P-WAVE REFLECTION MOVEOUT, SPREADING AND REFLECTION COEFFICIENT IN A HORIZONTALLY LAYERED MEDIUM OF ARBITRARY ANISOTROPY

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Outline

Introduction

A-parameters

Approximate P-wave traveltime formula

Approximate P-wave spreading formula

Approximate P-wave reflection coefficient

Conclusions

Possible extensions

Introduction

Moveout approximations

standard

- expansion of T^2 in terms of the squared offset x

hyperbolic, non-hyperbolic, ...

alternative

- expansion of T^2 in terms of A-parameters specifying

deviations of anisotropy from isotropy

Introduction

Spreading approximation

- based on the derivatives of moveout expressions
  expressed in terms of A-parameters

P-wave reflection coefficient approximation

- weak-anisotropy and weak-contrast approximations
  of the P-wave reflection coefficient

- approximate coefficient expressed in terms of A-parameters

A-parameters

- 21 A-parameters

- represent deviation from an isotropic reference

- generalization of *Thomsen's (1986)* parameters

- related to weak-anisotropy (WA) parameters

- an alternative to stiffness tensor c_{ijkl} or elastic parameters $C_{\alpha\beta}$

- linear relation to elements of c_{ijkl} or $C_{\alpha\beta}$

- natural combinations of $C_{\alpha\beta}$ taken into account

- applicable to anisotropy of any type, strength and orientation

A-parameters

- describe exactly any wave attribute

- simple transformation, equivalent of Bond transformation

- freedom in the choice of the reference velocities

- all 21 A-parameters dimensionless, of comparable size

- first-order P-wave attributes depend on only **15 A-parameters**

- first-order S-wave attributes depend on only **15 A-parameters**

A-parameters

6 P-wave A-parameters

6 S-wave A-parameters

9 common A-parameters

global A-parameters - in 3D space

profile A-parameters - along a profile - superscript P

crystal A-parameters - coincidence of symmetry
and coordinate elements - superscripts TI, OR, TR, ...

Bond transformation between various types of A-parameters
(linear transformation)

A-parameters

P-wave A-parameters (global coordinates)

$$\epsilon_x = \frac{A_{11}-\alpha^2}{2\alpha^2}\ , \qquad \epsilon_y = \frac{A_{22}-\alpha^2}{2\alpha^2}\ , \qquad \epsilon_z = \frac{A_{33}-\alpha^2}{2\alpha^2}\ ,$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha^2}\ , \qquad \chi_y = \frac{A_{25}+2A_{46}}{\alpha^2}\ , \qquad \chi_z = \frac{A_{36}+2A_{45}}{\alpha^2}$$

$A_{\alpha\beta}$ - density-normalized elastic parameters

α - P-wave velocity in a reference isotropic medium

A-parameters

S-wave A-parameters (global coordinates)

$$\gamma_x = \frac{A_{44}-\beta^2}{2\beta^2}\ , \quad \gamma_y = \frac{A_{55}-\beta^2}{2\beta^2}\ , \quad \gamma_z = \frac{A_{66}-\beta^2}{2\beta^2}\ ,$$

$$\epsilon_{45} = \frac{A_{45}}{\beta^2}\ , \quad \epsilon_{46} = \frac{A_{46}}{\beta^2}\ , \quad \epsilon_{56} = \frac{A_{56}}{\beta^2}$$

$A_{\alpha\beta}$ - density-normalized elastic parameters

β - S-wave velocity in a reference isotropic medium

A-parameters

Common A-parameters (global coordinates)

$$\eta_x = \frac{2(A_{23}+2A_{44})-A_{22}-A_{33}}{2\alpha^2}\,, \quad \eta_y = \frac{2(A_{13}+2A_{55})-A_{33}-A_{11}}{2\alpha^2}\,, \quad \eta_z = \frac{2(A_{12}+2A_{66})-A_{11}-A_{22}}{2\alpha^2}\,,$$

$$\xi_{15} = \frac{A_{25}+2A_{46}-A_{15}}{\alpha^2}\,, \quad \xi_{16} = \frac{A_{36}+2A_{45}-A_{16}}{\alpha^2}\,, \quad \xi_{24} = \frac{A_{14}+2A_{56}-A_{24}}{\alpha^2}\,,$$

$$\xi_{26} = \frac{A_{36}+2A_{45}-A_{26}}{\alpha^2}\,, \quad \xi_{34} = \frac{A_{14}+2A_{56}-A_{34}}{\alpha^2}\,, \quad \xi_{35} = \frac{A_{25}+2A_{46}-A_{35}}{\alpha^2}$$

$A_{\alpha\beta}$ - density-normalized elastic parameters

α - P-wave velocity in a reference isotropic medium

Approximate traveltime formula

$$T^2(x) = [\sum_{i=1}^{N} T_i(x_i)]^2 \quad , \qquad x = \sum_{i=1}^{N} x_i$$

T - traveltime

x - offset

N - number of layers

T_i - traveltime along down- and up-going elements
of ray in i-th layer

x_i - offset corresponding to down- and up-going elements
of ray in i-th layer

Approximate traveltime formula

$$T^2(x) = [\sum_{i=1}^{N} T_i(x_i)]^2 \quad , \qquad x = \sum_{i=1}^{N} x_i$$

Two approximations

- replacement of actual ray

  by a ray in the reference isotropic medium

- replacement of exact ray velocity

  by its first-order approximation

Approximate traveltime formula

$$T_i^2(\bar{x}_i) = T_{0i}^2(1+\bar{x}_i^2)^3/P(\bar{x}_i)$$

T_{0i} - two-way zero-offset traveltime in i-th layer

$\bar{x}_i$ - normalized offset in i-th layer

$$T_{0i} = 2h_i/\alpha_i \quad , \qquad \bar{x}_i = x_i/2h_i$$

h_i - thickness of i-th layer

α_i - P-wave reference velocity of i-th layer

$$P(\bar{x}) = (1+\bar{x}^2)^2 + 2\epsilon_x^P\bar{x}^4 + 2(\eta_y^P + \epsilon_x^P + \epsilon_z^P)\bar{x}^2 + 2\epsilon_z^P$$

