mathbf{x}, \mathbf{x}_0)] \]

Coupling-ray-theory Green tensor:

\[ G_{ij}^{\text{CRT}}(\mathbf{x}, \mathbf{x}_0, \omega) \]

Single-frequency approximation of the coupling-ray-theory Green tensor:

\[ G_{ij}^{\text{CRT}}(\mathbf{x}, \mathbf{x}_0, \omega) \approx G_{ij}^{\text{SF}}(\mathbf{x}, \mathbf{x}_0, \omega) \]

Single-frequency Green tensor:

\[ G_{ij}^{\text{SF}}(\mathbf{x}, \mathbf{x}_0, \omega) = \sum_{K=1}^{2} A^{(K)}(\mathbf{x}, \mathbf{x}_0) e_i^{(K)}(\mathbf{x}, \omega_0) e_j^{(K)}(\mathbf{x}_0, \omega_0) \exp[i\omega T^{(K)}(\mathbf{x}, \mathbf{x}_0, \omega_0)] \]

\( \omega_0 \) ... prevailing circular frequency.

\( T^{(K)}(\mathbf{x}, \mathbf{x}_0, \omega_0) \) ... coupling-ray-theory travel times.

\( e_i^{(K)}(\mathbf{x}, \omega_0), e_j^{(K)}(\mathbf{x}_0, \omega_0) \) ... coupling-ray-theory polarization vectors.
Single-frequency Green tensor — conditions:

\[ G_{ij}^{SF}(x, x_0, \omega_0) = G_{ij}^{CRT}(x, x_0, \omega_0) \]

\[ \frac{\partial G_{ij}^{SF}}{\partial \omega}(x, x_0, \omega_0) = \frac{\partial G_{ij}^{CRT}}{\partial \omega}(x, x_0, \omega_0) \]

These conditions uniquely determine coupling-ray-theory travel times \( T^{(K)}(x, x_0, \omega_0) \) and coupling-ray-theory polarization vectors \( e_i^{(K)}(x, \omega_0) \) and \( e_j^{(K)}(x_0, \omega_0) \).

We numerically calculate \( G_{ij}^{CRT}(x, x_0, \omega_0) \) using the algorithm by Bulant & Klimeš (2002).

We calculate \( \frac{\partial G_{ij}^{CRT}}{\partial \omega}(x, x_0, \omega_0) \) using the derivative of this algorithm.
Examples of singularities at the slowness surface

Along the reference ray, the slowness vector smoothly rotates according to Hamilton’s equations of rays, and thus moves along the slowness surface. Let us see how the slowness vector moves along the slowness surface.
INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE
INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE

ANISOTROPIC RAY

NO CHANGE OF POLARIZATION
SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE
SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE

ANISOTROPIC RAY

RAPID CHANGE OF POLARIZATION
SPLIT INTERSECTION SINGULARITY AT THE SLOWNESS SURFACE

COUPLING RAY

NO CHANGE OF POLARIZATION
S-WAVE CONVERSION
VICINITY OF CONICAL SINGULARITY AT THE SLOWNESS SURFACE
VICINITY OF CONICAL SINGULARITY AT THE SLOWNESS SURFACE

COUPLING RAY

NO CHANGE OF POLARIZATION
S-WAVE CONVERSION
Numerical comparison of seismograms

Single-frequency approximation.
Standard coupling ray theory.
Fourier pseudospectral method.

Source-receiver configuration:
Velocity model QIH, transverse (top) and vertical (bottom) component. Single-frequency approximation, standard coupling ray theory.
Velocity model QI, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.
Velocity model QI2, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.
Velocity model QI4, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.
Velocity model SC1-I, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.
Velocity model KISS, transverse (top) and vertical (bottom) component.

Single-frequency, coupling ray theory, Fourier pseudospectral method.
Velocity model ORT, transverse (top) and vertical (bottom) component. Single-frequency, coupling ray theory, Fourier pseudospectral method.
Conclusions

The single-frequency approximation of the coupling ray theory allows us to process the coupling-ray-theory wavefield in the same way as the anisotropic-ray-theory wavefield.

The single-frequency approximation of the coupling ray theory can thus be included in wavefront tracing and in the interpolation within ray cells in anisotropic media.

This will enable common-source Kirchhoff prestack depth migration with coupling-ray-theory $S$ waves.

We would like to further improve the accuracy of the coupling ray theory using the first-order perturbation of polarization vectors.
References:


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