Comparison of the FORT approximation of the coupling ray theory with the Fourier pseudospectral method

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3D Fourier pseudospectral method

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- S waves in inhomogeneous weakly anisotropic media
  and in vicinity of a singularity are coupled

- ray theories for S waves in inhomogeneous isotropic
  or anisotropic media do not describe coupling

Introduction

- coupling ray theory, quasi-isotropic approximation

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- common ray - artificial trajectory approximating rays
  of S1 and S2 waves

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  combination of the coupling ray theory
  and first-order ray tracing (FORT)

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- first-order common S-wave ray

- second-order coupling equations
  along the first-order common S-wave ray

Second-order coupling ray theory

First-order common S-wave ray tracing

$$dx_i/d\tau = \tfrac{1}{2}\partial G^{[\mathcal{M}]}/\partial p_i \ , \qquad dp_i/d\tau = -\tfrac{1}{2}\partial G^{[\mathcal{M}]}/\partial x_i$$

x_i - coordinates of the first-order ray

p_i - components of the first-order slowness vector $\mathbf{p}$

τ - first-order traveltime

$G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p})$ - S-wave first-order mean eigenvalue; $G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p}) = 1$

$$G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p}) = \tfrac{1}{2}[G^{(1)}_{S1}(\mathbf{x},\mathbf{p}) + G^{(1)}_{S2}(\mathbf{x},\mathbf{p})]$$

$G^{(1)}_{SI}(\mathbf{x},\mathbf{p})$ - S-wave first-order eigenvalues of Christoffel matrix $\mathbf{\Gamma}$

Second-order coupling ray theory

First-order dynamic ray-tracing

$$dX_i^{(I)}/d\tau = \frac{1}{2}\left(\frac{\partial^2 G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p})}{\partial p_i \partial x_j} X_j^{(I)} + \frac{\partial^2 G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p})}{\partial p_i \partial p_j} Y_j^{(I)}\right)$$

$$dY_i^{(I)}/d\tau = -\frac{1}{2}\left(\frac{\partial^2 G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p})}{\partial x_i \partial x_j} X_j^{(I)} + \frac{\partial^2 G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p})}{\partial x_i \partial p_j} Y_j^{(I)}\right)$$

$$X_i^{(I)} = [\partial x_i / \partial \gamma^{(I)}]_{\tau=const} \, , \qquad Y_i^{(I)} = [\partial p_i / \partial \gamma^{(I)}]_{\tau=const}$$

$\gamma^{(I)}$ - ray parameters (e.g., take-off angles)

Second-order coupling ray theory

Displacement vector of coupled S waves

$$\mathbf{u}(\tau,\omega) = [\mathcal{A}(\tau)\mathbf{f}^{[1]}(\tau) + \mathcal{B}(\tau)\mathbf{f}^{[2]}(\tau)]\; D^{[\mathcal{M}]}(\tau)\, \exp(\mathrm{i}\omega\tau)$$

$\mathbf{u}$ - displacement vector of coupled S waves

τ - traveltime along common S-wave ray

$\mathcal{A}$, $\mathcal{B}$ - amplitude terms

$D^{[\mathcal{M}]}(\tau) = [\rho(\tau)c^{[\mathcal{M}]}(\tau)]^{-1/2}[\mathcal{L}^{[\mathcal{M}]}(\tau)]^{-1}$ - common S-wave ray amplitude

ρ - density, $c^{[\mathcal{M}]}$ - common S-wave phase velocity

$\mathcal{L}^{[\mathcal{M}]}$ - common S-wave geometrical spreading

Second-order coupling ray theory

Second-order coupling equations

$$\begin{pmatrix} d\mathcal{A}/d\tau \\ d\mathcal{B}/d\tau \end{pmatrix} = -\mathrm{i}\omega/2 \begin{pmatrix} M_{11}-1 & M_{12} \\ M_{12} & M_{22}-1 \end{pmatrix} \begin{pmatrix} \mathcal{A} \\ \mathcal{B} \end{pmatrix}$$

$$M_{KL} = B_{KL} - B_{K3}B_{L3}/(B_{33}-1)$$

$$M_{KL} = M_{KL}(\mathbf{x},\mathbf{p}) \; , \qquad B_{kl} = B_{kl}(\mathbf{x},\mathbf{p})$$

Second-order coupling ray theory

$$B_{mn} = B_{mn}(\mathbf{x}, \mathbf{p}) = \Gamma_{ik}(\mathbf{x}, \mathbf{p}) e_i^{[m]}(\mathbf{x}) e_k^{[n]}(\mathbf{x})$$

Γ_{ik} - elements of Christoffel matrix $\mathbf{\Gamma}$

$\mathbf{e}^{[m]}$ - triplet of orthonormal vectors

$\mathbf{e}^{[3]} = c^{[\mathcal{M}]}\mathbf{p},$ $\mathbf{e}^{[1]}$, $\mathbf{e}^{[2]}$ arbitrarily in the plane $\perp \mathbf{e}^{[3]}$

$\mathbf{p}$ - first-order common S-wave slowness vector

$c^{[\mathcal{M}]}$ - common S-wave phase velocity

Second-order coupling ray theory

$$\mathbf{f}^{[K]} = \mathbf{e}^{[K]} - (c^{[\mathcal{M}]})^2 (V_P^2 - V_S^2)^{-1} B_{K3}\, \mathbf{e}^{[3]}$$

$\mathbf{f}^{[K]}$ define S-wave polarization plane perpendicular to

$$\mathbf{f}^{[3]} = \mathbf{e}^{[3]} + (c^{[\mathcal{M}]})^2 (V_P^2 - V_S^2)^{-1} (B_{13}\mathbf{e}^{[1]} + B_{23}\mathbf{e}^{[2]})$$

$\mathbf{f}^{[i]}$ - non-perpendicular, non-unit vectors

$V_S^2(x_m) = (c^{[\mathcal{M}]})^2$, $\qquad V_P^2(x_m) = (c^{[\mathcal{M}]})^2 B_{33}$

