P-wave reflection moveout in a layered medium of moderate anisotropy

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- Introduction
- Weak-anisotropy parameters
- Approximate traveltime formula
- Tests of formula
- Conclusions

Introduction

Moveout approximations

common

- Taylor series expansion of T^2 in terms of x^2

hyperbolic, non-hyperbolic, ...

alternative

- expansion of T^2 in terms of the deviations

of anisotropy from isotropy $=>$

weak anisotropy (WA) parameters

Introduction

Moveout approximations with WA parameters

- replacement of the actual primary reflected P-wave ray
  from the bottom of a stack of horizontal anisotropic layers
  of arbitrary symmetry and orientation by a ray of a primary
  reflected P-wave in the reference isotropic medium;
  while the actual ray may be 3D, the reference ray is 2D

- approximation of the exact P-wave ray velocity
  by the first-order phase velocity in each layer

Weak-anisotropy parameters

- 21 weak-anisotropy (WA) parameters

  slightly modified *Mensch & Rasolofosaon (1997)*

- generalization of *Thomsen's (1986)* parameters

- characterize deviations from an isotropic reference

- zero WA parameters $\rightarrow$ moveout in the isotropic reference

- an alternative to stiffness tensor or Voigt's $C_{\alpha\beta}$ or $A_{\alpha\beta}$

- applicable to anisotropy of any type, strength and orientation

- can describe exactly any wave attribute

Weak-anisotropy parameters

- natural combinations of $C_{\alpha\beta}$ or $A_{\alpha\beta}$ taken into account

- linear relation of WA to $C_{\alpha\beta}$ or $A_{\alpha\beta}$ parameters

- simple linear transformation of WA parameters
  from one coordinate system to another

- definable in coordinate systems independent of symmetry elements
  of studied anisotropy symmetry

- optional choice of the reference velocity α

- all 21 WA parameters dimensionless, of comparable size

- **for P waves, only 15 WA parameters sufficient**

Weak-anisotropy parameters

$$\boldsymbol{\epsilon_x = \frac{A_{11}-\alpha^2}{2\alpha^2}}, \qquad \epsilon_y = \frac{A_{22}-\alpha^2}{2\alpha^2}, \qquad \boldsymbol{\epsilon_z = \frac{A_{33}-\alpha^2}{2\alpha^2}}$$

$$\delta_x = \frac{A_{23}+2A_{44}-\alpha^2}{\alpha^2}, \qquad \boldsymbol{\delta_y = \frac{A_{13}+2A_{55}-\alpha^2}{\alpha^2}}, \qquad \delta_z = \frac{A_{12}+2A_{66}-\alpha^2}{\alpha^2}$$

$$\epsilon_{15} = \frac{A_{15}}{\alpha^2}, \quad \epsilon_{16} = \frac{A_{16}}{\alpha^2}, \quad \epsilon_{24} = \frac{A_{24}}{\alpha^2}, \quad \epsilon_{26} = \frac{A_{26}}{\alpha^2}, \quad \epsilon_{34} = \frac{A_{34}}{\alpha^2}, \quad \epsilon_{35} = \frac{A_{35}}{\alpha^2}$$

$$\chi_x = \frac{A_{14}+2A_{56}}{\alpha^2}, \qquad \chi_y = \frac{A_{25}+2A_{46}}{\alpha^2}, \qquad \chi_z = \frac{A_{36}+2A_{45}}{\alpha^2}$$

α - P-wave reference velocity

ϵ_x, ϵ_z, δ_y **- WA parameters related to the profile along the** x_1 **axis**

Approximate traveltime formula

Reflection traveltime

$$T^2(x) = [\sum_{i=1}^{N} T_i(x_i)]^2 \qquad x = \sum_{i=1}^{N} x_i$$

N - number of layers

$T(x)$ - complete traveltime

x - source-receiver offset

x_i - local offset corresponding to two-way
reference ray in the i-th layer

$T_i(x_i)$ - approximate two-way traveltime in the i-th layer

Approximate traveltime formula

Reflection traveltime

$$T_i^2(\bar{x}_i) = T_{0i}^2(1+\bar{x}_i^2)^3 P^{-1}(\bar{x}_i) \qquad \bar{x}_i = x_i/2h_i$$

T_{0i} - two-way zero-offset traveltime along the reference ray

in the i-th layer: $T_{0i} = 2h_i/\alpha_i$

h_i - thickness, α_i - P-wave reference velocity of the i-th layer

$$P(\bar{x}_i) = (1+\bar{x}_i^2)^2 + 2\epsilon_x^{(i)}\bar{x}_i^4 + 2\delta_y^{(i)}\bar{x}_i^2 + 2\epsilon_z^{(i)}$$

