S–WAVE COUPLING IN HETEROGENEOUS ANISOTROPIC MEDIA

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Seismic Waves in Complex 3-D Structures
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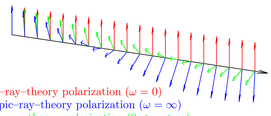
Coupling ray theory

Traditionally, there are two different high–frequency asymptotic ray theories: the *isotropic ray theory* assuming equal velocities of both S waves, and the *anisotropic ray theory* assuming both S waves strictly decoupled. In the isotropic ray theory, the S–wave polarization vectors do not rotate about the ray, whereas in the anisotropic ray theory they may rotate rapidly about the ray.



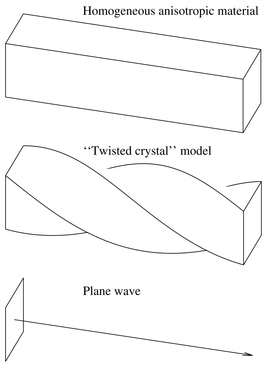
Isotropic–ray–theory polarization ($\omega = 0$)
Anisotropic–ray–theory polarization ($\omega = \infty$)
Coupling–ray–theory polarization ($0 < \omega < \infty$)

In weakly anisotropic velocity models, at moderate frequencies, the S–wave polarization vector tends to remain unrotated about the ray, but is partly attracted by the rotation of the eigenvectors of the Christoffel matrix. The intensity of the attraction increases with frequency. This behaviour of the S–wave polarization vector is described by the *coupling ray theory* proposed by Coates & Chapman (1990). The coupling ray theory is applicable at all degrees of anisotropy, from isotropic velocity models to considerably anisotropic ones. The frequency–dependent coupling ray theory is the generalization of both the zero–order isotropic and anisotropic ray theories and provides continuous transition between them. The coupling ray theory is particularly important for calculation of S waves at degrees of anisotropy and frequencies typical in seismic exploration and structural seismology on all scales, because isotropic ray theory does not describe the two S waves from the principle, and the anisotropic ray theory often fails to determine correct S–wave polarization.

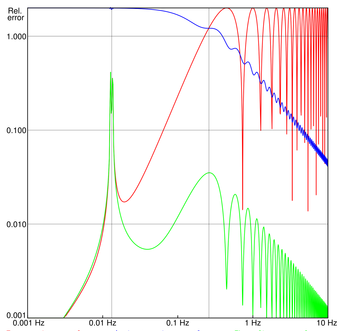
Both S waves can be calculated along a single reference ray. The numerical algorithm for calculating the frequency–dependent complex–valued S–wave polarization vectors of the coupling ray theory is described in Bulant & Klimeš (2002) and Klimeš (2006). For a concise overview of the coupling ray theory refer to Červený, Klimeš & Pšenčík (2007).

Errors of isotropic, anisotropic and coupling–ray–theory in the 1-D anisotropic "simplified twisted crystal" velocity model

The 1-D "simplified twisted crystal" velocity model is created of a homogeneous anisotropic elastic material by uniformly helicoidally twisting the x_1x_2 coordinate plane along the x_3 axis. The exact analytical solution for the plane S wave, propagating along the axis of spirality in the velocity model, is known (Klimeš, 2004).



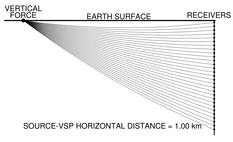
The frequency dependencies of the relative differences of the isotropic, anisotropic, and coupling ray theory from the exact solution are plotted on a log–log scale. The two vertical lines denote the resonant and coupling frequencies. A relative error of 200% occurs, e.g., for an opposite polarization, or an opposite phase. The coupling ray theory yields excellent results in this velocity model, except for the resonant frequency, which is far outside the validity region of the ray theories. For details refer to Bulant & Klimeš (2004).