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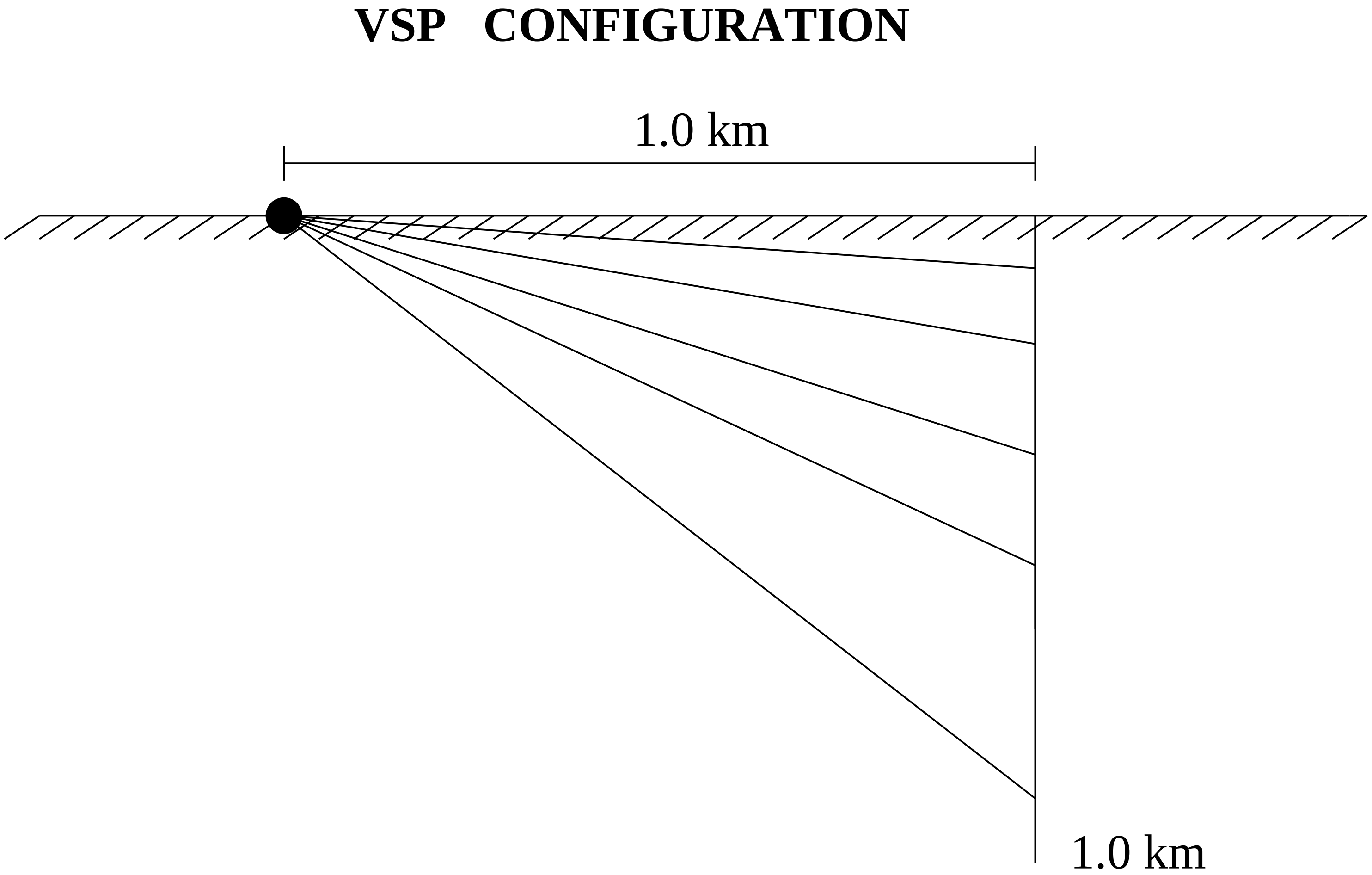
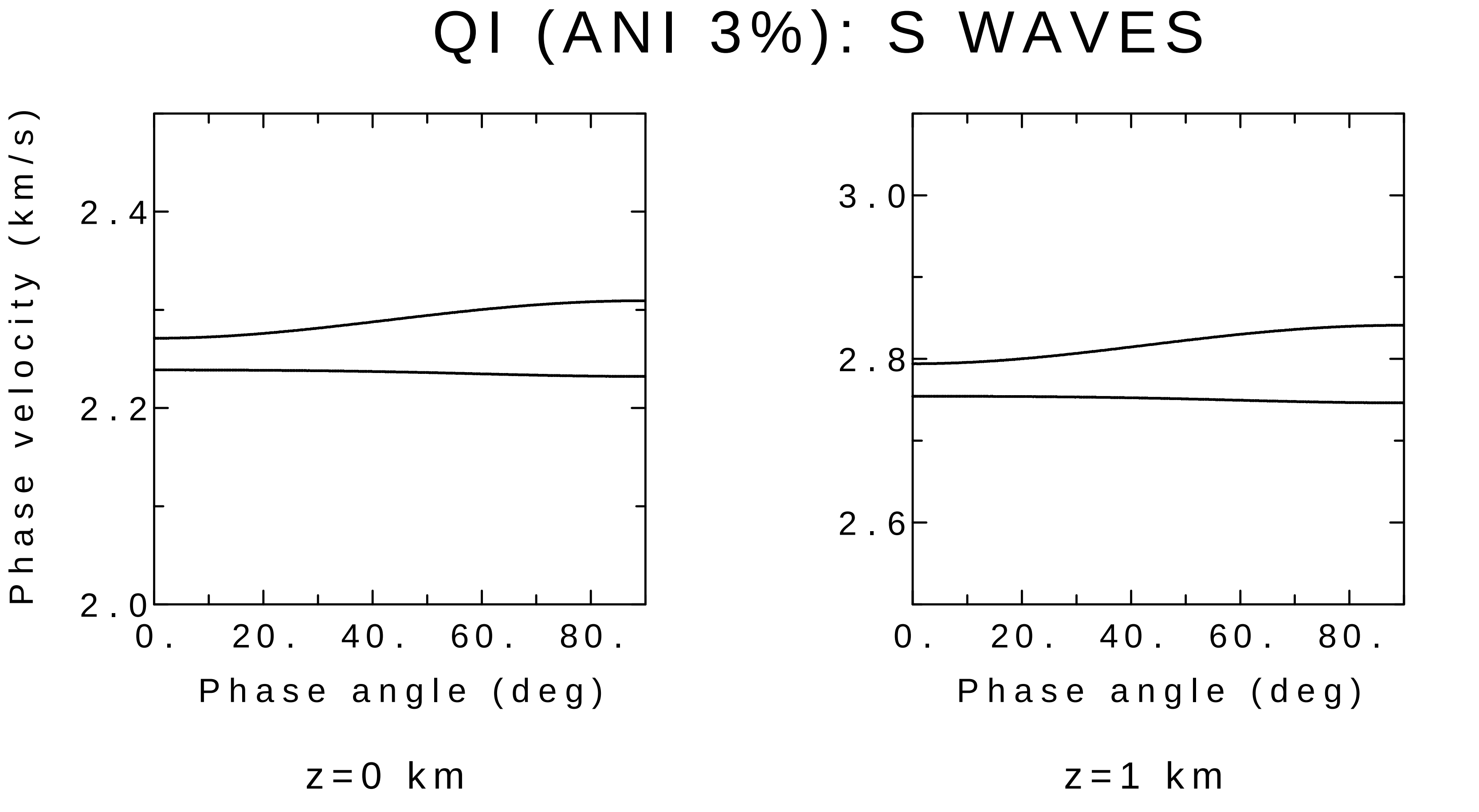
- high accuracy, suppressed numerical anisotropy
- large grid spacing allowed
- more expensive than FD
- inherent periodicity of model boundaries $\Rightarrow$
  special treatment to avoid reflections/wrap-around

Numerical examples



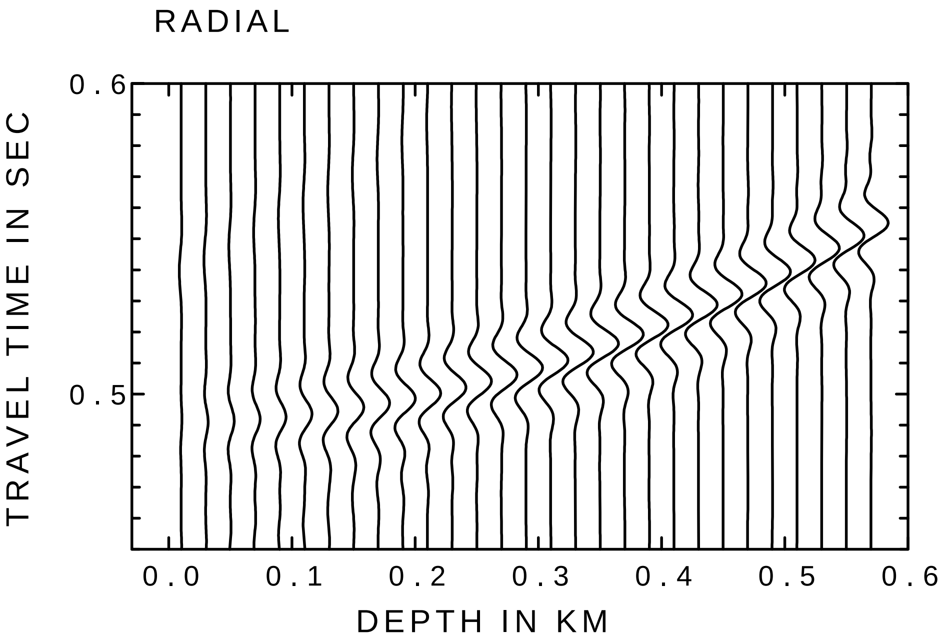
Numerical examples

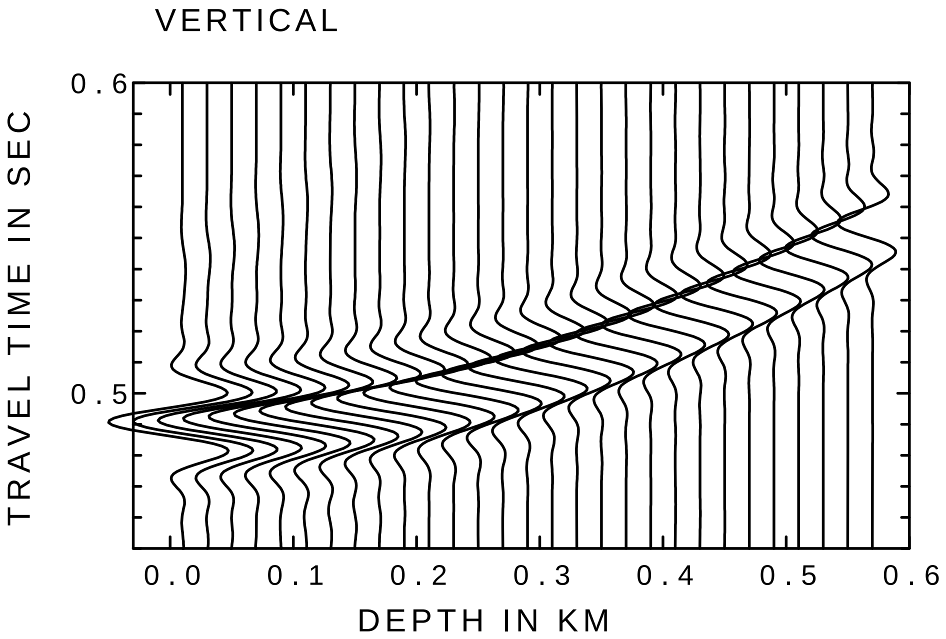
HTI rotated by 45 degrees (Klimeš & Bulant, 2004)