Approximate traveltime formula

Reflection traveltime

Determination of the reduced local offset $\bar{x}_i$

in the reference layered isotropic medium

$\bar{x}_i = p/|q_i|$

p - ray parameter of the reference ray;

given or determined for a given offset x

q_i - vertical component of the slowness vector

of the reference ray in the i-th layer: $q_i = \pm(\bar{\alpha}_i^{-2} - p^2)^{1/2}$

$\bar{\alpha}_i$ - P-wave reference velocity for the reference ray

$\bar{\alpha}_i$ may differ from α_i specifying WA parameters

Tests of formula

N-layered model

H - total depth, $H = h_1 + h_2 + ... + h_N$, $h_i = h_j$

Exact reference - ANRAY program package

Relative traveltime errors - $(T - T_{ex})/T_{ex} \times 100\%$

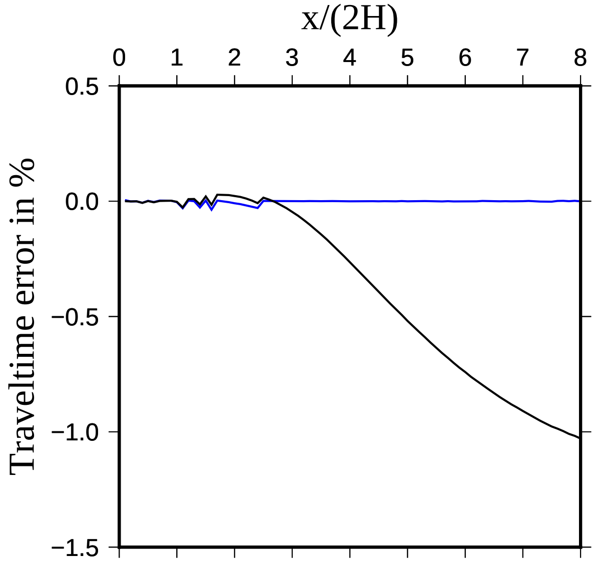
T - approximate traveltime, T_{ex} - exact (ANRAY) traveltime

Tests of formula

Two-layer isotropic model ($\bar{\alpha}_1$=2.5, $\bar{\alpha}_2 = 3$ km/s)

blue - WA formula

black - rational approximation (Tsvankin & Thomsen, 1994)



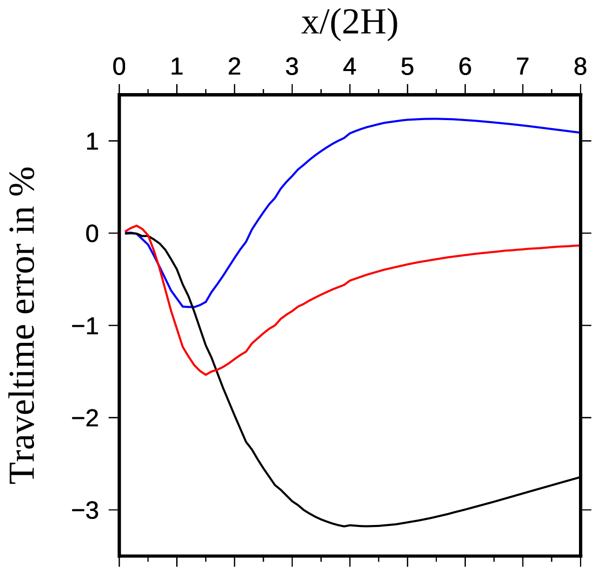
Tests of formula

Two-layer VTI model

Limestone (8% - top), Greenhorn shale (26% - bottom)

WA - $\bar{\alpha}_i$ along symmetry axis (blue), along source-receiver line (red)