ϵ_x^P, ϵ_z^P, η_y^P - profile A-parameters of the i-th layer

Approximate traveltime formula

$$T_i^2(\bar{x}_i) = T_{0i}^2(1 + \bar{x}_i^2)^3 / P(\bar{x}_i)$$

$$\bar{x}_i = p q_i^{-1} \quad , \qquad q_i = (\bar{\alpha}_i^{-2} - p^2)^{1/2}$$

p - horizontal component of slowness vector; ray parameter

q_i - vertical component of slowness vector; ray parameter

$\bar{\alpha}_i$ - P-wave reference velocity of i-th layer

used for construction of reference rays; generally $\bar{\alpha}_i \neq \alpha_i$

Approximate traveltime formula - tests

Relative traveltime errors $(T - T_{ex})/T_{ex} \times$ 100%

as a function of $x/2H$

T - approximate traveltime; T_{ex} - ANRAY traveltime

x - offset; H - thickness of the model

Models: TTI limestone (8%), TTI Greenhorn shale (26%)

$H = h_1 + h_2$, $h_1 = h_2$

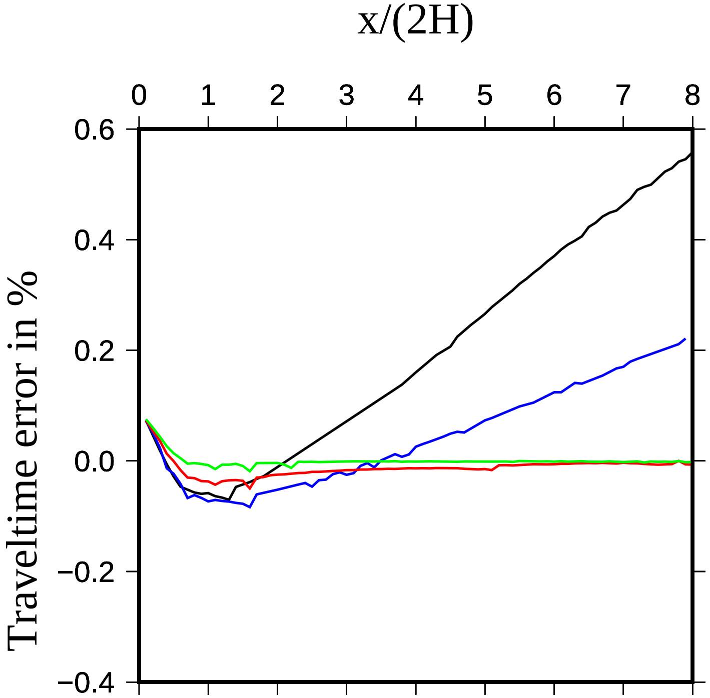
Axes of symmetry specified by polar angles θ_i and azimuth angles φ_i

Approximate traveltime formula - tests

Two-layer TTI limestone: $\bar{\alpha}_i$ - phase velocity along axis of symmetry

polar angles fixed: $\theta_1 = 60°$ and $\theta_2 = 30°$

azimuth angles: $\varphi_1 = \varphi_2$, $\varphi_i = 0°, 30°, 60°, 90°$

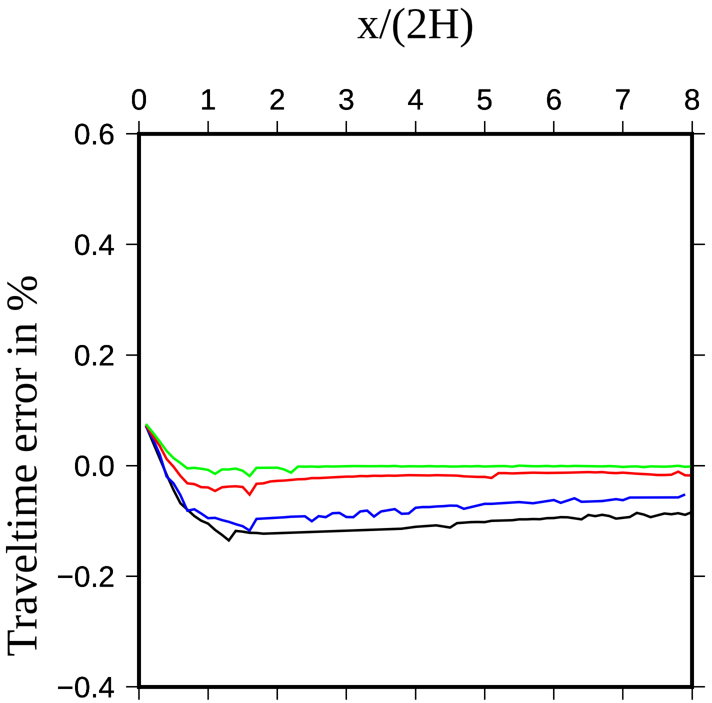


Approximate traveltime formula - tests

Two-layer TTI limestone: $\bar{\alpha}_i$ - phase velocity along source-receiver line

