Sensitivity of electromagnetic waves to a heterogeneous bianisotropic structure

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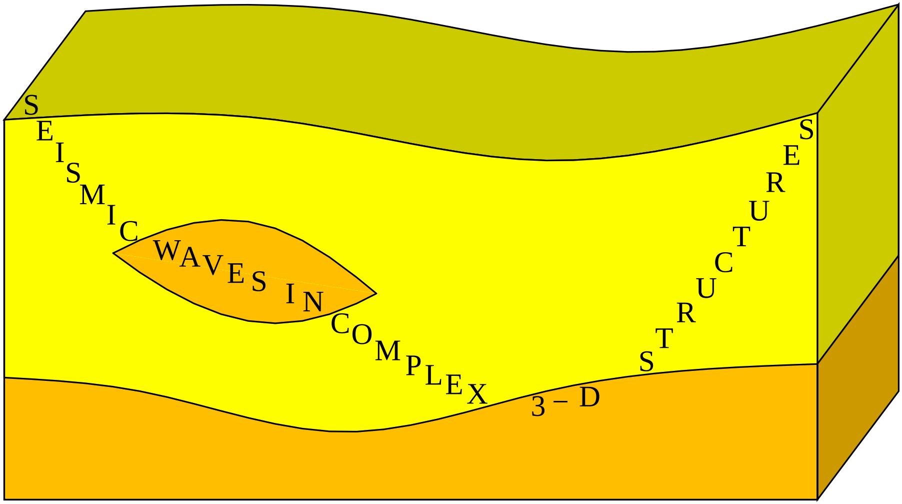

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Notation

Einstein summation. Indices $i, j, ... = 1, 2, 3$; $\alpha, \beta, ... = 1, 2, 3, 4$.

Maxwell equations

System of 8 first-order partial differential equations for 12 quantities:
E_i ... electric field
B^i ... magnetic induction
D^i ... electric displacement
H_i ... magnetic field

System of 6 constitutive equations

Tellegen representation:

$$D^i = D^i(E_m, H_n)$$
$$B^i = B^i(E_m, H_n)$$

Boys-Post representation:

$$D^i = D^i(E_m, B^n)$$
$$H_i = H_i(E_m, B^n)$$

Special cases of the constitutive equations in the Boys-Post representation

Linear isotropic medium:

$$D^i = \varepsilon E_i$$
$$H_i = \mu^{-1} B^i$$

ε ... permitivity

μ^{-1} ... inverse permeability

Linear isotropic chiral medium:

$$D^i = \varepsilon E_i + \alpha B^i$$
$$H_i = \alpha E_i + \mu^{-1} B^i$$

α ... chirality parameter

Linear anisotropic medium:

$$D^i = \varepsilon^{ij} E_j$$
$$H_i = \mu^{-1} B^i$$

Counterpart of elastic anisotropy:

$$D^i = \varepsilon E_i$$
$$H_i = \mu_{ij}^{-1} B^j$$

Linear bianisotropic medium

$$D^i = \varepsilon^{ij} E_j + \alpha^i{}_j B^j$$
$$H_i = \beta_i{}^j E_j + \mu_{ij}^{-1} B^j$$

$\alpha^i{}_j$, $\beta_i{}^j$ ... magnetoelectric matrices

Vector potential

We may express 6 components E_i and B^i in terms of 6 skew combinations $A_{\alpha,\beta} - A_{\beta,\alpha}$ of the derivatives of the components of covariant 4-vector potential A_α.

Then 4 Maxwell equations for E_i and B^i are identically satisfied.

Remaining 4 Maxwell equations:
for $D^i = D^i(A_{\alpha,\beta} - A_{\beta,\alpha})$ and $H_i = H^i(A_{\alpha,\beta} - A_{\beta,\alpha})$.

In this case, the Boys-Post representation is superior to the Tellegen representation.

Aharonov-Bohm experiment

Electrons propagate around a solenoid through a region where $E_i = 0$ and $B^i = 0$, but $A_\alpha \neq 0$.

Interference of electrons depends on A_α.

The electromagnetic field cannot be completely described by E_i and B^i.
The electromagnetic field is better described by A_α.

Electromagnetic wave equation

Maxwell equations for a linear bianisotropic medium in the Boys-Post representation:

$$[\chi^{\alpha\beta\gamma\delta}(x^\mu)\, A_{\gamma,\delta}(x^\nu)]_{,\beta} + J^\alpha(x^\mu) = 0 \quad . \tag{2}$$

Constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ is a contravariant tensor density of weight -1. Current density J^α is a contravariant 4-vector density of weight -1.