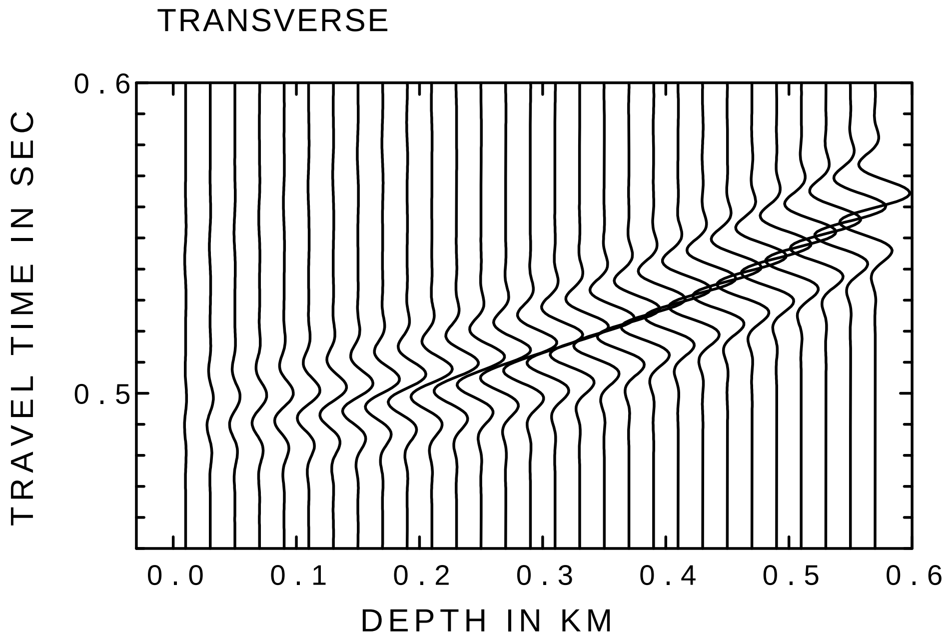


Numerical examples

QI: FM







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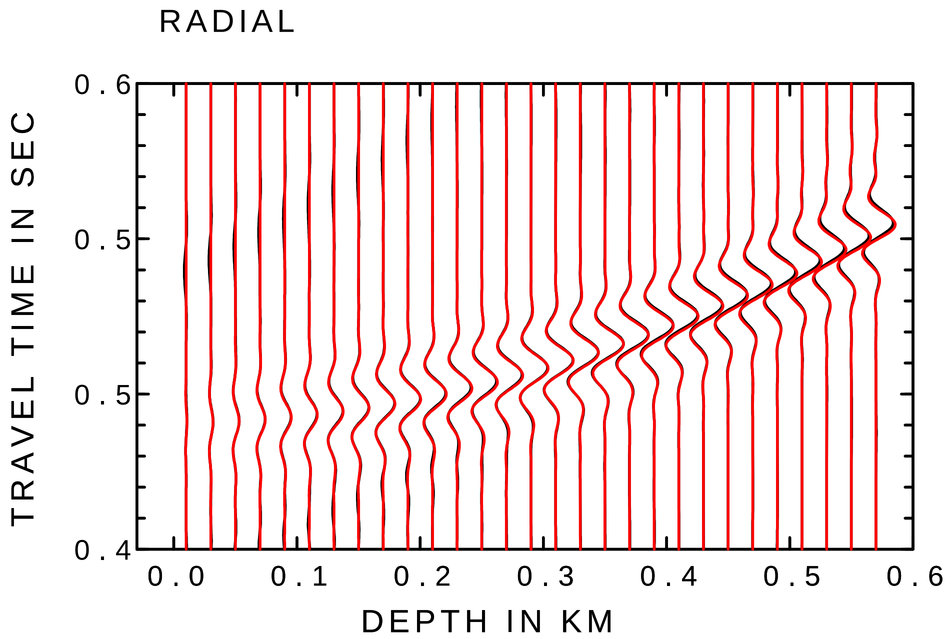

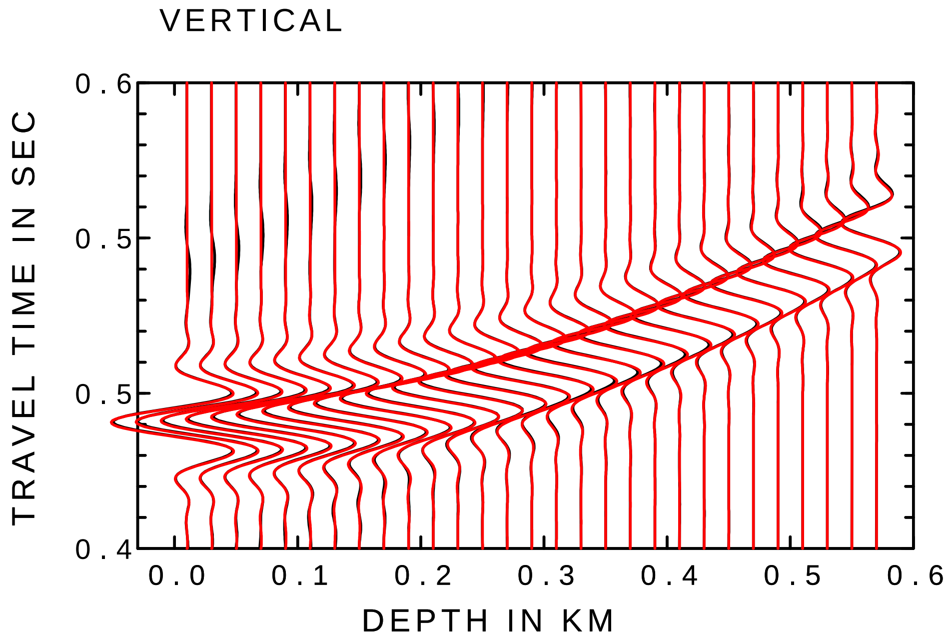

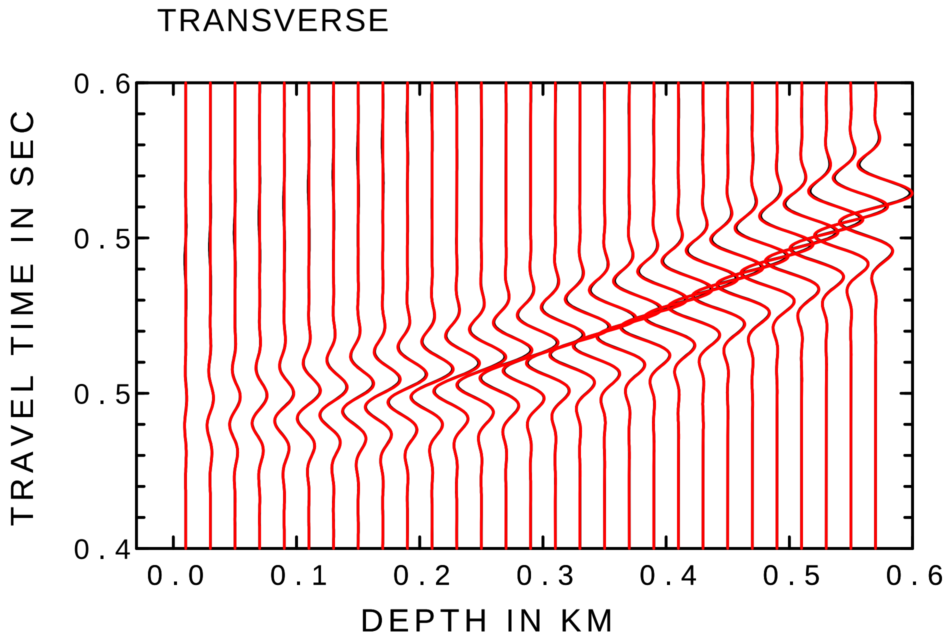

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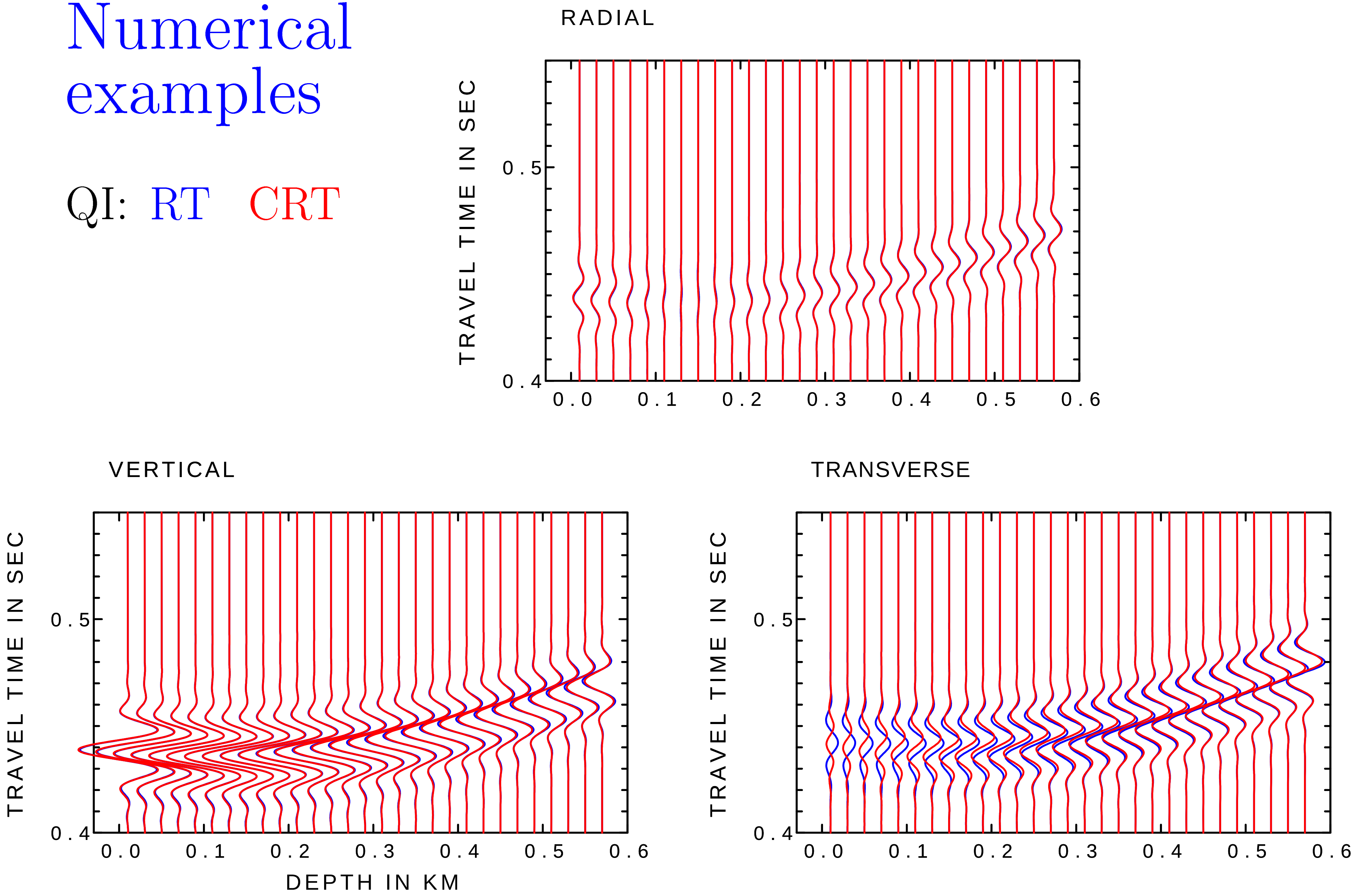
QI: RT CRT