rational approximation (Tsvankin & Thomsen, 1994) - black



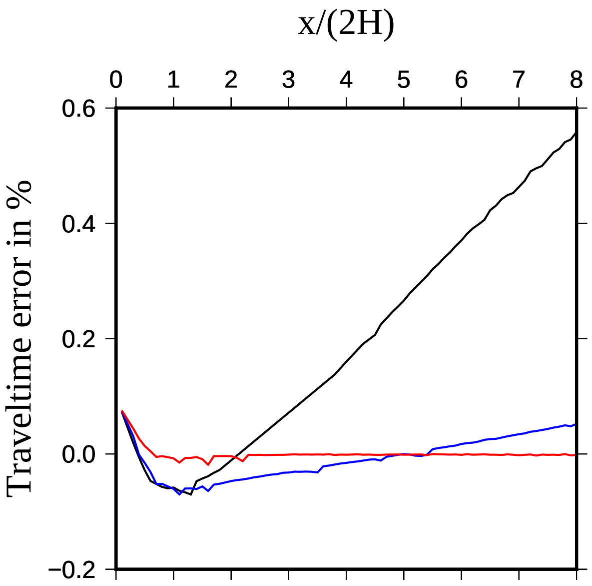
Tests of formula

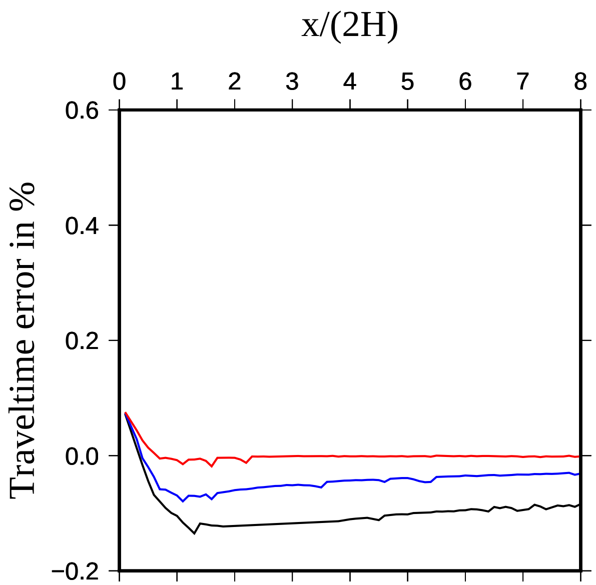
Two-layer TTI Limestone model (8%)

Symmetry axes dip: top (30°), bottom (60°)

Axes azimuths (from x_1): 0° - black, 45° - blue, 90° - red

$\bar{\alpha}_i$ along symmetry axis (left), along source-receiver line (right)





Tests of formula

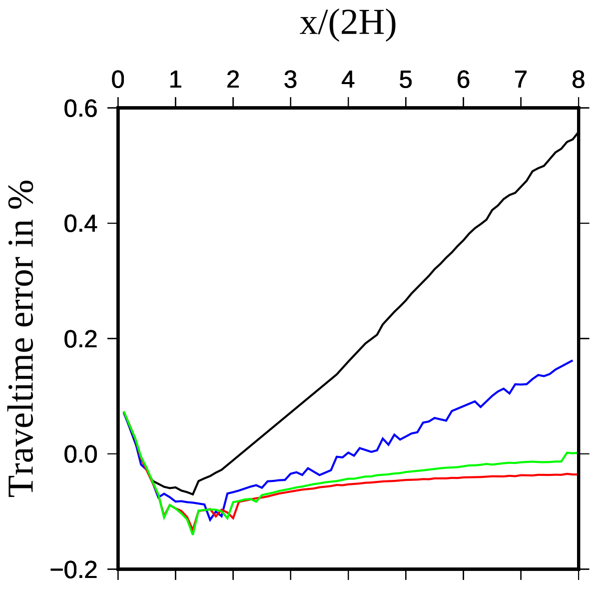
Two-layer TTI Limestone model (8%)

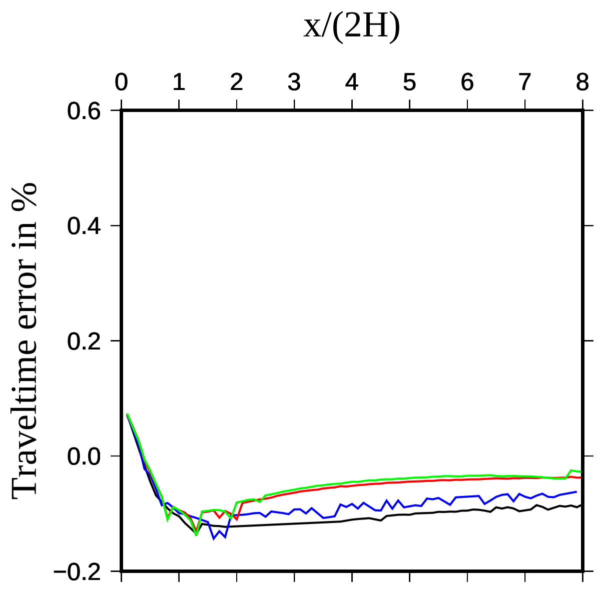
Symmetry axis in the top layer: dip: 30°

azimuths (from x_1): 0° - black, 30° - blue, 60° - red, 90° - green

Symmetry axis in the bottom layer - dip: 60° in the plane (x_1, x_3)