polar angles fixed: $\theta_1 = 60°$ and $\theta_2 = 30°$

azimuth angles: $\varphi_1 = \varphi_2$, $\varphi_i = 0°$, $30°$, $60°$, $90°$

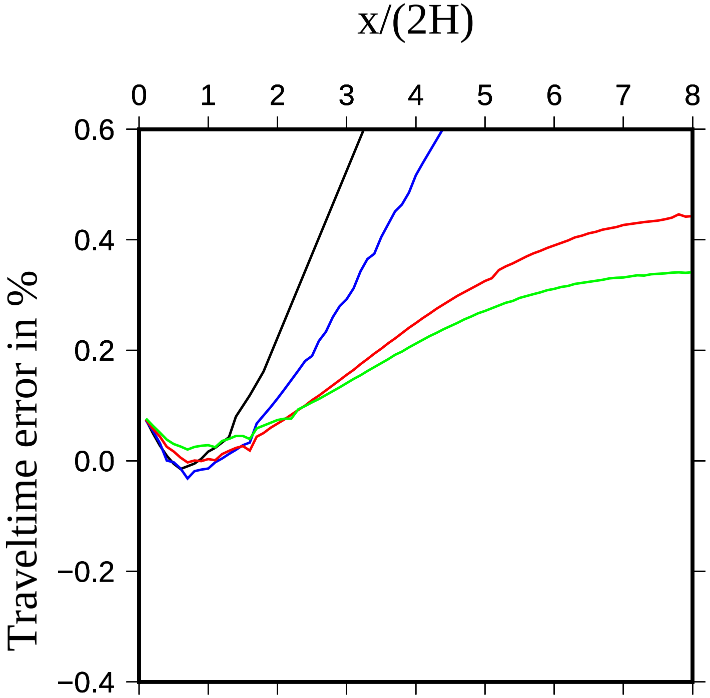


Approximate traveltime formula - tests

Two-layer TTI limestone: $\bar{\alpha}_i$ - phase velocity along the vertical

polar angles fixed: $\theta_1 = 60^\circ$ and $\theta_2 = 30^\circ$

azimuth angles: $\varphi_1 = \varphi_2$, $\varphi_i = 0^\circ$, 30°, 60°, 90°

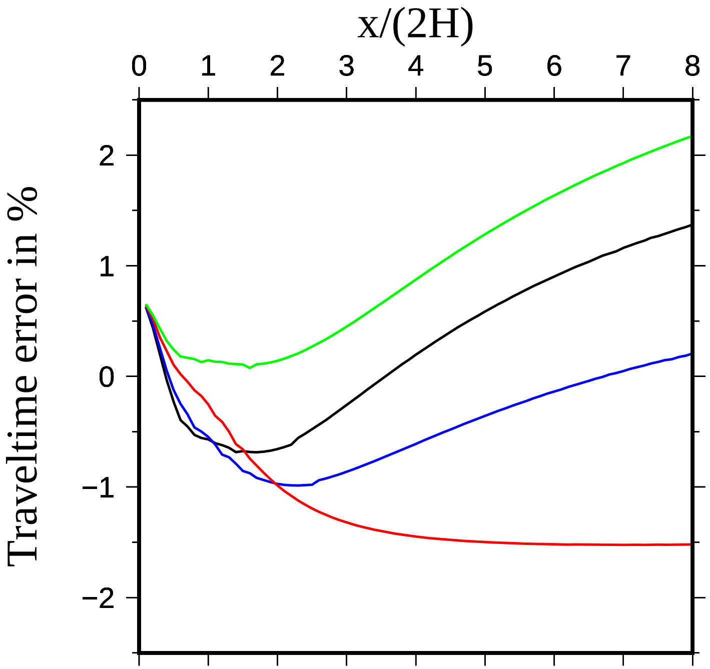


Approximate traveltime formula - tests

Two-layer TTI Greenhorn shale: $\bar{\alpha}_i$ - phase velocity along axis of symmetry

polar angles fixed: $\theta_1 = 60°$ and $\theta_2 = 30°$

azimuth angles: $\varphi_1 = 0°$, $30°$, $60°$, $90°$, $\varphi_2 = 0°$

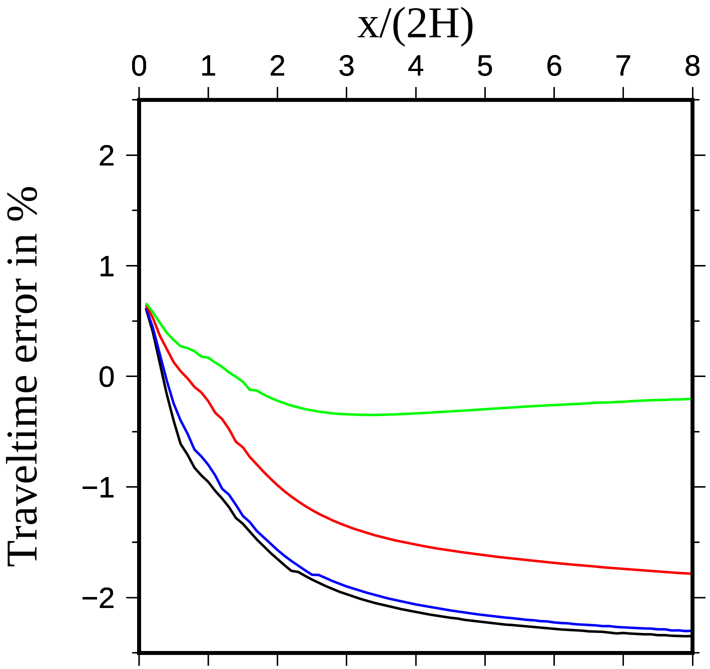


Approximate traveltime formula - tests

Two-layer TTI Greenhorn shale: $\bar{\alpha}_i$ - phase velocity along source-receiver line