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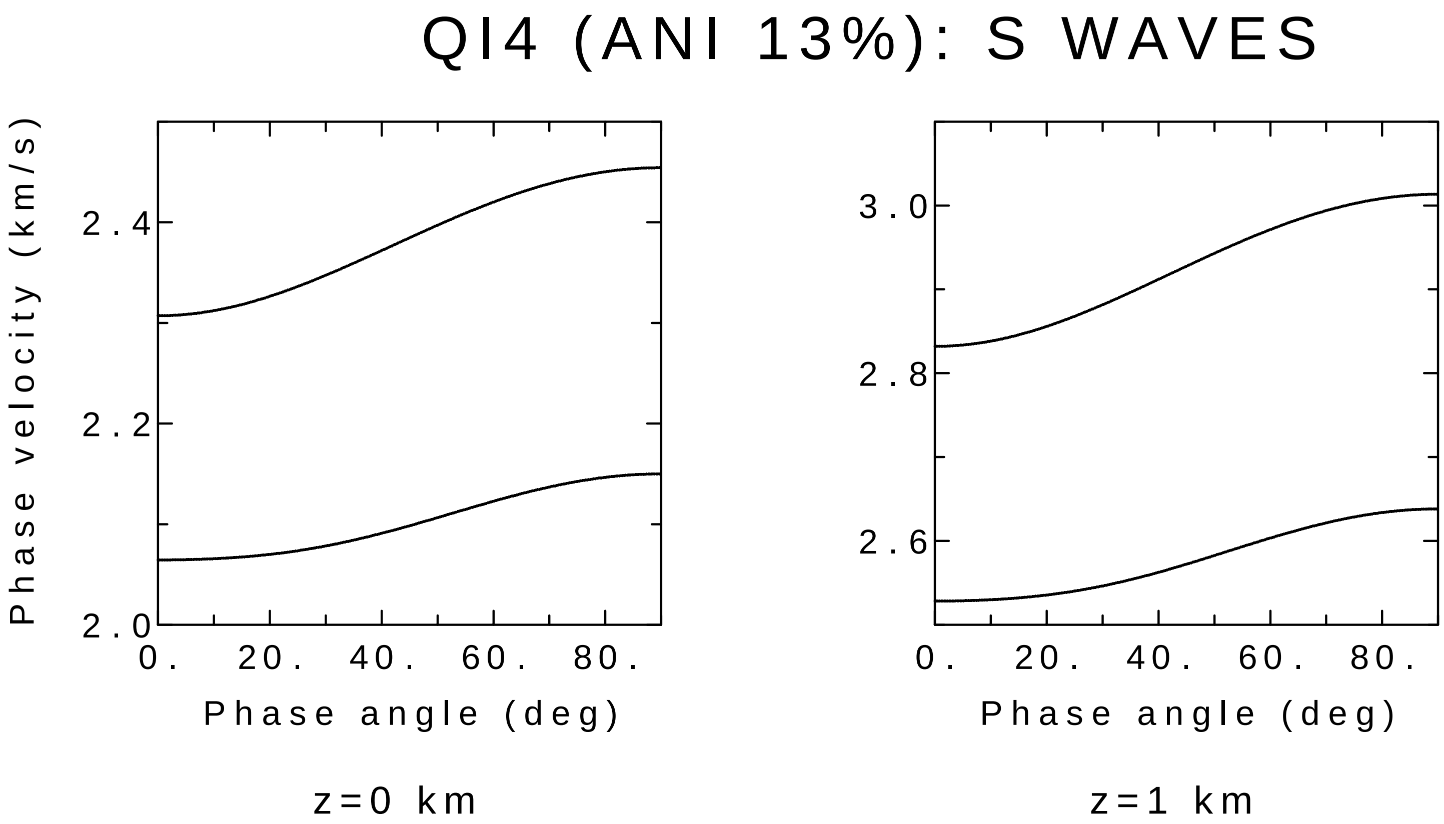
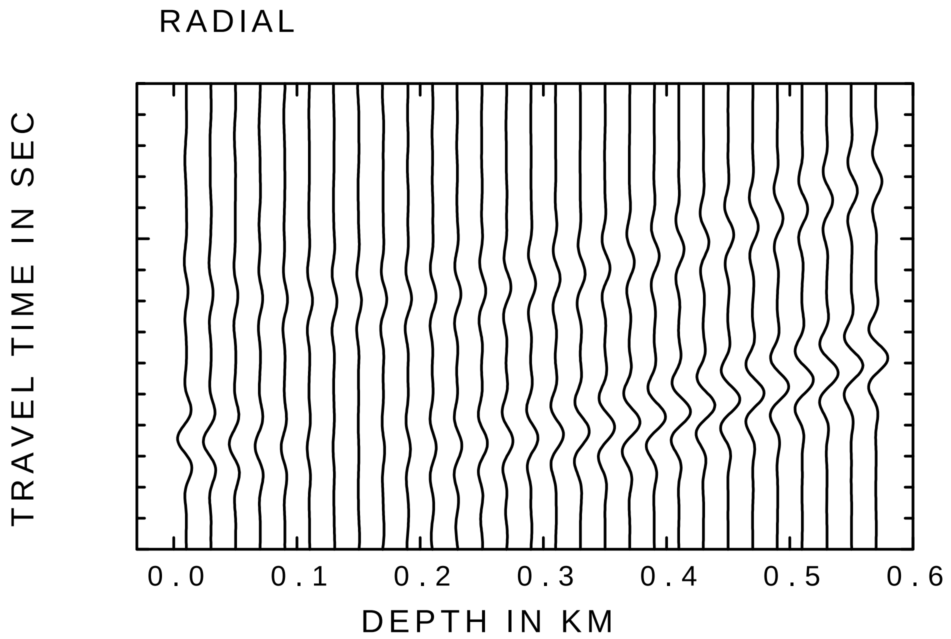
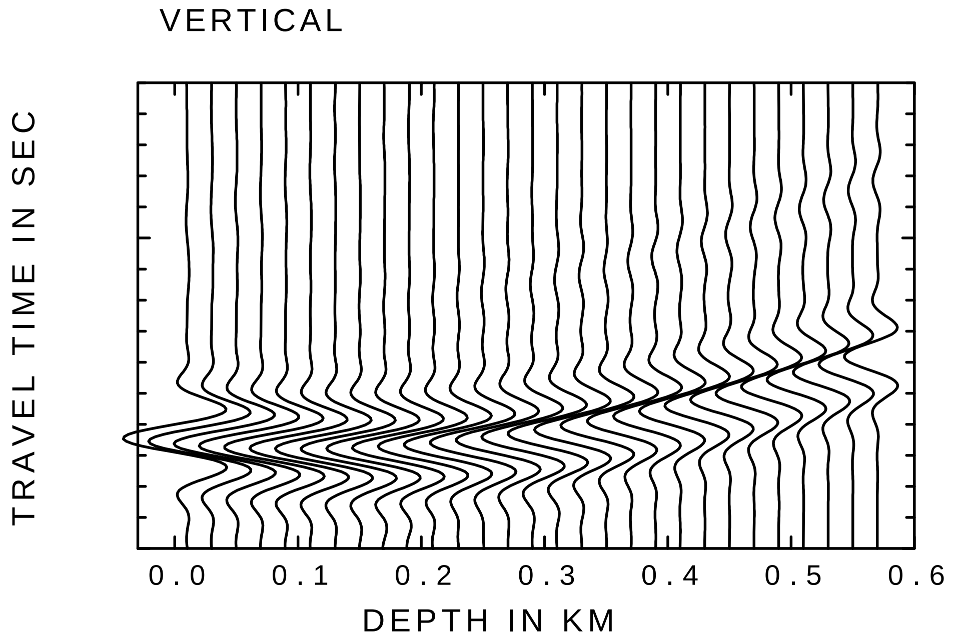
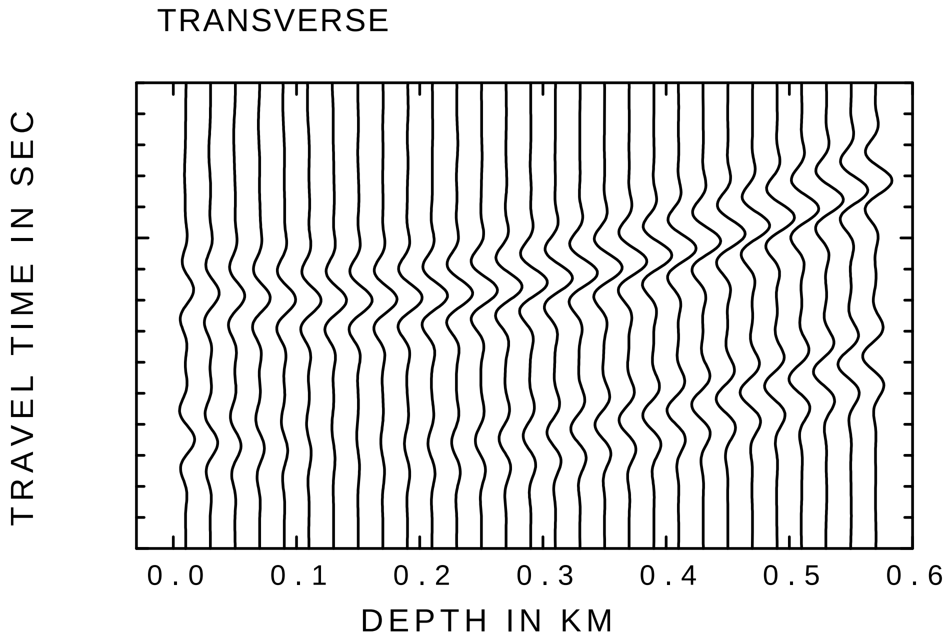