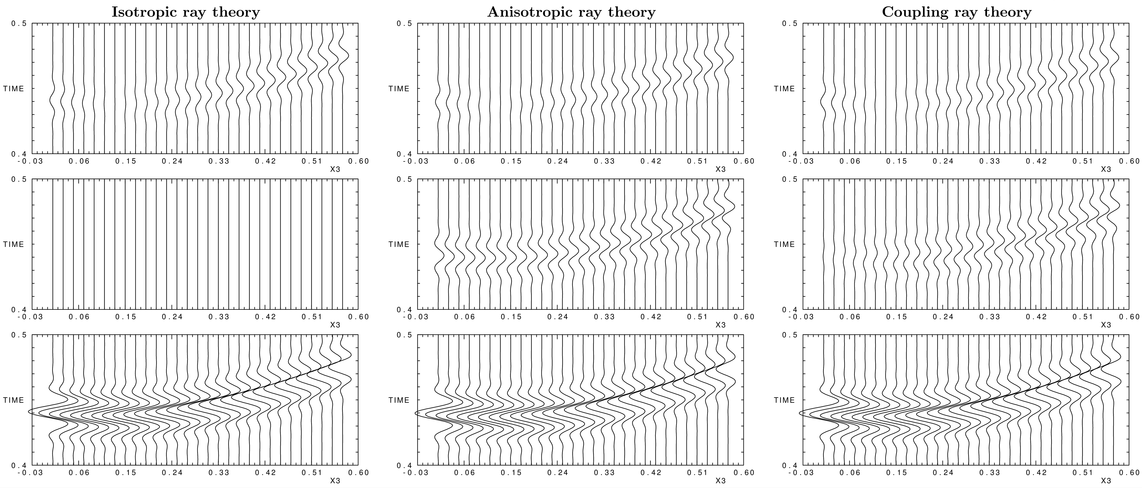


Isotropic ray theory. Anisotropic ray theory. Coupling ray theory.

Comparison of isotropic, anisotropic and coupling–ray–theory seismograms in velocity model QIH

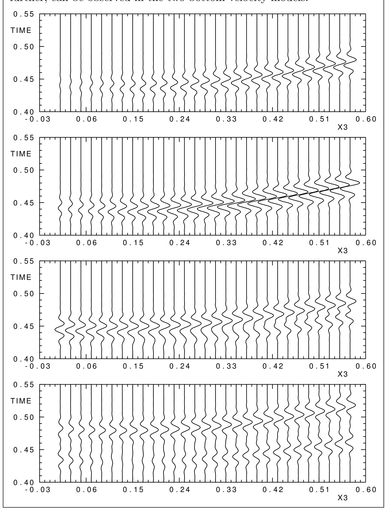
Ray–theory seismograms are calculated in weakly anisotropic velocity model QIH. From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component. In the isotropic ray theory, the S–wave polarization vectors do not rotate about the ray. In the anisotropic ray theory, the S–wave polarization vectors coincide with the eigenvectors of the Christoffel matrix and may rotate rapidly about the ray. In the coupling ray theory, the S–wave polarization vectors are only partly attracted by the rotation of the eigenvectors of the Christoffel matrix.





Development of S–wave splitting with increasing anisotropy

S–wave coupling decreases and S–wave splitting increases with increasing anisotropy and frequency. This behaviour is illustrated on the coupling–ray–theory seismograms in velocity models QIH, QI, QI2 and QI4 of different degrees of anisotropy. The vertically heterogeneous 1–D anisotropic velocity model QI was used by Pšenčík & Dellinger (2001) for comparison of the coupling–ray–theory synthetic seismograms with the reflectivity method. For a description of velocity models QI, QI2 and QI4 and measurement configuration refer to Bulant & Klimeš (2008). Anisotropy increases from the top seismograms to the bottom seismograms in the ratio 1:2:4:8. Only the second (transverse) component is shown. Note that this component vanishes in the analogous isotropic medium. For weak anisotropy, the change of polarization with increasing anisotropy is indicated by a clear increment of the transverse amplitudes in the two upper velocity models. The clear development of S–wave splitting, if anisotropy is increased further, can be observed in the two bottom velocity models.



