Sensitivity Gaussian packets

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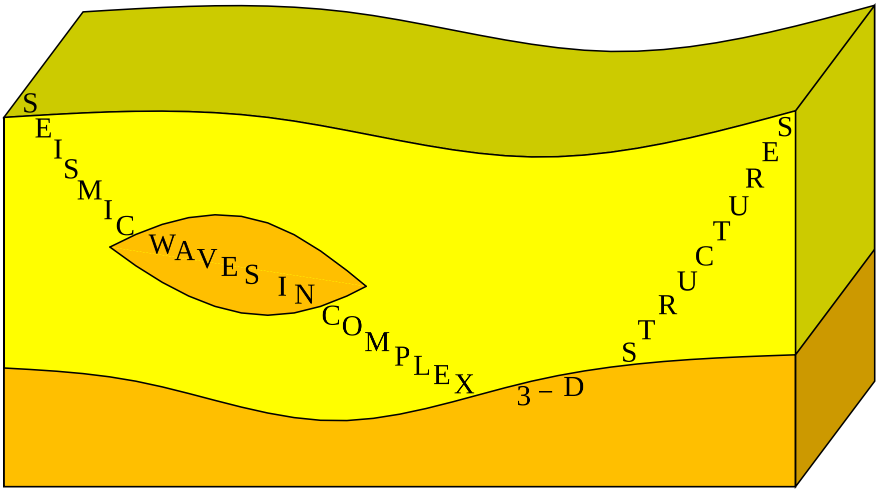

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Gabor representation of medium perturbations

We consider infinitesimally small perturbations $\delta c_{ijkl}(\mathbf{x})$ and $\delta\varrho(\mathbf{x})$ of elastic moduli $c_{ijkl}(\mathbf{x})$ and density $\varrho(\mathbf{x})$.

We decompose the perturbations into Gabor functions $g^{\Omega}(\mathbf{x})$ indexed here by Ω:

$$\delta c_{ijkl}(\mathbf{x}) = \sum_{\Omega} c^{\Omega}_{ijkl}\, g^{\Omega}(\mathbf{x}) \quad , \qquad \delta\varrho(\mathbf{x}) = \sum_{\Omega} \varrho^{\Omega}\, g^{\Omega}(\mathbf{x}) \quad ,$$

$$g^{\Omega}(\mathbf{x}) = \exp[\mathrm{i}\,\mathbf{k}^{\Omega\,\mathrm{T}}(\mathbf{x}-\mathbf{x}^{\Omega}) - \tfrac{1}{2}(\mathbf{x}-\mathbf{x}^{\Omega})^{\mathrm{T}}\mathbf{K}^{\Omega}(\mathbf{x}-\mathbf{x}^{\Omega})] \quad .$$

Gabor functions $g^{\Omega}(\mathbf{x})$ are centred at various spatial positions $\mathbf{x}^{\Omega}$ and have various structural wavenumber vectors $\mathbf{k}^{\Omega}$.

The wavefield scattered by the perturbations is then composed of waves $u^{\Omega}_i(\mathbf{x},t)$ scattered by individual Gabor functions:

$$\delta u_i(\mathbf{x},t) = \sum_{\Omega} u^{\Omega}_i(\mathbf{x},t) \quad .$$

Applied approximations

Short-duration broad-band wavefield with a smooth frequency spectrum incident at the Gabor function, expressed in terms of the amplitude and travel time.

First-order Born approximation of each wave $u_i^\Omega(\mathbf{x}, t)$ scattered by one Gabor function.

Ray-theory approximation of the Green tensor in the Born approximation.

High-frequency approximation of spatial derivatives of both the incident wave and the Green tensor. In this high-frequency approximation, we neglect the derivatives of the amplitude, which are of order $1/frequency$ with respect to the derivatives of the travel time.

Paraxial ray approximation of the incident wave in the vicinity of central point $\mathbf{x}^\Omega$ of the Gabor function.