Numerical examples

QI4: FM







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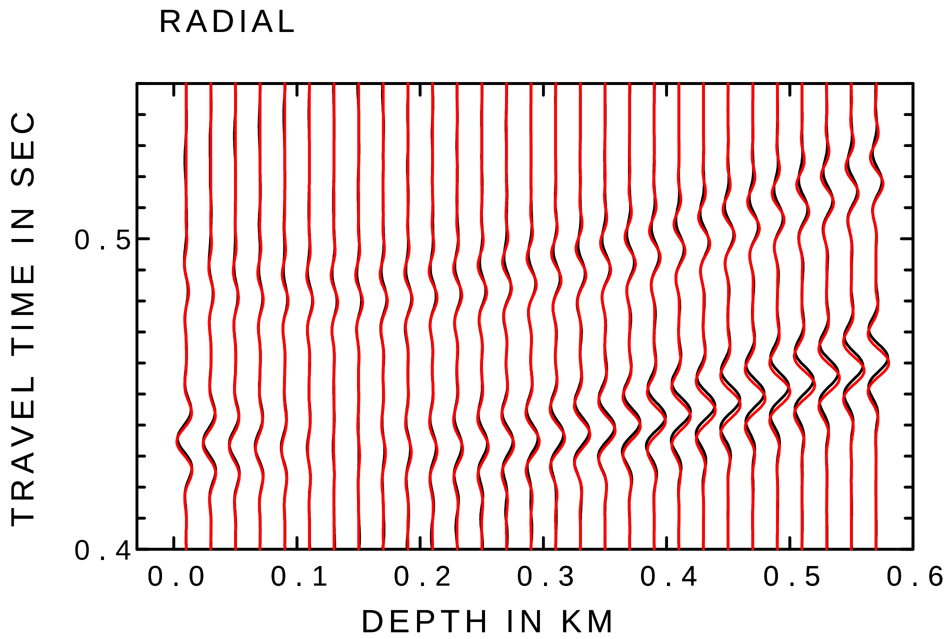

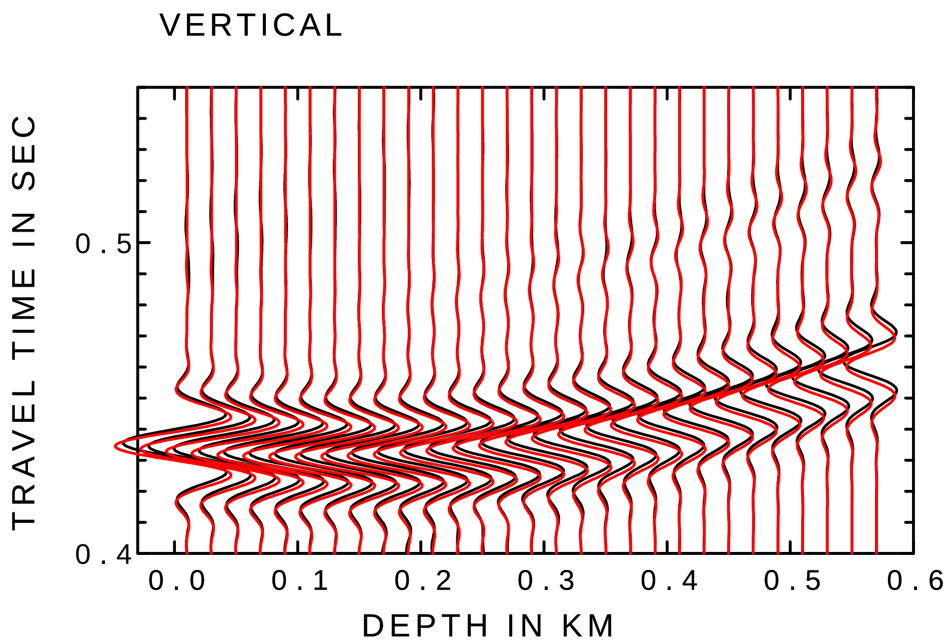

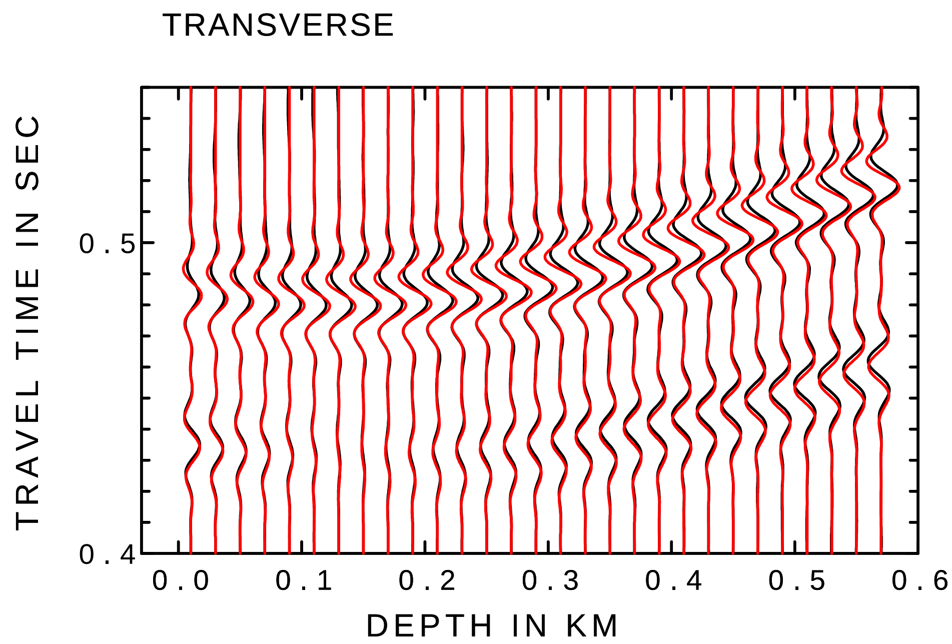

Numerical examples

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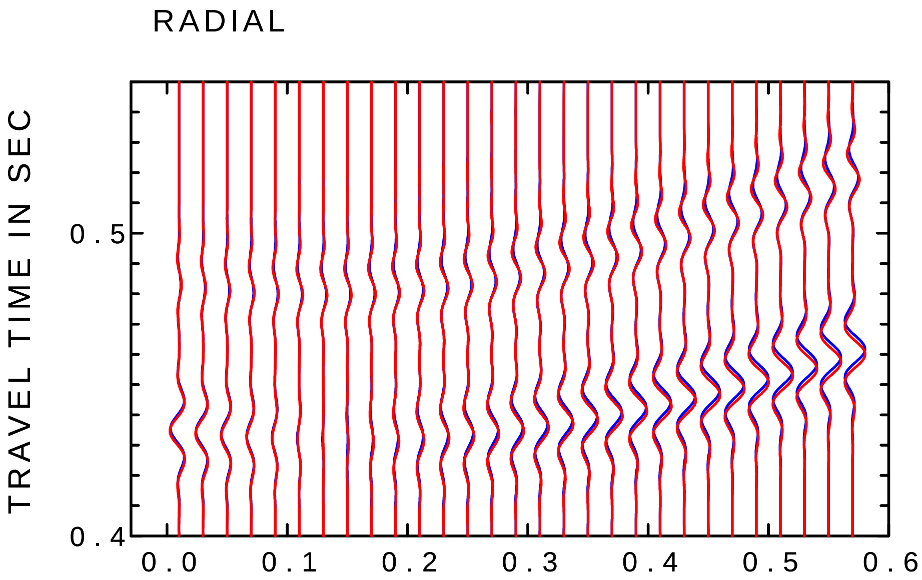
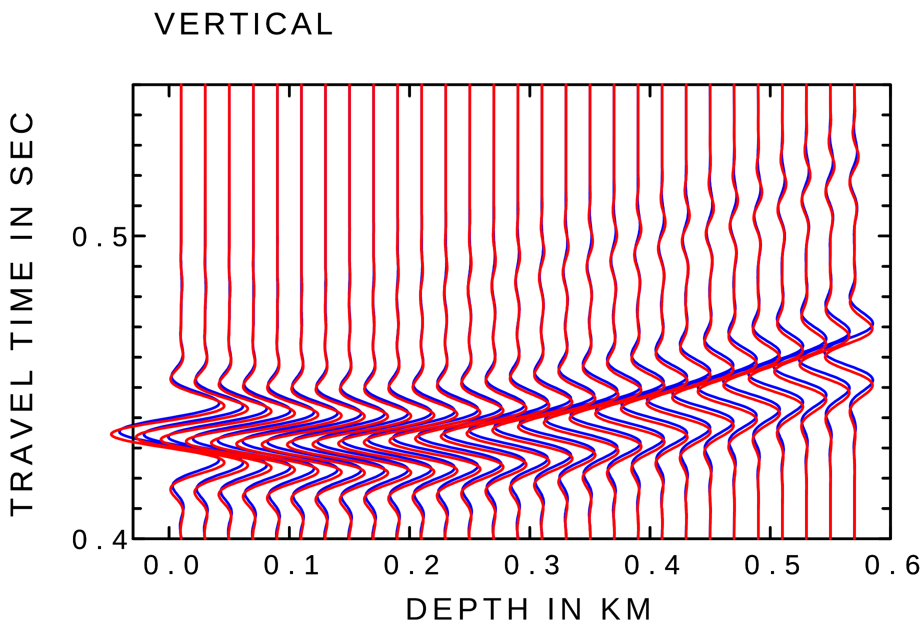
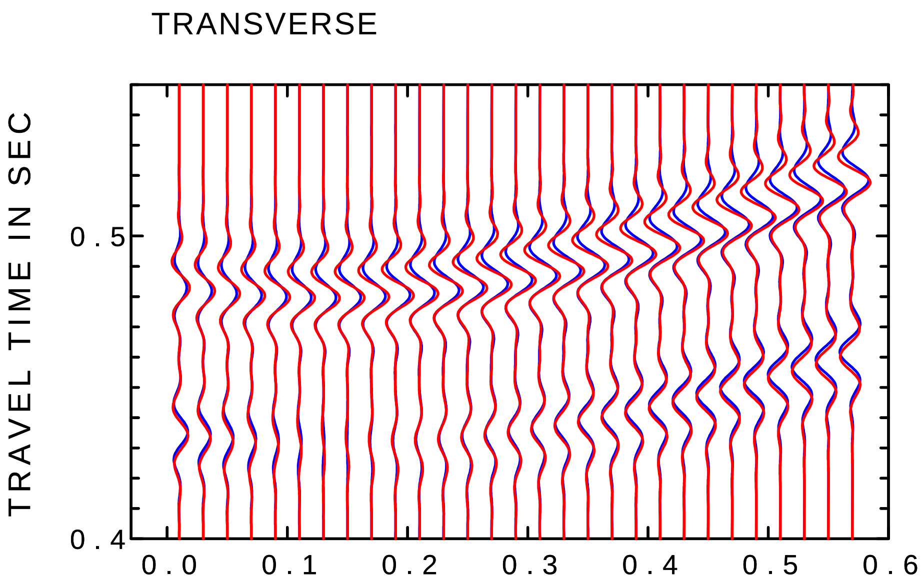


