Gaussian beams in inhomogeneous layered anisotropic media (DRT in Cartesian coordinates)

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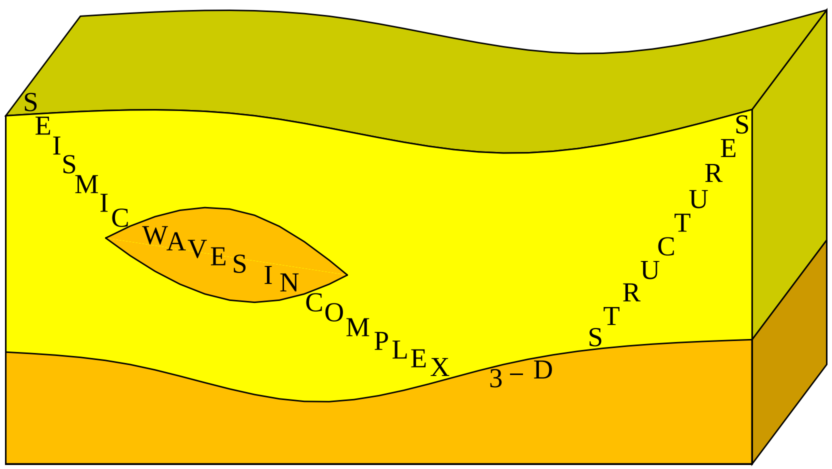

Outline:

Introduction

Dynamic ray tracing

Transformation relation

Paraxial approximation

Gaussian beams

Conclusions

Introduction

Gaussian beams - solutions of elastodyn. equation concentrated to ray

exact solutions $\Rightarrow$ Gaussian beams

approximate solutions $\Rightarrow$ paraxial Gaussian beams

Gaussian beams in isotropic media - broad applications

Gaussian beams in anisotropic media - Hanyga, 1986; Alkhalifah, 1995

ray tracing (RT) $\Rightarrow$ central ray Ω

dynamic ray tracing (DRT) $\Rightarrow$ amplitudes,

second derivatives of traveltime along Ω

Dynamic ray tracing

DRT in ray-centred coordinates (last year)

DRT in Cartesian coordinates (now)

DRT in global Cartesian coordinates x_i

DRT in local Cartesian (WOC) coordinates y_i

simplified DRT in global Cartesian coordinates x_i

Solution of DRT

using ray propagator matrix along Ω

- plane-wave and point-source initial conditions

specified initial conditions - real-valued $\times$ complex-valued

DRT in Cartesian coordinates

6 linear ordinary differential equations of first order

simple structure of ordinary differential equations

ray propagator matrix $6 \times 6 \quad \Rightarrow \quad$ solution of DRT $6\times$

evaluation of GB at any point in a vicinity of a point on Ω

simplified DRT - ray propagator matrix 4×4

$\Rightarrow \quad$ solution of DRT $4\times+$ transformation

DRT in ray-centred coordinates

4 linear ordinary differential equations of first order

more complicated structure of ordinary differential equations

ray propagator matrix $4 \times 4 \quad \Rightarrow \quad$ solution of DRT $4\times$

evaluation of GB only in plane tangent to wavefront

at its intersection with Ω

Transformation of 2nd traveltime derivatives

$$\mathbf{M}^{(x)} = \mathbf{f}\ \mathbf{M}^{(y)}\ \mathbf{f}^T + \mathbf{p}\boldsymbol{\eta}^T + \boldsymbol{\eta}\mathbf{p}^T - \mathbf{p}\mathbf{p}^T(\boldsymbol{\mathcal{U}}^T\boldsymbol{\eta})$$

$\mathbf{M}^{(x)}$ - 6 $\times$6 matrix with $\partial^2 T/\partial x_i \partial x_j$

$\mathbf{M}^{(y)}$ - 4 $\times$4 matrix with $\partial^2 T/\partial y_N \partial y_M$

$\mathbf{p}$ - slowness vector

$\boldsymbol{\mathcal{U}}$ - ray-velocity vector

$\boldsymbol{\eta}$ - eta vector

$\mathbf{f} = (\mathbf{f}_1, \mathbf{f}_2)$ - $\mathbf{f}_K$ vectors perpendicular to ray Ω

$\mathbf{p}$, $\boldsymbol{\mathcal{U}}$, $\boldsymbol{\eta}$ obtained from tracing ray Ω

$\mathbf{f}_K$ - covariant basis vectors in ray-centred coordinates

(require solution of one vectorial differential equation along Ω)

Paraxial approximation of displacement vector (WOC)

Initial conditions: 2×2 **real-valued** matrix $\mathbf{M}^y(\tau_0)$

$$\mathbf{u}(\mathbf{y},\tau) = \mathbf{U}^{\Omega}(\tau)[\det \mathbf{Q}^y(\tau_0)/ \det \mathbf{Q}^y(\tau)]^{1/2} \ \exp[-\mathrm{i}\omega(t - T(\tau) - \tfrac{1}{2}\mathbf{y}^T\mathbf{M}^y(\tau)\mathbf{y})]$$

$\mathbf{u}(\mathbf{y},\tau)$ - displacement vector

$\mathbf{U}^{\Omega}(\tau)$ - spreading-free vectorial amplitude

$\det \mathbf{Q}^y(\tau_0)/ \det \mathbf{Q}^y(\tau)$, $\mathbf{M}^y(\tau)$ - **real valued**,

obtained from ray propagator matrix and $\mathbf{M}^y(\tau_0)$ specified at $\tau = \tau_0$

T - traveltime

Gaussian beams (WOC)

Initial conditions: 2×2 **complex-valued** matrix $\mathbf{M}^y(\tau_0)$

Existence conditions:

$\mathbf{M}^y(\tau_0)$ is symmetric and finite, $\mathrm{Im}\mathbf{M}^y(\tau_0)$ is positive definite

$$\mathbf{u}(\mathbf{y},\tau) = \mathbf{U}^{\Omega}(\tau)[\det \mathbf{Q}^y(\tau_0)/\det \mathbf{Q}^y(\tau)]^{1/2} \exp[-\tfrac{1}{2}\omega \mathbf{y}^T \mathrm{Im}\mathbf{M}^y(\tau)\mathbf{y}]$$
$$\times \exp[-\mathrm{i}\omega(t - T(\tau) - \tfrac{1}{2}\mathbf{q}^T \mathrm{Re}\mathbf{M}^y(\tau)\mathbf{q})]$$

$\mathbf{U}^{\Omega}(\tau)$ - spreading-free vectorial amplitude

$\det \mathbf{Q}^y(\tau_0)/\det \mathbf{Q}^y(\tau)$, $\mathbf{M}^y(\tau)$ - **complex valued**

obtained from ray propagator matrix and $\mathbf{M}^y(\tau_0)$ specified at $\tau = \tau_0$

T - traveltime

Conclusions

- various options for constructing GB in anisotropic media
- efficiency to be studied
- use of ray propagator matrix $\Rightarrow$ great flexibility
- advantageous to use $\mathbf{M}^{(x)}$ at termination point of GB
- possible simplifications in specific cases