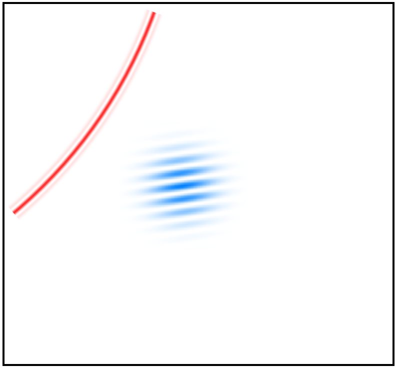
Two-point paraxial ray approximation of the Green tensor at point $\mathbf{x}^\Omega$ and at the receiver. The paraxial ray approximation consists in a constant amplitude and in the second-order Taylor expansion of the travel time.

Sensitivity Gaussian packets

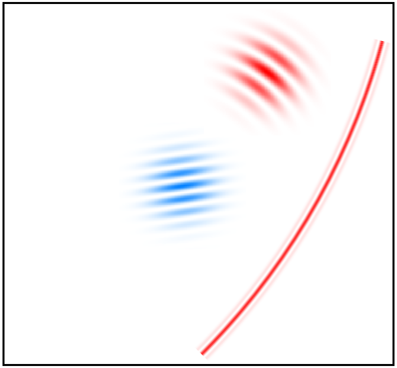
The mentioned approximations enable us to calculate the waves scattered by Gabor functions analytically (Klimeš, 2007).

Wave $u_i^\Omega(\mathbf{x},t)$ scattered by one Gabor function is composed of a few (i.e., 0 to 5 as a rule) Gaussian packets. Each of these "sensitivity" Gaussian packets has a specific frequency and propagates from point $\mathbf{x}^\Omega$ in a specific direction:

A single Gabor function $g^\Omega(\mathbf{x})$ centred at point $\mathbf{x}^\Omega$.



Broad-band wave incident at the Gabor function.



Scattered wave $u_i^\Omega(\mathbf{x},t)$ composed of one sensitivity Gaussian packet.

Each of these sensitivity Gaussian packets scattered by Gabor function $g^{\Omega}(\mathbf{x})$ is sensitive to just a single linear combination

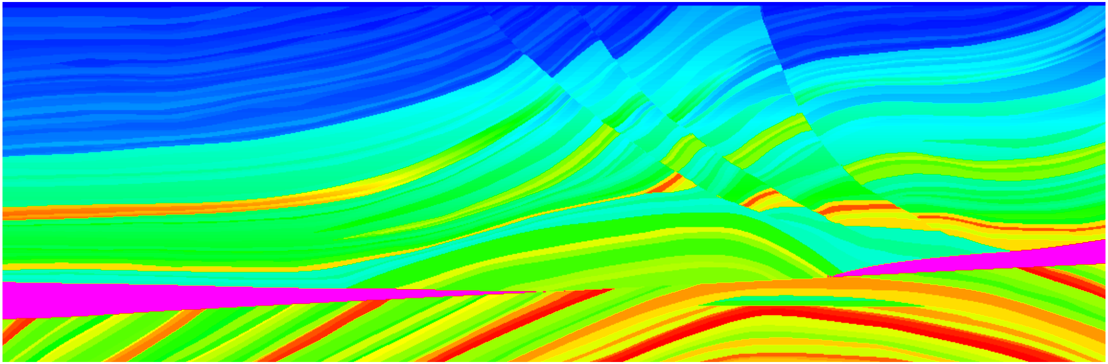
$$\sum_{ijkl} c^{\Omega}_{ijkl} \, E_i \, P_j \, e_k \, p_l - \varrho^{\Omega} \sum_i E_i \, e_i$$

of coefficients c^{Ω}_{ijkl} and ϱ^{Ω} corresponding to the Gabor function.