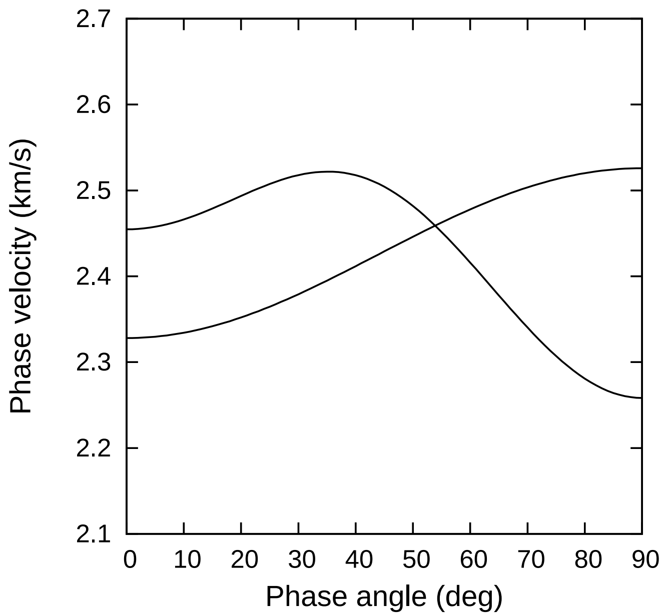




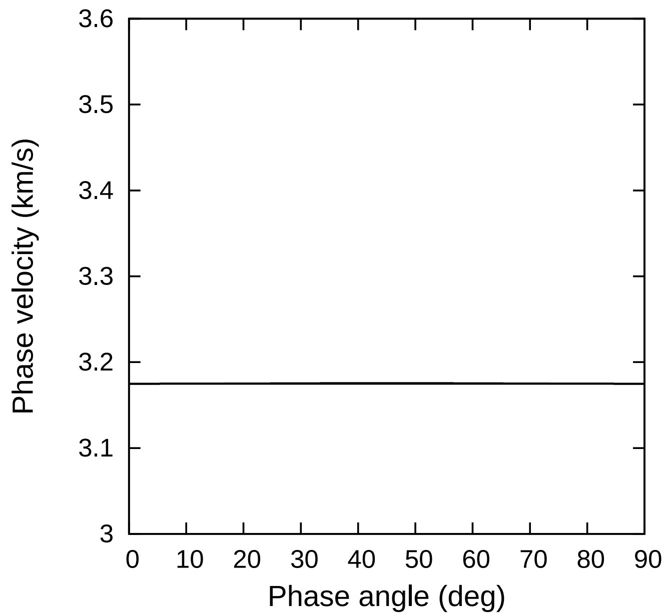
Numerical examples (modified Shearer & Chapman, 1989)

MODEL SC1G I (ANI 11% at z=0 km)

S WAVES

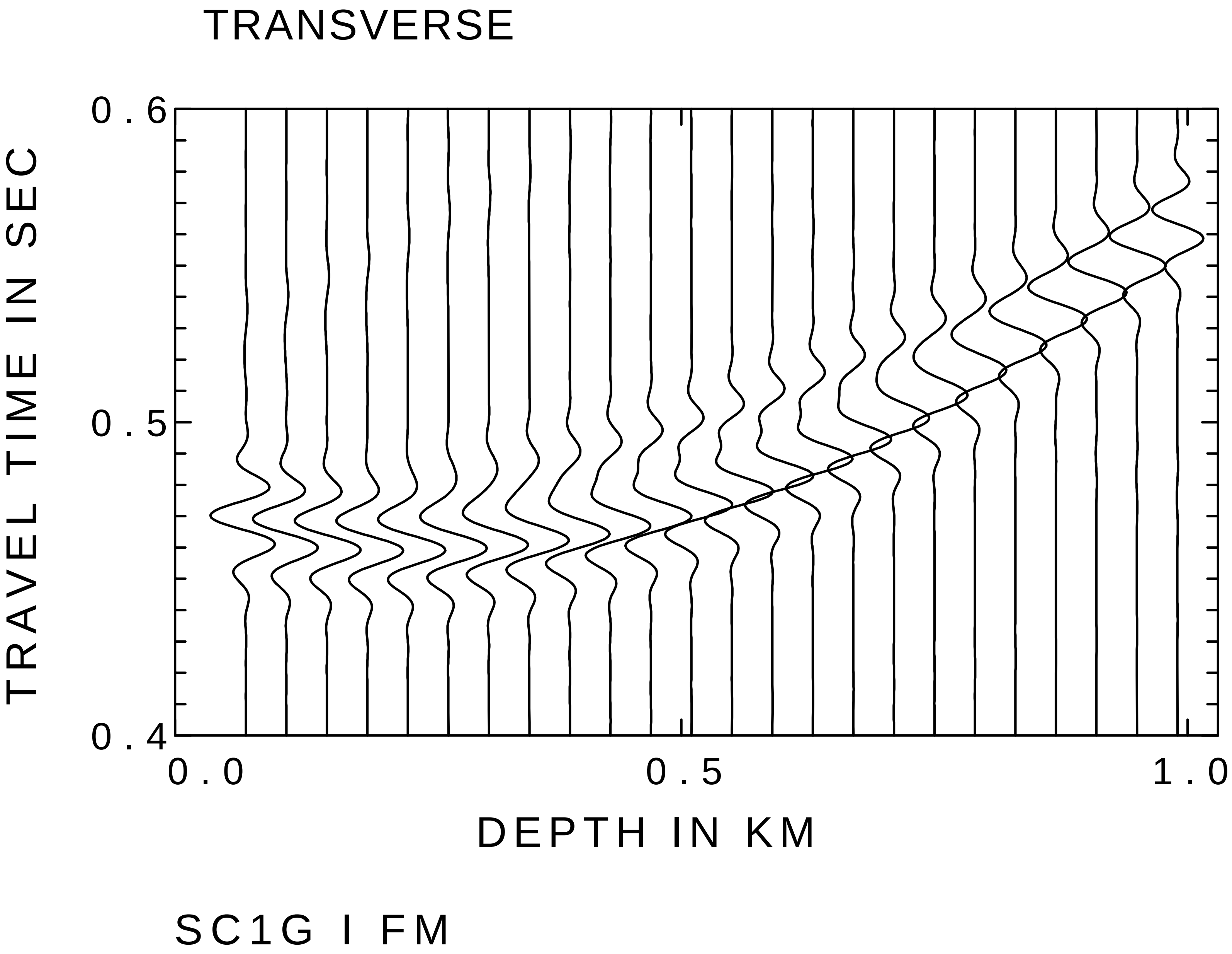


z=0 km



z=1.5 km

Numerical examples



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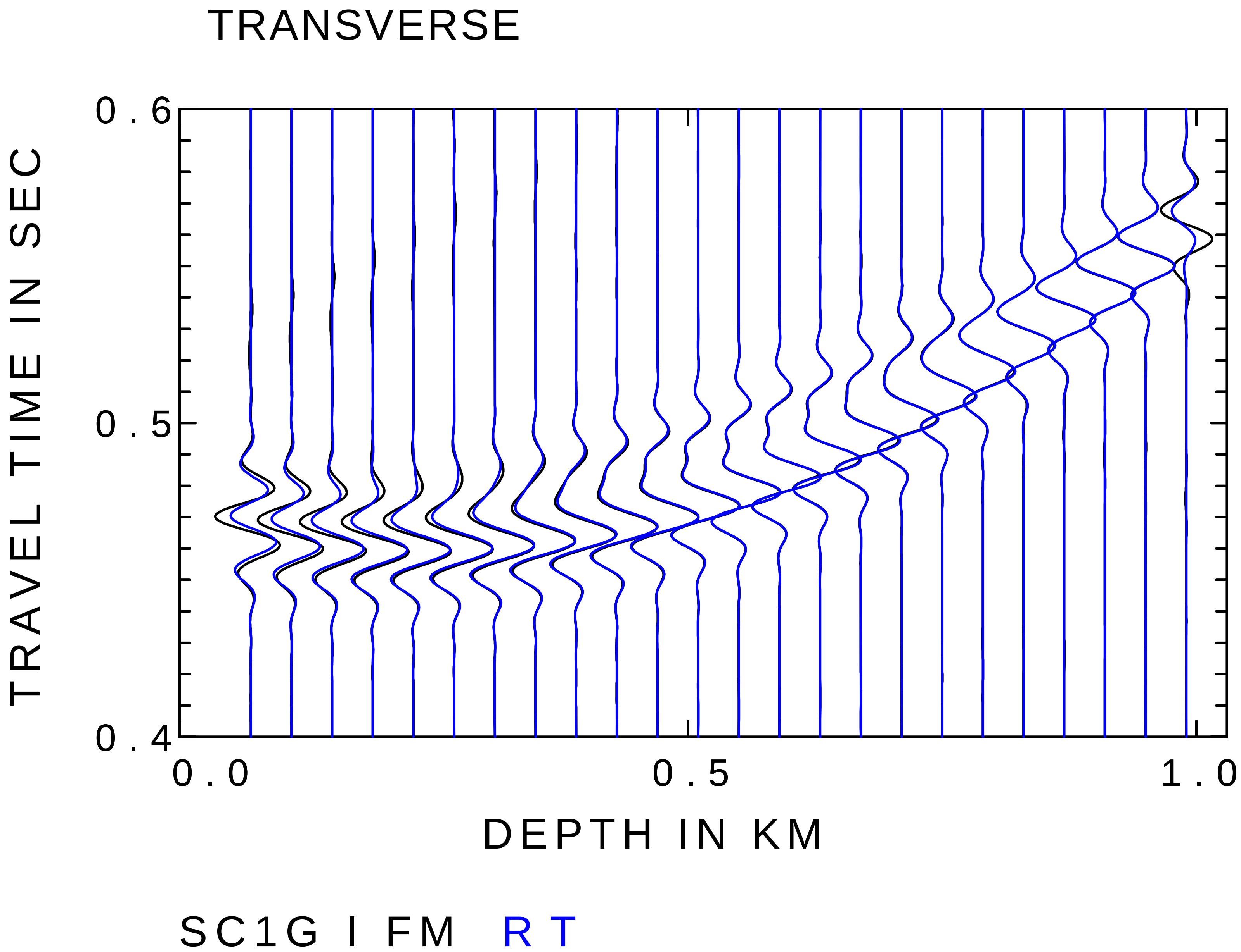