$\bar{\alpha}_i$ along symmetry axis (left), along source-receiver line (right)





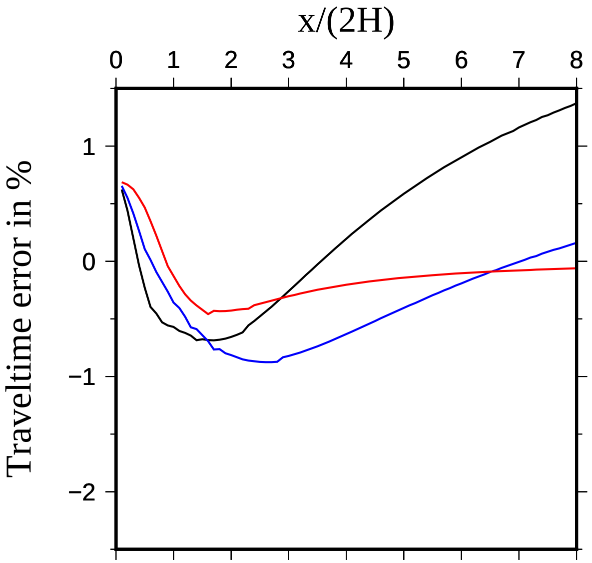
Tests of formula

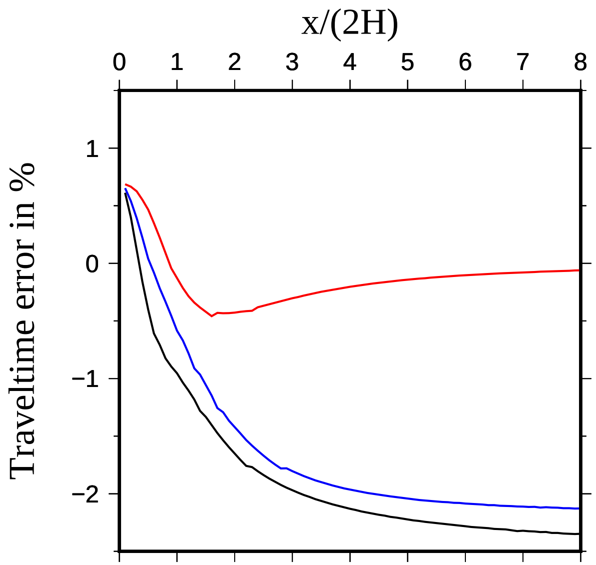
Two-layer TTI Greenhorn shale model (26%)

Symmetry axes dip: top ($30°$), bottom ($60°$)

Axes azimuths (from x_1): $0°$ - black, $45°$ - blue, $90°$ - red

$\bar{\alpha}_i$ along symmetry axis (left), along source-receiver line (right)





Tests of formula

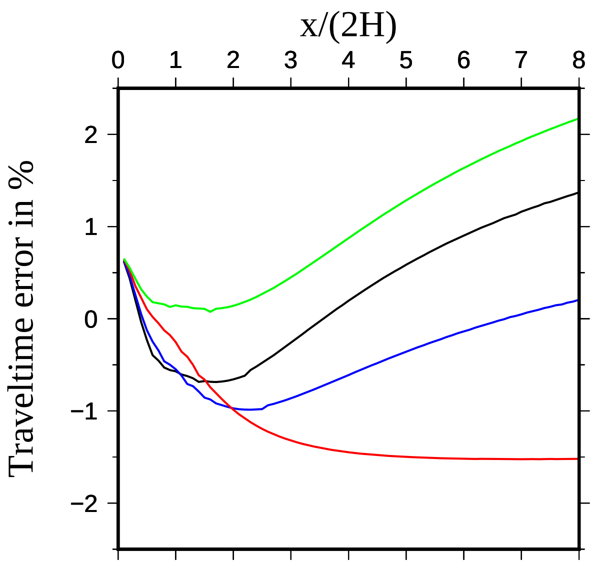
Two-layer TTI Greenhorn shale model (26%)