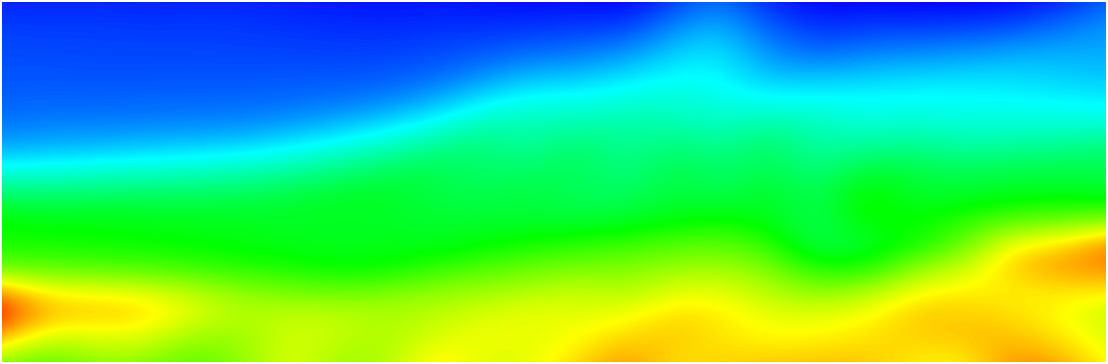
P_i, E_i... slowness vector and polarization vector of the incident wave,
p_i, e_i... slowness vector and unit polarization vector of the sensitivity Gaussian packet.

This information about the Gabor function is lost if the sensitivity Gaussian packet does not fall into the aperture covered by the receivers and into the legible frequency band. The situation improves with the increasing number of differently positioned sources. If we have many sources, the sensitivity Gaussian packets propagating from a Gabor function may be lost during the measurement corresponding to one source, but recorded during the measurement corresponding to another, differently positioned source. However, the problem is not only to record the Gaussian packets from a Gabor function, but to record them in as many different measurement configurations as to resolve perturbation coefficients c^{Ω}_{ijkl} and ϱ^{Ω}.

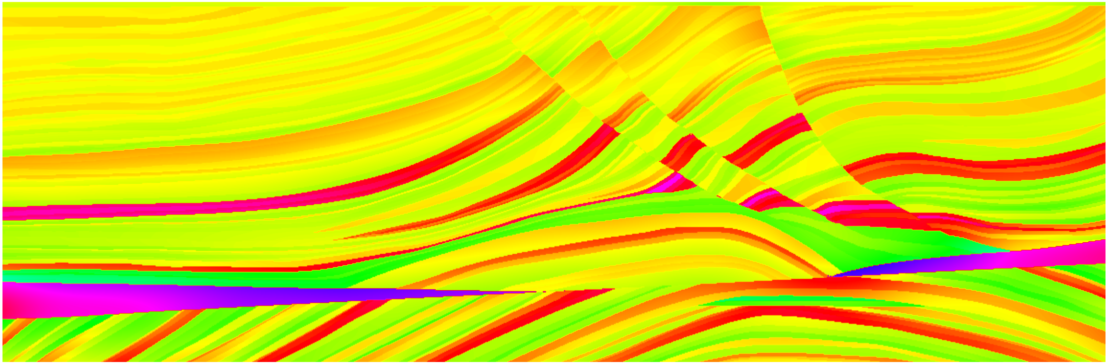
Marmousi example (Klimeš, 2010a)



P-wave velocity in the Marmousi structure.

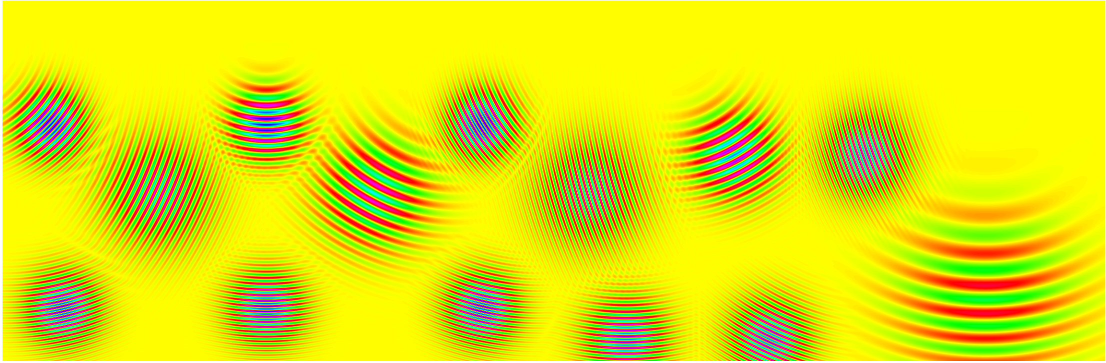


P-wave velocity in the velocity model for ray tracing.