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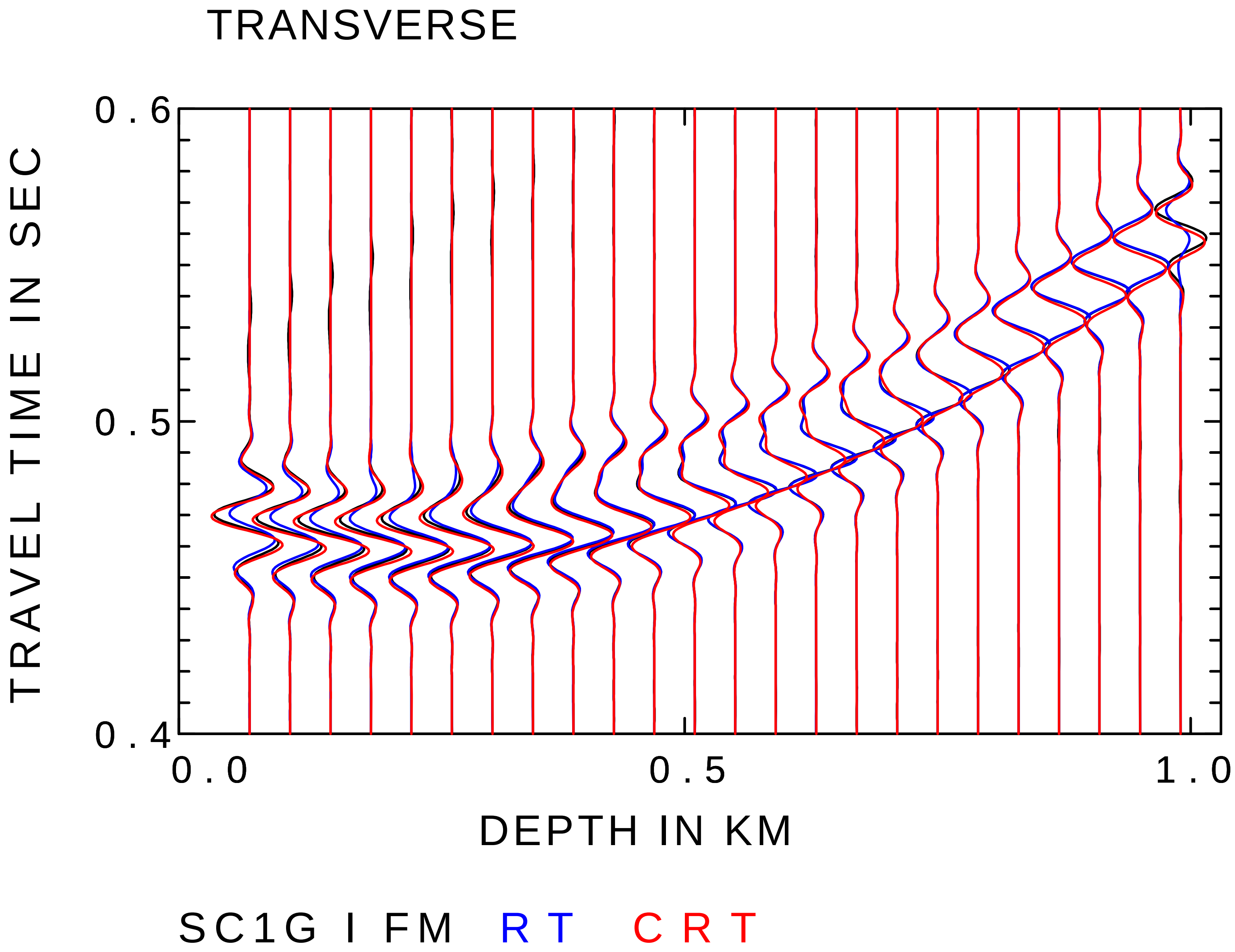
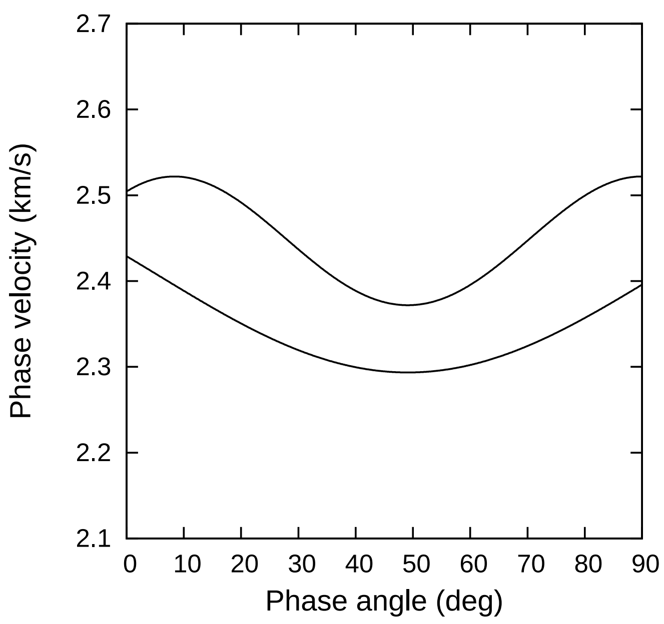


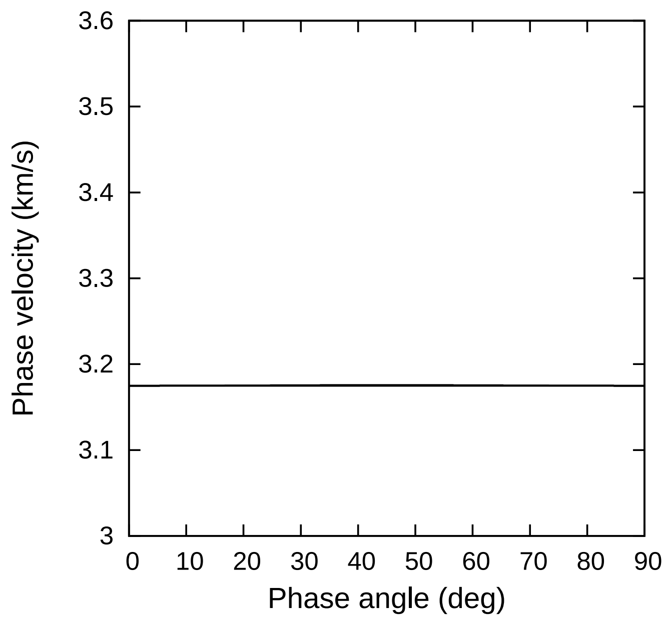
Numerical examples **(modified Shearer & Chapman, 1989)**

MODEL SC1G II (ANI 11% at z=0 km)