The constitutive tensor is skew with respect to its first and second indices, and with respect to its third and fourth indices:

$$\chi^{\alpha\beta\gamma\delta} = -\chi^{\beta\alpha\gamma\delta} = -\chi^{\alpha\beta\delta\gamma} \quad . \tag{3}$$

Components of the constitutive tensor

Constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ has 36 independent components represented by 36 constitutive parameters ε^{ij}, $\alpha^{i}{}_{j}$, $\beta_{i}{}^{j}$, μ^{-1}_{ij}:

$$\begin{pmatrix} \chi^{1414} & \chi^{1424} & \chi^{1434} \\ \chi^{2414} & \chi^{2424} & \chi^{2434} \\ \chi^{3414} & \chi^{3424} & \chi^{3434} \end{pmatrix} = -\begin{pmatrix} \varepsilon^{11} & \varepsilon^{12} & \varepsilon^{13} \\ \varepsilon^{21} & \varepsilon^{22} & \varepsilon^{23} \\ \varepsilon^{31} & \varepsilon^{32} & \varepsilon^{33} \end{pmatrix}$$

$$\begin{pmatrix} \chi^{1423} & \chi^{1431} & \chi^{1412} \\ \chi^{2423} & \chi^{2431} & \chi^{2412} \\ \chi^{3423} & \chi^{3431} & \chi^{3412} \end{pmatrix} = -\begin{pmatrix} \alpha^{1}{}_{1} & \alpha^{1}{}_{2} & \alpha^{1}{}_{3} \\ \alpha^{2}{}_{1} & \alpha^{2}{}_{2} & \alpha^{2}{}_{3} \\ \alpha^{3}{}_{1} & \alpha^{3}{}_{2} & \alpha^{3}{}_{3} \end{pmatrix}$$

$$\begin{pmatrix} \chi^{2314} & \chi^{2324} & \chi^{2334} \\ \chi^{3114} & \chi^{3124} & \chi^{3134} \\ \chi^{1214} & \chi^{1224} & \chi^{1234} \end{pmatrix} = \begin{pmatrix} \beta_{1}{}^{1} & \beta_{1}{}^{2} & \beta_{1}{}^{3} \\ \beta_{2}{}^{1} & \beta_{2}{}^{2} & \beta_{2}{}^{3} \\ \beta_{3}{}^{1} & \beta_{3}{}^{2} & \beta_{3}{}^{3} \end{pmatrix}$$

$$\begin{pmatrix} \chi^{2323} & \chi^{2331} & \chi^{2312} \\ \chi^{3123} & \chi^{3131} & \chi^{3112} \\ \chi^{1223} & \chi^{1231} & \chi^{1212} \end{pmatrix} = \begin{pmatrix} \mu^{-1}_{11} & \mu^{-1}_{12} & \mu^{-1}_{13} \\ \mu^{-1}_{21} & \mu^{-1}_{22} & \mu^{-1}_{23} \\ \mu^{-1}_{31} & \mu^{-1}_{32} & \mu^{-1}_{33} \end{pmatrix}$$

Our additional assumptions about the constitutive tensor

In order to simplify the application of the ray-theory approximation, we are assuming here that the constitutive tensor is real-valued, and is symmetric with respect to its first and second pairs of indices:

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\gamma\delta\alpha\beta} \quad . \tag{4}$$

For the sake of simplicity, we are also assuming here that the structure is time-independent:

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\alpha\beta\gamma\delta}(x^m) \quad . \tag{5}$$

Gabor representation of medium perturbations

We consider infinitesimally small perturbations $\delta\chi^{\alpha\beta\gamma\delta}$ of the constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ in the electromagnetic wave equation.

We decompose the perturbations of the constitutive tensor into Gabor functions $g^{\Omega}(x^m)$ indexed here by Ω:

$$\delta\chi^{\alpha\beta\gamma\delta}(x^m) = \sum_{\Omega} \chi_{\Omega}^{\alpha\beta\gamma\delta}\, g^{\Omega}(x^m) \quad , \tag{7}$$

$$g^{\Omega}(x^m) = \exp\left[\mathrm{i}\, k_i^{\Omega}\,(x^i - x_{\Omega}^i) - \tfrac{1}{2}(x^i - x_{\Omega}^i)\, K_{ij}^{\Omega}\,(x^j - x_{\Omega}^j)\right] \quad . \tag{8}$$

Gabor functions $g^{\Omega}(x^m)$ are centred at various spatial positions x_{Ω}^i and have various structural wavenumber vectors k_i^{Ω}.