polar angles fixed: $\theta_1 = 60^\circ$ and $\theta_2 = 30^\circ$

azimuth angles: $\varphi_1 = 0^\circ$, 30°, 60°, 90°, $\varphi_2 = 0^\circ$

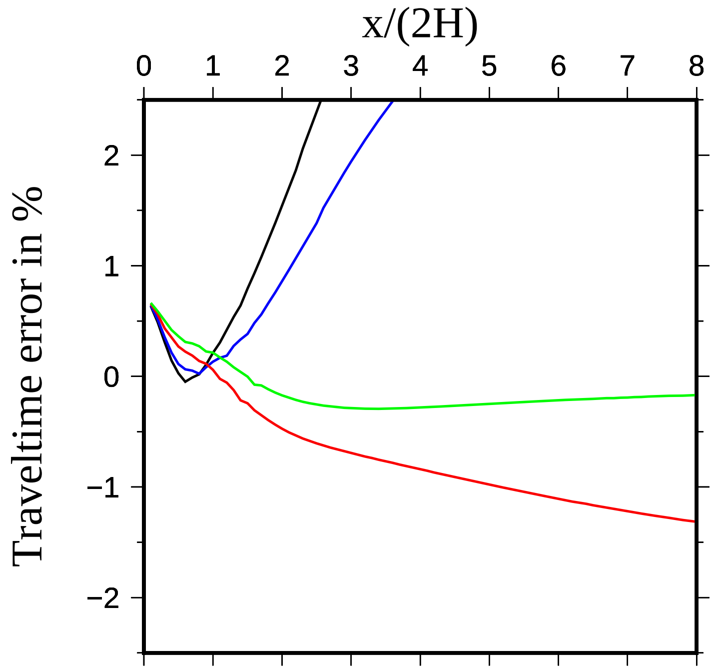


Approximate traveltime formula - tests

Two-layer TTI Greenhorn shale: $\bar{\alpha}_i$ - phase velocity along the vertical

polar angles fixed: $\theta_1 = 60°$ and $\theta_2 = 30°$

azimuth angles: $\varphi_1 = 0°$, $30°$, $60°$, $90°$, $\varphi_2 = 0°$



Approximate spreading formula

$$L(x,\varphi) = x\cos\psi \left[\partial^2 T/\partial x^2 \left(\partial^2 T/\partial\varphi^2 + x\partial T/\partial x\right)\right]^{-1/2}$$

T - traveltime

x - offset

$$T^2(x,\varphi) = [\sum_{i=1}^{N} T_i(x_i,\varphi)]^2 \quad , \qquad x = \sum_{i=1}^{N} x_i$$

ψ - ray angle of the symmetric reference ray

φ - azimuth of source-receiver profile

Approximate spreading formula

Factorization in arbitrary anisotropy

$$L(x,\varphi) = L_{\|}(x,\varphi) L_{\perp}(x,\varphi)$$

$$L_{\|}(x,\varphi) = \cos\psi \left(\frac{\partial^2 T}{\partial x^2}\right)^{-1/2}$$

$$L_{\perp}(x,\varphi) = x \left(\frac{\partial^2 T}{\partial \varphi^2} + x \frac{\partial T}{\partial x}\right)^{-1/2}$$

$L_{\|}$ - in-plane spreading, $L_{\perp}$ - out-of-plane spreading

Approximate spreading formula - tests

Relative spreading errors $(L - L_{ex})/L_{ex} \times$ 100%

as a function of $x/2H$

L - approximate spreading; L_{ex} - ANRAY spreading

x - offset; H - thickness of the model

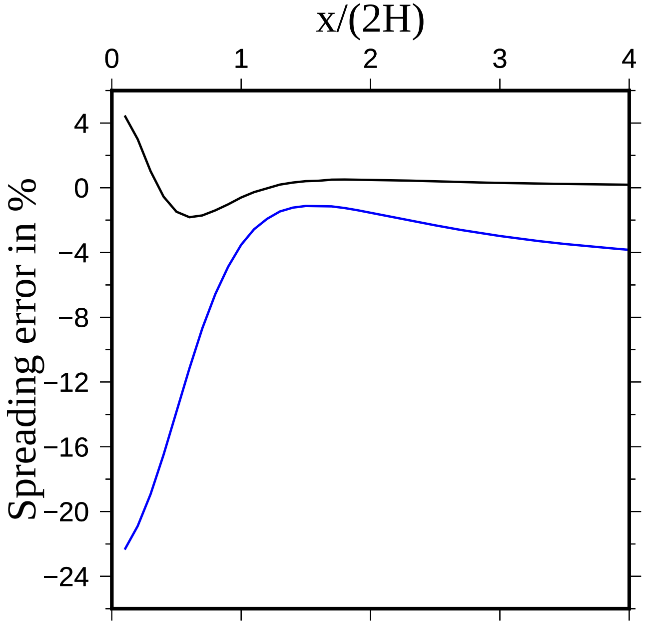
Models: TTI limestone (8%), TTI Greenhorn shale (26%)

$H = \sum_{i=1}^{N} h_i$, $h_1 = h_2 = ... = h_N$, N - number of layers

Axes of symmetry specified by polar angles θ_i and azimuth angles φ_i

Approximate spreading formula - tests

Single-layer VTI limestone: $\bar{\alpha}_1$ - phase velocity along source-receiver line



—— relative spreading error

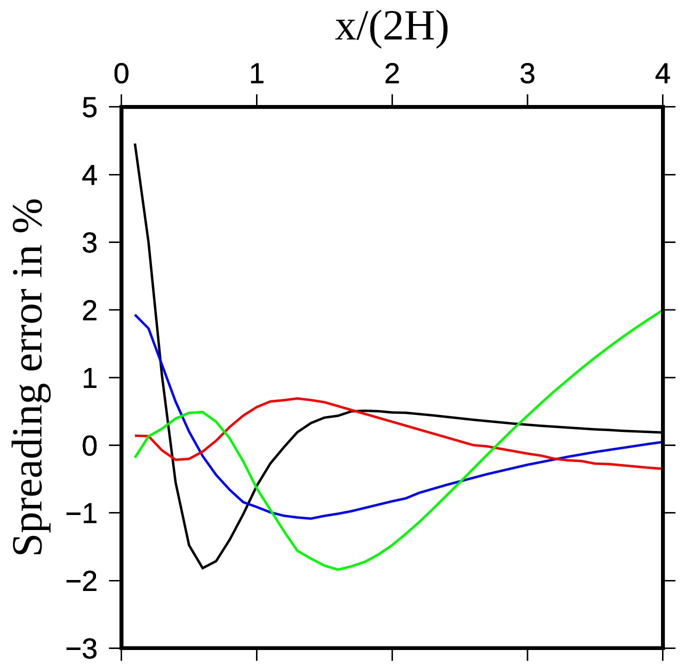
—— **relative difference of spreading in isotropic layer (vertical VTI phase velocity) and in VTI layer**