S WAVES



z=0 km



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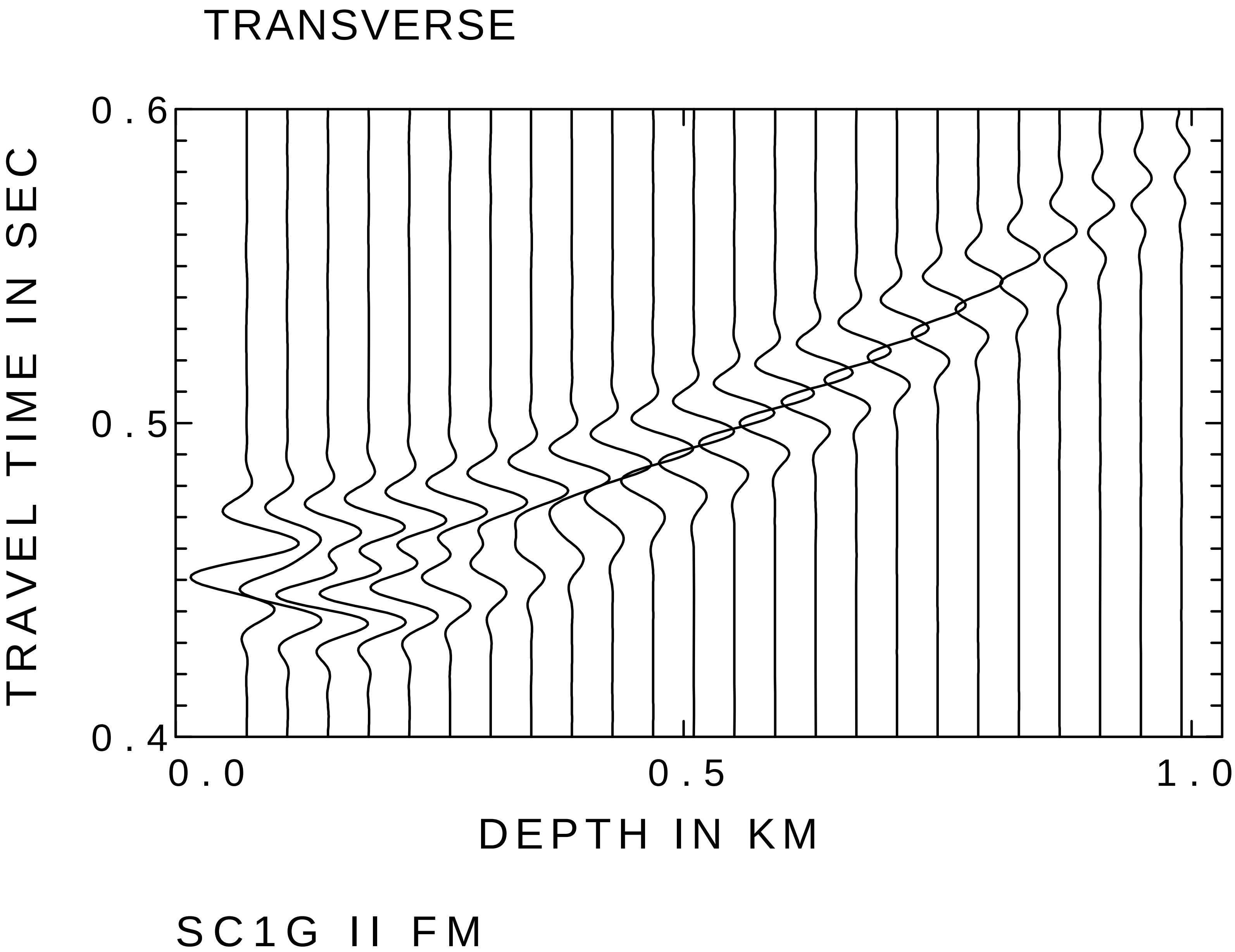

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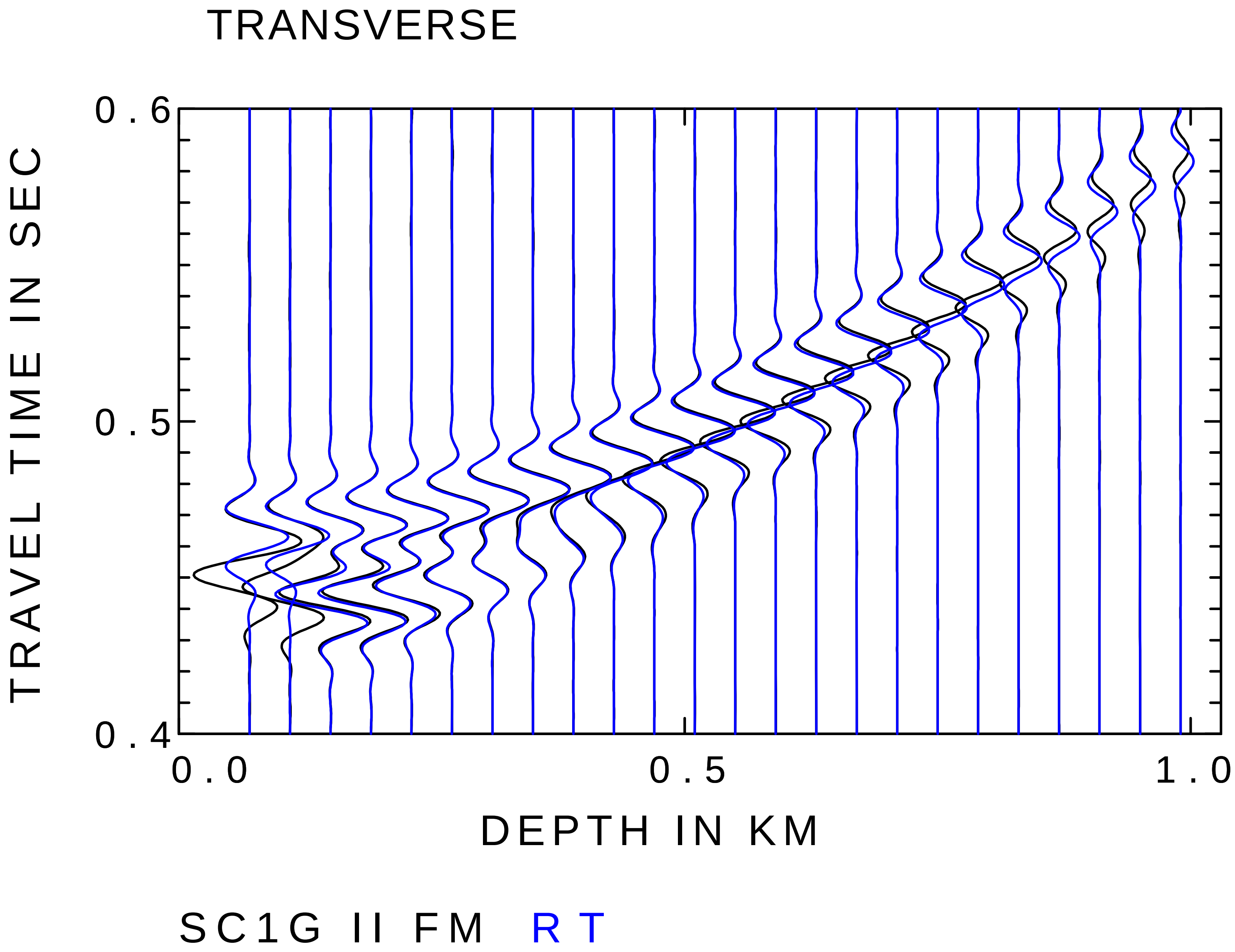

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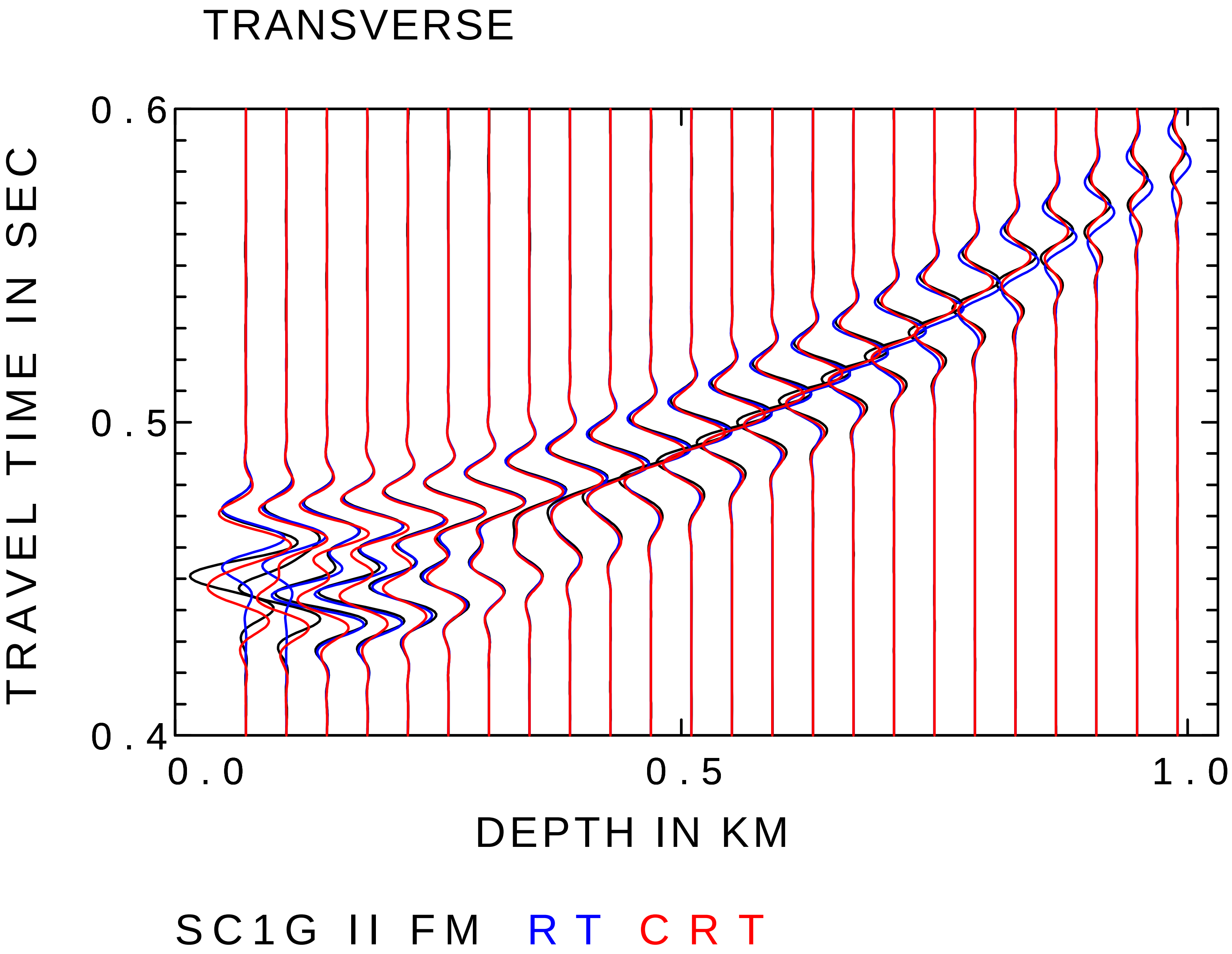

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