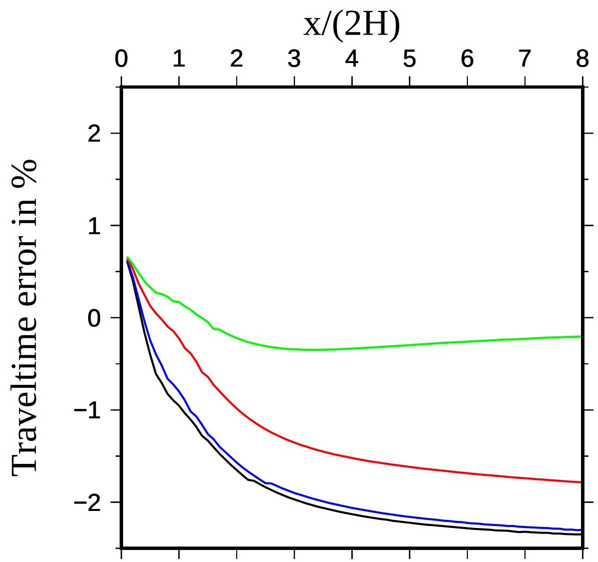
Symmetry axis in the top layer: dip: 30°

azimuths (from x_1): 0° - black, 30° - blue, 60° - red, 90° - green

Symmetry axis in the bottom layer - dip: 60° in the plane (x_1, x_3)

$\bar{\alpha}_i$ along symmetry axis (left), along source-receiver line (right)





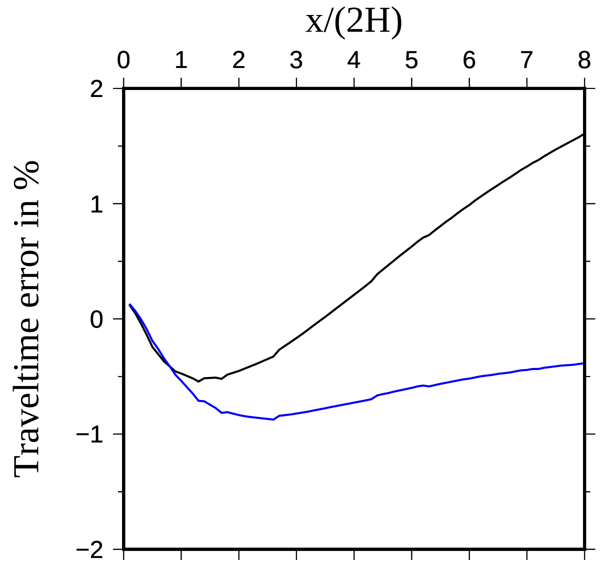
Tests of formula

Four-layer TTI Greenhorn shale model (26%)

Dip of symmetry axes in all layers: 60°

Symmetry axes azimuths (from x_1) from top to bottom: 0°, 30°, 60°, 90°

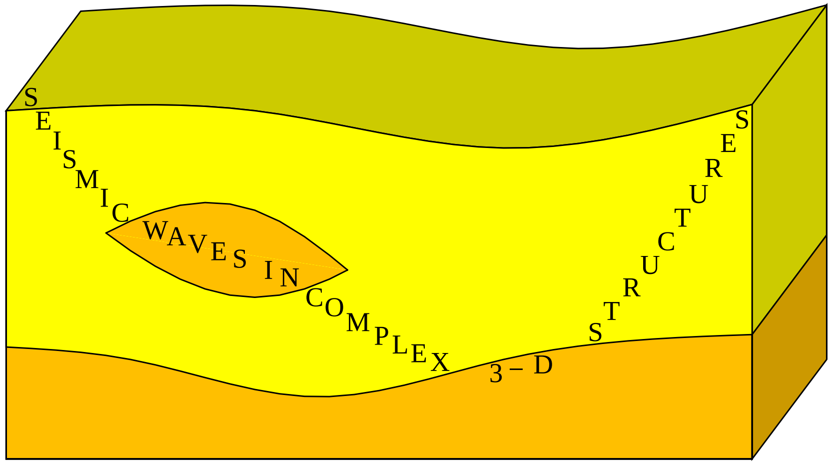
$\bar{\alpha}_i$ along axis (black) and along source-receiver line (blue)



Conclusions

- moveout formula based on WA approximation
- applicable to horizontally layered media of weak
  or moderate anisotropy of arbitrary symmetry and orientation
- simple and transparent, applicable to any offset
- accuracy within the first-order WA approximation
- accuracy depends on the choice of reference velocities
- inaccuracies caused by deviations of $\mathbf{n}$ and $\mathbf{N}$
- accuracy comparable with single-layer moveout
- possible to rewrite into the form of the Taylor series expansion
- possible to generalize for converted waves

Acknowledgement



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