Velocity difference
between the Marmousi structure and the velocity model.

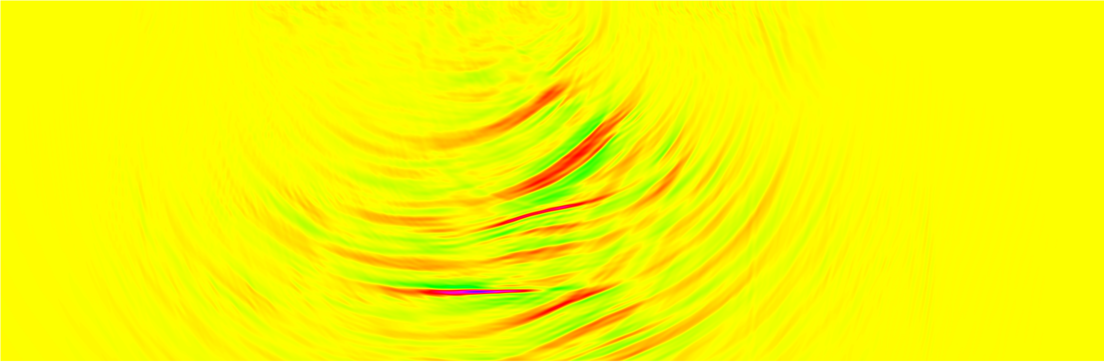
For the decomposition of the 2-D velocity difference, we generate the set of 2-D Gabor functions $g^{\Omega}(\mathbf{x})$ with matrices $\mathbf{K}^{\Omega}$ optimized according to Klimeš (2008b). We obtain 67014 2-D Gabor functions within the selected wavenumber domain.



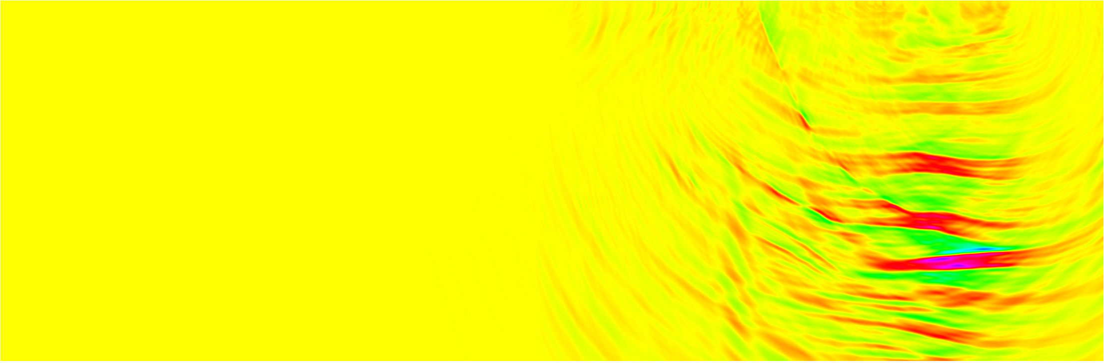
Example showing 14 ones of 67014 optimized 2-D Gabor functions used to decompose the 2-D velocity difference.

We decompose the velocity difference into the sum of Gabor functions. For each shot, we calculate the quantities describing the paraxial approximation of the incident P wave at all central points of Gabor functions. For each shot and each Gabor function, we calculate the initial conditions for the corresponding sensitivity Gaussian packets which form the scattered wave. We consider Gaussian packets corresponding to the given frequency band only. We then trace the central ray of each sensitivity Gaussian packet. If a sensitivity Gaussian packet arrives to the receiver array within the registration time, the recorded wavefield contains information on the corresponding Gabor function.