Approximate spreading formula - tests

Single-layer TTI limestone: $\bar{\alpha}_1$ - phase velocity along source-receiver line

azimuth angle: $\varphi_1 = 0^\circ$

polar angles: $\theta_1 = 0^\circ$, 30°, 60°, 90°

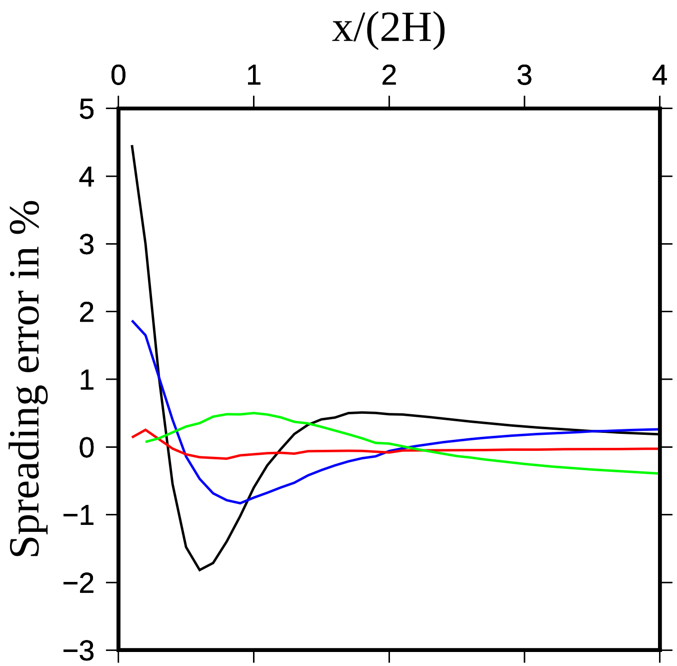


Approximate spreading formula - tests

Single-layer TTI limestone: $\bar{\alpha}_1$ - phase velocity along source-receiver line

azimuth angle: $\varphi_1 = 45^\circ$

polar angles: $\theta_1 = 0^\circ$, 30°, 60°, 90°

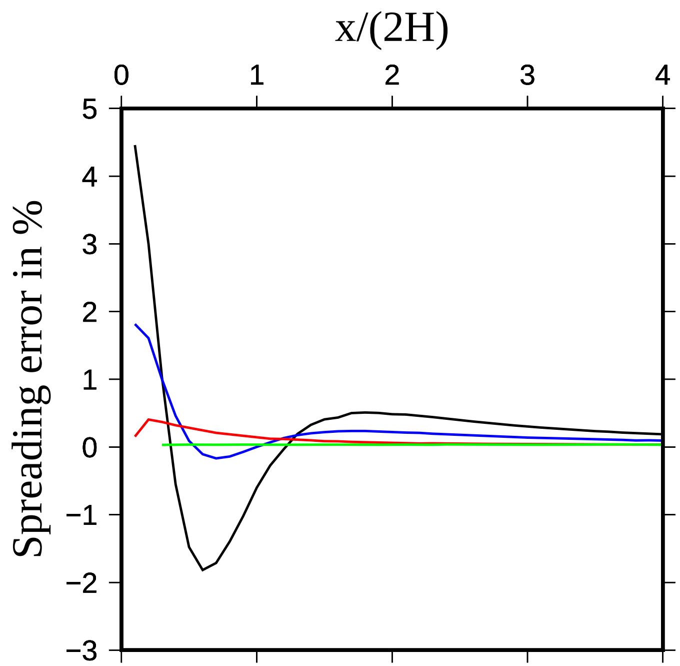


Approximate spreading formula - tests

Single-layer TTI limestone: $\bar{\alpha}_1$ - phase velocity along source-receiver line