Acknowledgements

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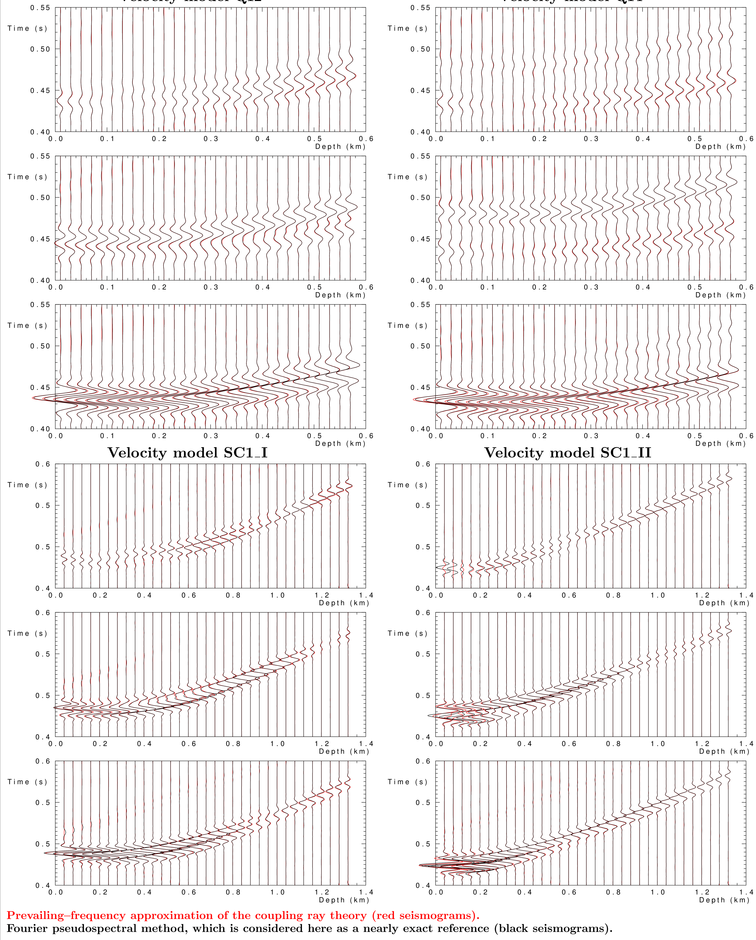
Prevailing–frequency approximation of the coupling ray theory

The coupling–ray–theory Green tensor is frequency dependent, and is usually calculated for many frequencies. This frequency dependence represents no problem for calculating the Green tensor, but may represent a great problem when storing the Green tensor at the nodes of dense grids, typical for applications such as seismic migrations. To eliminate this frequency dependence, the frequency–domain coupling–ray–theory Green tensor may be approximated in the vicinity of a given prevailing frequency by two frequency–independent Green tensors corresponding to two S waves described by their travel times and amplitudes, refer to Klimeš & Bulant (2012). This "prevailing–frequency approximation" of the coupling ray theory allows us to process the coupling–ray–theory wavefield in the same way as the anisotropic–ray–theory wavefield.

Numerical comparison with the Fourier pseudospectral method

For the elastic moduli in velocity models QI2 and QI4 refer to Bulant & Klimeš (2008). For the description of velocity models SC1_I and SC1_II, and for the description of the Fourier pseudospectral method, refer to Pšenčík, Farra & Tessmer (2012).
From top to bottom: the first (radial) component, the second (transverse) component, the third (vertical) component of the seismograms.



Prevailing–frequency approximation of the coupling ray theory (red seismograms).
Fourier pseudospectral method, which is considered here as a nearly exact reference (black seismograms).

The seismograms in velocity models QI2, QI4 and SC1_I are in overall good agreement. The seismograms from the shallow receivers in velocity model SC1_II probably indicate problems with the approximation of the S–wave eigenvectors of the Christoffel matrices by the reference eigenvectors. This problem will require a further study.