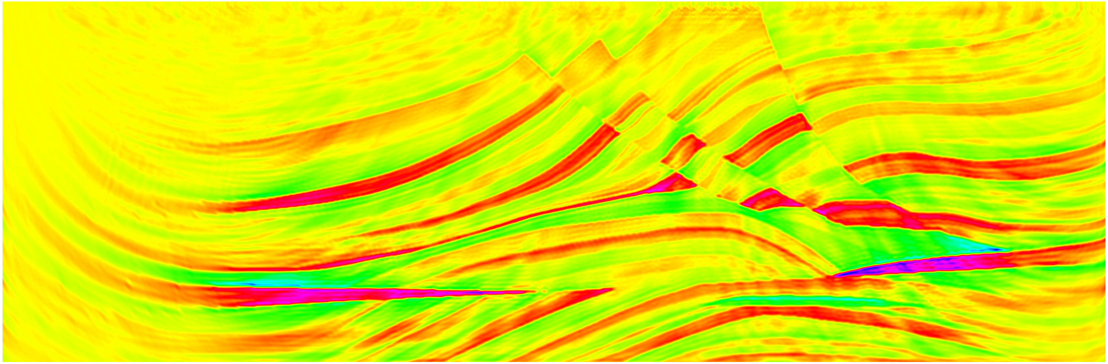
The velocity difference can thus be decomposed into the part to which the recorded seismograms are sensitive and into the part to which the recorded seismograms are not sensitive.



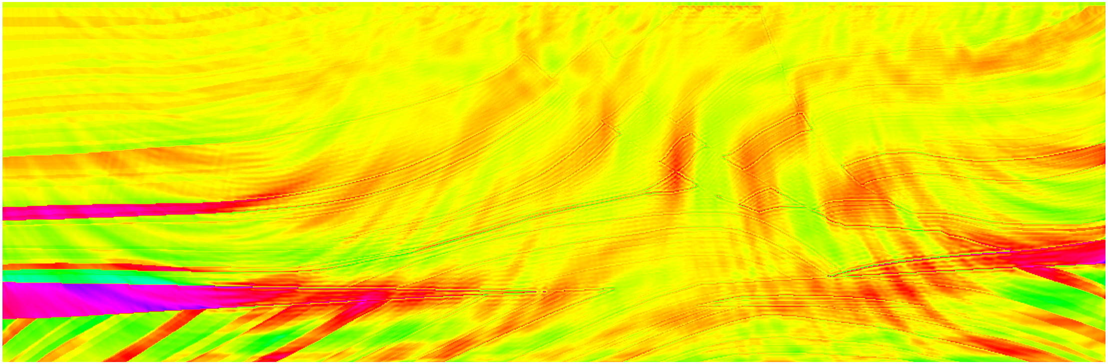
Sum of the Gabor functions
to which the seismograms recorded for shot 70 are sensitive.



Sum of the Gabor functions
to which the seismograms recorded for shot 220 are sensitive.



Sum of the Gabor functions
to which the seismograms collected from all shots are sensitive.



Part of the velocity difference
influencing no recorded seismogram.

Electromagnetic wave equation

Maxwell equations for a linear bianisotropic medium with the local constitutive relations in the Boys-Post representation:

$$[\chi^{\alpha\beta\gamma\delta}(x^\mu)\, A_{\gamma,\delta}(x^\nu)]_{,\beta} + J^\alpha(x^\mu) = 0 \quad .$$

Einstein summation. Indices $i, j, ... = 1, 2, 3$; $\alpha, \beta, ... = 1, 2, 3, 4$.

Electromagnetic vector potential A_α is a covariant 4-vector.
Constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ is a contravariant tensor density of weight -1.
Current density J^α is a contravariant 4-vector density of weight -1.