azimuth angle: $\varphi_1 = 90^\circ$

polar angles: $\theta_1 = 0^\circ$, 30°, 60°, 90°



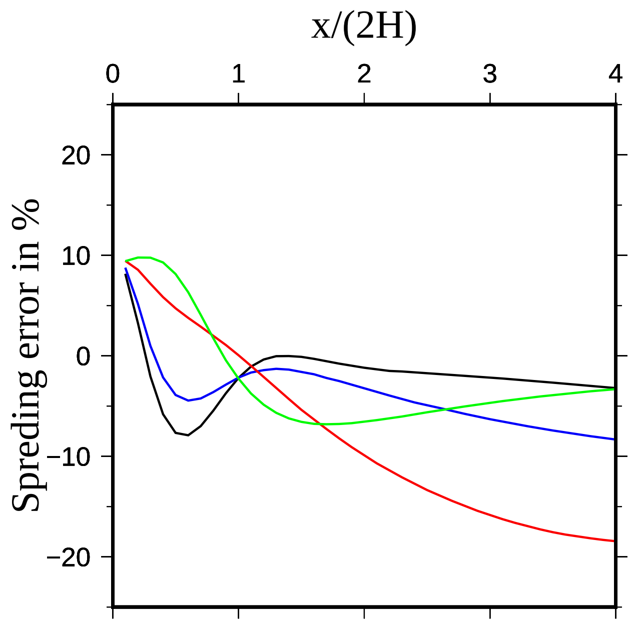
Approximate spreading formula - tests

Two-layer TTI Greenhorn shale

$\bar{\alpha}_i$: phase velocities along source-receiver line

azimuth angles: $\varphi_1 = 0°$, $30°$, $60°$, $90°$, $\varphi_2 = \varphi_1$

polar angles: $\theta_1 = 60°$, $\theta_2 = 30°$



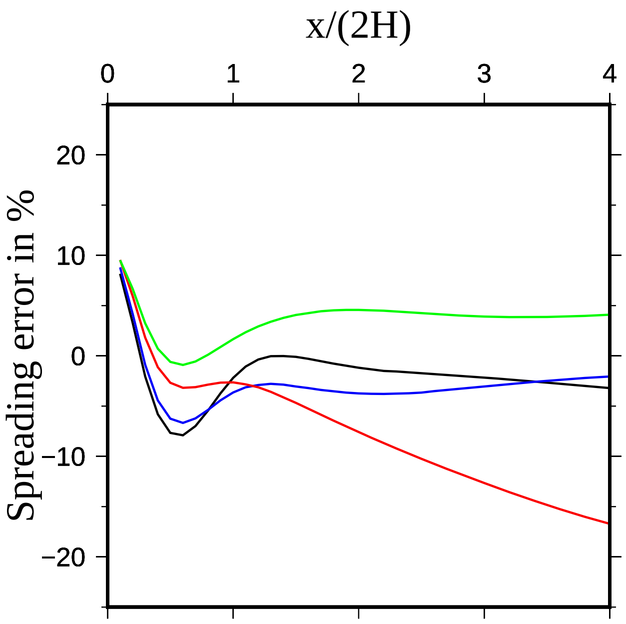
Approximate spreading formula - tests

Two-layer TTI Greenhorn shale

$\bar{\alpha}_i$: phase velocities along source-receiver line

azimuth angles: $\varphi_1 = 0^\circ$, 30°, 60°, 90°, $\varphi_2 = 0^\circ$

polar angles: $\theta_1 = 60^\circ$, $\theta_2 = 30^\circ$



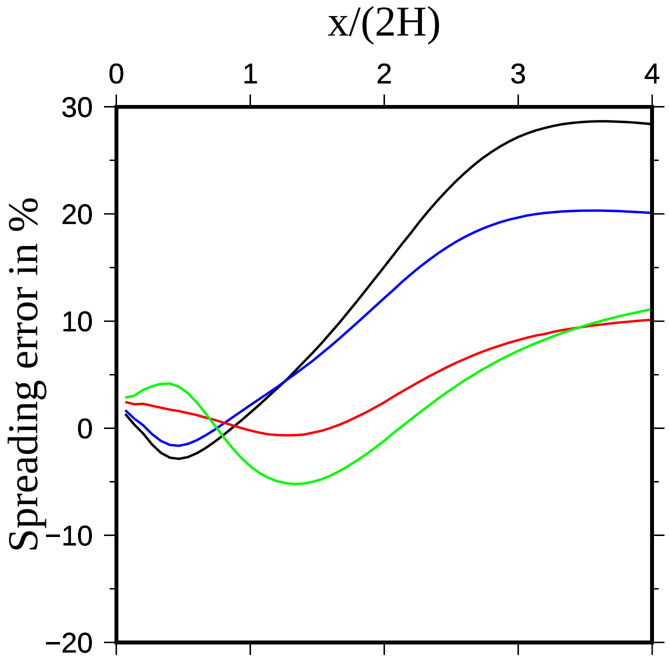
Approximate spreading formula - tests

Three-layer model: TTI limestone/ISO (3 km/s)/TTI Greenhorn shale

$\bar{\alpha}_1$, $\bar{\alpha}_3$: phase velocities along source-receiver line

azimuth angles: $\varphi_1 = 0°$, $30°$, $60°$, $90°$, $\varphi_3 = \varphi_1$

polar angles: $\theta_1 = \theta_3 = 30°$



Approximate P-wave reflection coefficient

$$R_{PP}(\theta_i) = R_{PP}^{iso}(\theta_i) + \frac{1}{2}\Delta\epsilon_z^P + \frac{1}{2}(\Delta\eta_y^P + \Delta\epsilon_x^P - 8\frac{\bar{\beta}^2}{\bar{\alpha}^2}\Delta\gamma_y^P)\sin^2\theta_i$$

$$+\frac{1}{2}\Delta\epsilon_x^P \sin^2\theta_i \tan^2\theta_i$$

$$R_{PP}^{iso}(\theta_i) = \frac{1}{2}\frac{\Delta Z}{\bar{Z}} + \frac{1}{2}[\frac{\Delta\alpha}{\bar{\alpha}} - 4\frac{\bar{\beta}^2}{\bar{\alpha}^2}\frac{\Delta G}{\bar{G}}]\sin^2\theta_i + \frac{1}{2}\frac{\Delta\alpha}{\bar{\alpha}}\sin^2\theta_i\tan^2\theta_i$$

$R_{PP}(\theta_i) = R_{PP}(-\theta_i) \quad \Rightarrow \quad R_{PP}$ - reciprocal

$Z = \rho\alpha$, $\quad G = \rho\beta^2$, $\quad \Delta X = X_2 - X_1$, $\quad \bar{X} = 1/2(X_1 + X_2)$

Approximate P-wave reflection coefficient

Models

TTI/TOR: TI (24%), OR (24%)

velocity contrast for normal incidence $\sim 40\%$

HTI/TRI: TI (8%), TRI (17%)

velocity contrast for normal incidence $\sim 14\%$

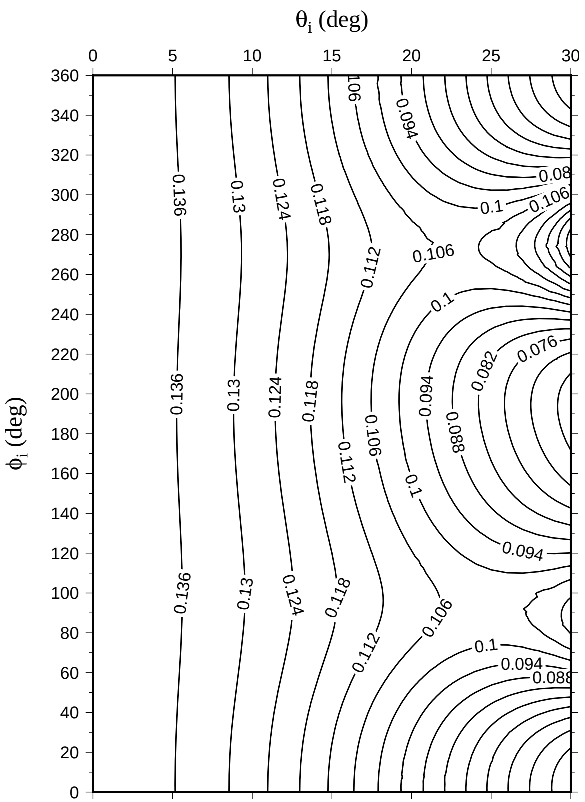
φ_i, θ_i - angles of incidence; $\alpha^2 = A_{33}$, $\beta^2 = A_{55}$