The wavefield scattered by the perturbations is then composed of waves $A_{\alpha}^{\Omega}(x^{\mu})$ scattered by the individual Gabor functions,

$$\delta A_{\alpha}(x^{\mu}) = \sum_{\Omega} a_{\alpha}^{\Omega}(x^{\mu}) \quad . \tag{9}$$

Applied approximations

Short-duration broad-band wavefield with a smooth frequency spectrum incident at the Gabor function, expressed in terms of the amplitude and travel time.

First-order Born approximation of each wave $A_{\alpha}^{\Omega}(x^{\mu})$ scattered by one Gabor function.

Ray-theory approximation of the Green tensor in the Born approximation.

High-frequency approximation of spatial derivatives of both the incident wave and the Green tensor. In this high-frequency approximation, we neglect the derivatives of the amplitude, which are of order $1/\mathit{frequency}$ with respect to the derivatives of the travel time.

Paraxial ray approximation of the incident wave in the vicinity of central point x_{Ω}^{i} of the Gabor function.

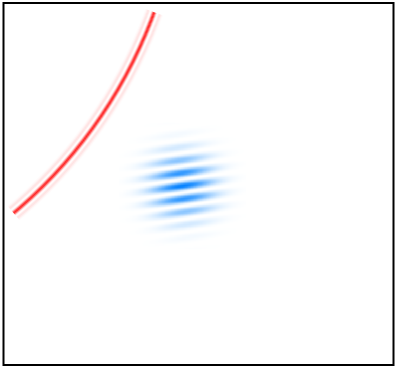
Two-point paraxial ray approximation of the Green tensor at point x_{Ω}^{i} and at the receiver. The paraxial ray approximation consists in a constant amplitude and in the second-order Taylor expansion of the travel time.

Sensitivity Gaussian packets

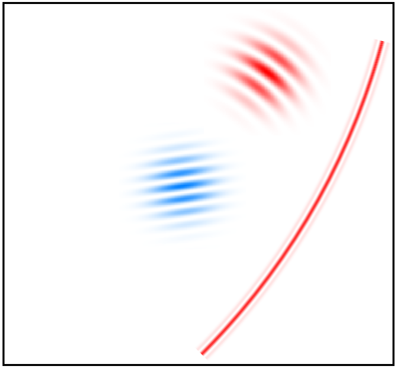
The mentioned approximations enable us to calculate the waves scattered by Gabor functions analytically.

Wave $A^{\Omega}_{\alpha}(x^{\mu})$ scattered by one Gabor function is composed of a few (i.e., 0 to 3 as a rule) Gaussian packets. Each of these "sensitivity" Gaussian packets has a specific frequency and propagates from point x^i_{Ω} in a specific direction:

A single Gabor function $g^{\Omega}(x^i_{\Omega})$ centred at point x^i_{Ω}.



Broad-band wave incident at the Gabor function.



Scattered wave $A^{\Omega}_{\alpha}(x^{\mu})$ composed of one sensitivity Gaussian packet.

Each of these sensitivity Gaussian packets scattered by Gabor function $g^{\Omega}(x^m)$ is sensitive to just a single linear combination

$$R^{\Omega} = \frac{\chi_{\Omega}^{\alpha\beta\gamma\delta} \, E_{\alpha} \, P_{\beta} \, e_{\gamma} \, p_{\delta}}{-2 \, \chi^{\alpha\beta\gamma\delta}(x_{\Omega}^{n}) \, e_{\alpha} \, P_{\beta} \, e_{\gamma} \, p_{\delta}} \tag{41}$$

of coefficients $\chi_{\Omega}^{\alpha\beta\gamma\delta}$ corresponding to the Gabor function.

P_i ... slowness vector of the incident wave; $P_4 = -1$
E_{α} ... polarization vector of the incident wave
p_i ... slowness vector of the sensitivity Gaussian packet; $p_4 = -1$
e_{α} ... polarization vector of the sensitivity Gaussian packet

Coefficient R^{Ω} represents the weak-contrast reflection-transmission coefficient at the interface at which the constitutive tensor changes by $\chi_{\Omega}^{\alpha\beta\gamma\delta}$.