The constitutive tensor is skew with respect to its first and second indices, and with respect to its third and fourth indices:

$$\chi^{\alpha\beta\gamma\delta} = -\chi^{\beta\alpha\gamma\delta} = -\chi^{\alpha\beta\delta\gamma} \quad .$$

Simplifications of the electromagnetic wave equation

$$[\chi^{\alpha\beta\gamma\delta}(x^\mu)\,A_{\gamma,\delta}(x^\nu)]_{,\beta} + J^\alpha(x^\mu) = 0 \quad .$$

In order to simplify the application of the ray-theory approximation, we are assuming here that constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ is:

Symmetric with respect to its first and second pairs of indices (to simplify the transport equation for amplitude):

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\gamma\delta\alpha\beta} \quad .$$

Time-independent (to apply the ray-theory approximation in the frequency domain):

$$\chi^{\alpha\beta\gamma\delta} = \chi^{\alpha\beta\gamma\delta}(x^m) \quad .$$

Gabor representation of medium perturbations (electromagnetic)

We consider infinitesimally small perturbations $\delta\chi^{\alpha\beta\gamma\delta}$ of the constitutive tensor $\chi^{\alpha\beta\gamma\delta}$ in the electromagnetic wave equation.

We decompose the perturbations of the constitutive tensor into Gabor functions $g^{\Omega}(x^m)$ indexed here by Ω:

$$\delta\chi^{\alpha\beta\gamma\delta}(x^m) = \sum_{\Omega} \chi_{\Omega}^{\alpha\beta\gamma\delta}\, g^{\Omega}(x^m) \quad ,$$

$$g^{\Omega}(x^m) = \exp\left[\mathrm{i}\, k_i^{\Omega}\,(x^i - x_{\Omega}^i) - \tfrac{1}{2}(x^i - x_{\Omega}^i)\, K_{ij}^{\Omega}\,(x^j - x_{\Omega}^j)\right] \quad .$$

Gabor functions $g^{\Omega}(x^m)$ are centred at various spatial positions x_{Ω}^i and have various structural wavenumber vectors k_i^{Ω}.

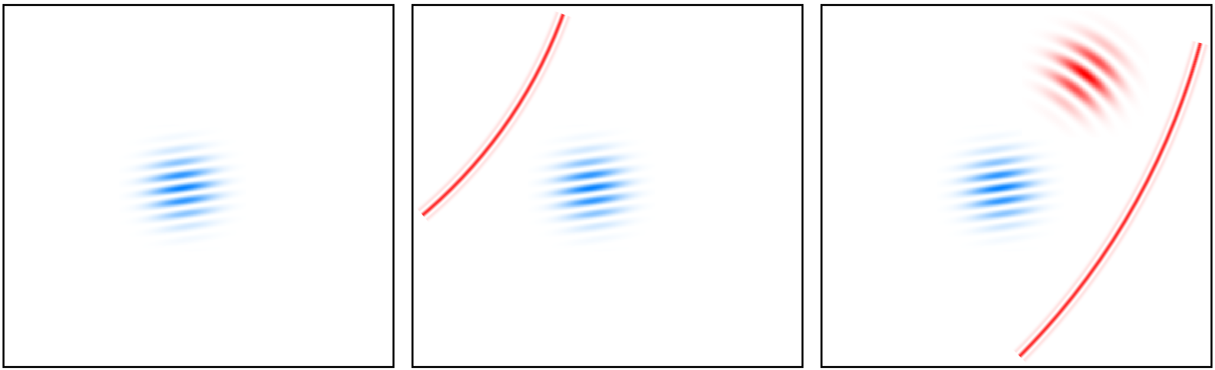
The wavefield scattered by the perturbations is then composed of waves $A_{\alpha}^{\Omega}(x^{\mu})$ scattered by the individual Gabor functions:

$$\delta A_{\alpha}(x^{\mu}) = \sum_{\Omega} A_{\alpha}^{\Omega}(x^{\mu}) \quad .$$

Sensitivity Gaussian packets (electromagnetic)

The mentioned approximations enable us to calculate the waves scattered by Gabor functions analytically (Klimeš, 2010b).