R_{PP}-coefficient absolute error: $R_{PP} - R_{PP}^{ex}$

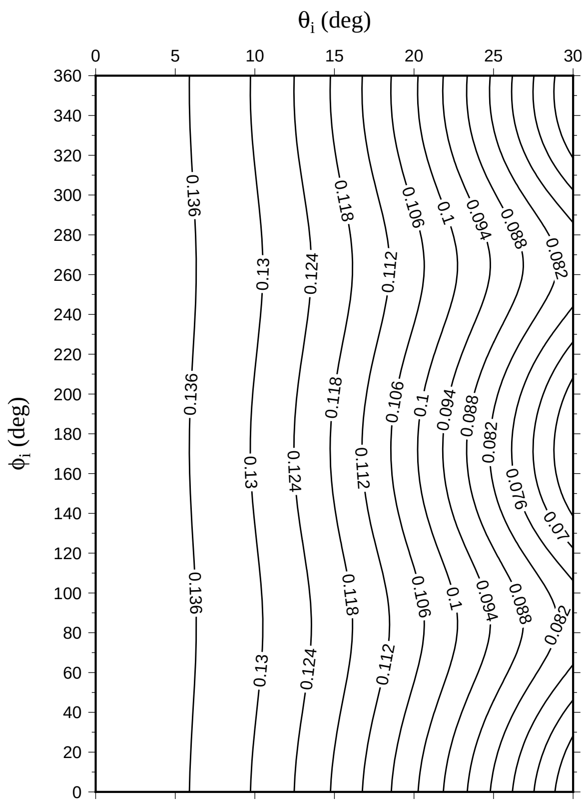
R_{PP} - approximate coefficient; R_{PP}^{ex} - ANRAY coefficient