Wave $A^{\Omega}_{\alpha}(x^{\mu})$ scattered by one Gabor function is composed of a few (i.e., 0 to 3 as a rule) Gaussian packets. Each of these "sensitivity" Gaussian packets has a specific frequency and propagates from point x^{i}_{Ω} in a specific direction:



A single Gabor function $g^{\Omega}(x^{i}_{\Omega})$ centred at point x^{i}_{Ω}.

Broad-band wave incident at the Gabor function.

Scattered wave $A^{\Omega}_{\alpha}(x^{\mu})$ composed of one sensitivity Gaussian packet.

Each of these sensitivity Gaussian packets scattered by Gabor function $g^{\Omega}(x^m)$ is sensitive to just a single linear combination

$$\sum_{\alpha\beta\gamma\delta} \chi_{\Omega}^{\alpha\beta\gamma\delta} E_{\alpha} \, P_{\beta} \, e_{\gamma} \, p_{\delta}$$

of coefficients $\chi_{\Omega}^{\alpha\beta\gamma\delta}$ corresponding to the Gabor function.

P_i ... slowness vector of the incident wave; $P_4 = -1$
E_α ... polarization vector of the incident wave
p_i ... slowness vector of the sensitivity Gaussian packet; $p_4 = -1$
e_α ... unit polarization vector of the sensitivity Gaussian packet

This information about the Gabor function is lost if the sensitivity Gaussian packet does not fall into the aperture covered by the receivers and into the legible frequency band.

Conclusions

Perturbations of elastic moduli and density can be decomposed into Gabor functions.

A short-duration broad-band wave with a smooth frequency spectrum incident at each Gabor function generates at most a few scattered Gaussian packets.

Each Gaussian packet has a specific frequency and propagates in a specific direction. Refer to Klimeš (2007) for the relevant equations.

Each Gaussian packet is sensitive to a single linear combination of the perturbations of elastic moduli and density corresponding to the Gabor function. This information about the Gabor function is lost if the Gaussian packet does not fall into the aperture covered by the receivers and into the legible frequency band.

The sensitivity Gaussian packets can enable to replace migrations by true linearized inversion of reflection seismic data. For the algorithm of the linearized inversion of the complete set of seismograms recorded for all shots refer to Klimeš (2008a).

References (online at “http://sw3d.cz”)

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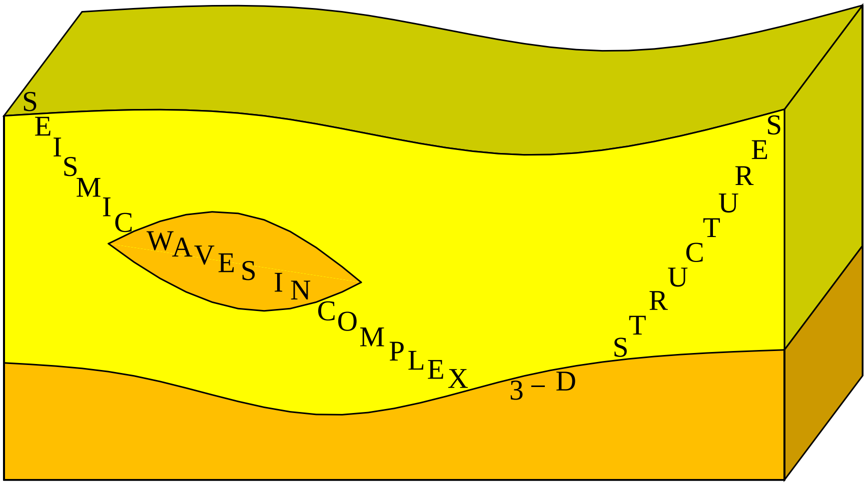
Acknowledgements

The research has been supported:

by the Grant Agency of the Czech Republic under contract P210/10/0736,

by the Ministry of Education of the Czech Republic within research project MSM0021620860,

and by the consortium "Seismic Waves in Complex 3-D Structures"



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