Approximate P-wave reflection coefficient

TTI/TOR model



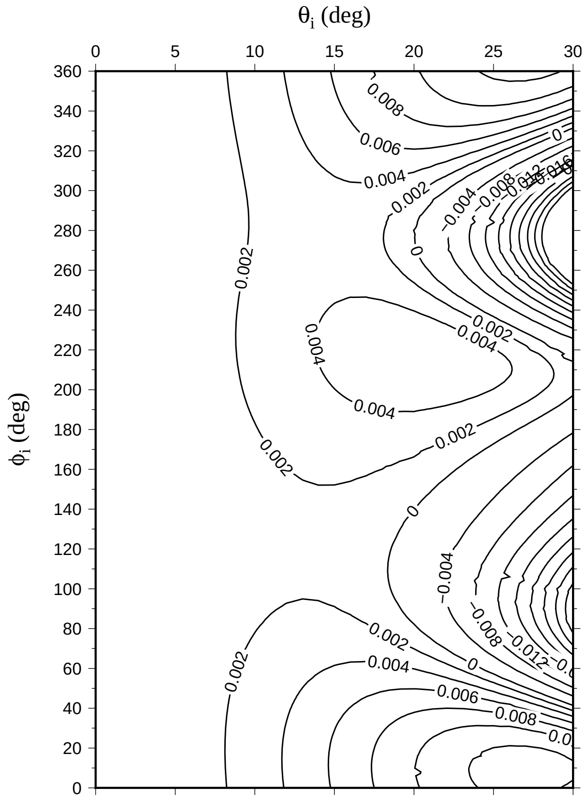
EXACT



APPROXIMATE

Approximate P-wave reflection coefficient

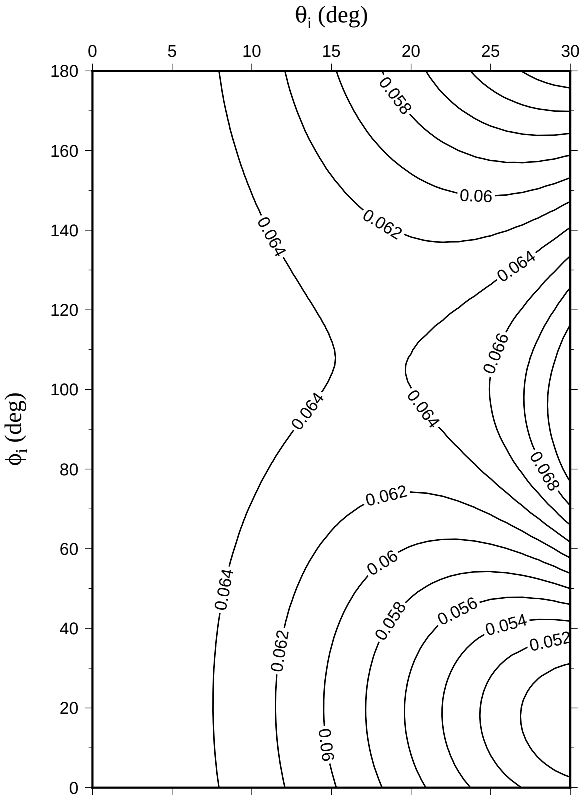
TTI/TOR model



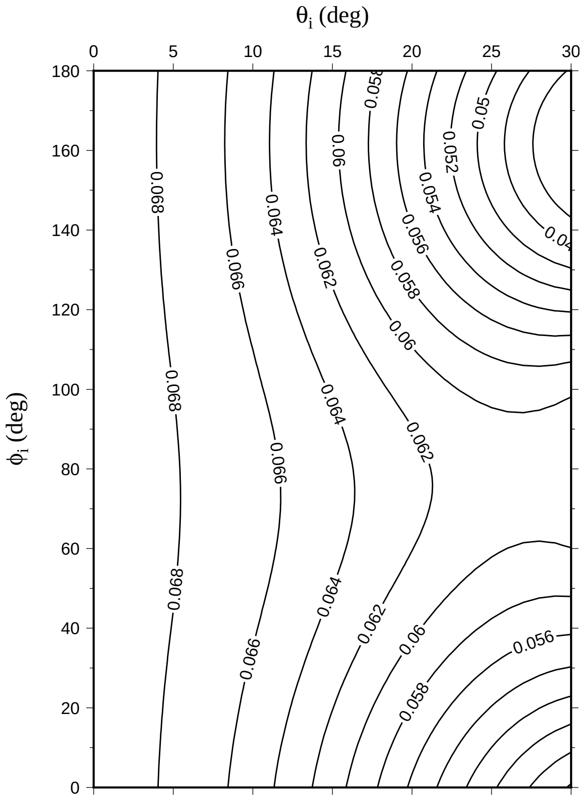
ABSOLUTE ERROR

Approximate P-wave reflection coefficient

HTI/TRI model



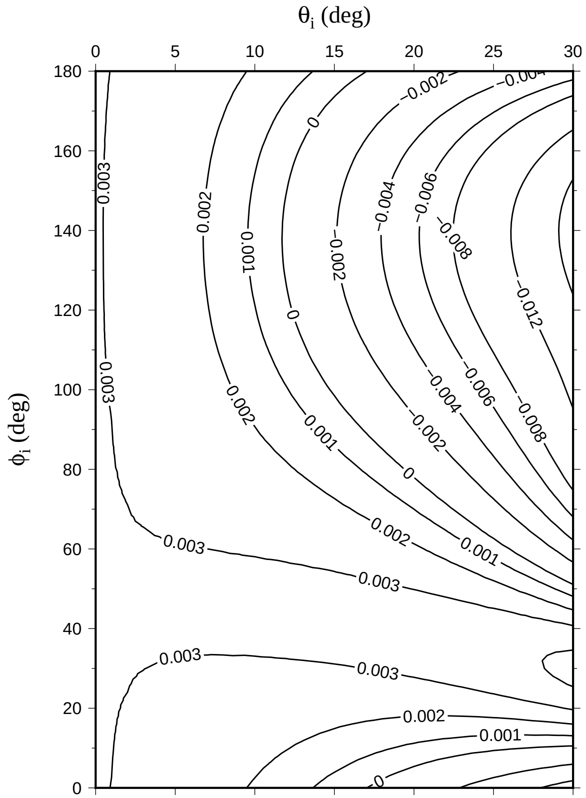
EXACT



APPROXIMATE

Approximate P-wave reflection coefficient

HTI/TRI model



ABSOLUTE ERROR

Conclusions

A-parameters

- applicable to anisotropy of any symmetry, strength and orientation
- linear transformation - analog of Bond transformation
- only 15 A-parameters for P-wave attributes in WA approximation
- only 15 A-parameters for S-wave attributes in WA approximation

Conclusions

P-wave moveout, spreading

- based on weak-anisotropy approximation

- applicable to anisotropy of any symmetry, strength and orientation

- decrease of accuracy for increasing strength of anisotropy
  and increasing deviation of phase- and ray-velocity directions

- no non-physical assumptions (no acoustic approximation)

- accuracy for all offsets within the order of WA approximation

Conclusions

P-wave moveout, spreading

- factorization of spreading in arbitrary anisotropy
- moveout and in-plane spreading:

  dependence on 3 profile A-parameters
- out-of-plane spreading: dependence on 6 profile A-parameters
- dependence of moveout and spreading on 9 global A-parameters

  through the Bond transformation

Conclusions

P-wave reflection coefficient

- based on weak-contrast and weak-anisotropy approximation

- dependence on contrast of 2 P-wave, 1 S-wave and 1 common profile A-parameters

- dependence on contrast of 4 P-wave, 3 S-wave and 5 common global A-parameters through the Bond transformation

- decrease of accuracy for increasing strength of anisotropy and of velocity contrast

Possible extensions

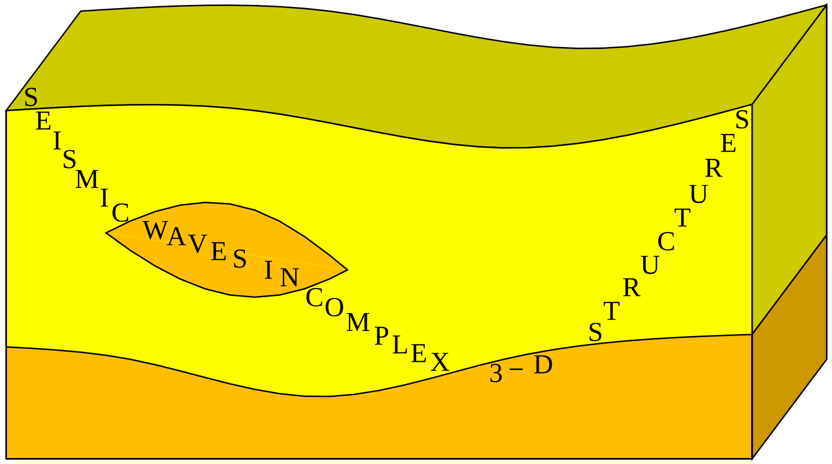
- reflection moveout of converted PS or common S waves

- inclined reflector

- reflection coefficients of converted or unconverted S waves

- transmission coefficients

Acknowledgement



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