Boundary attenuation angles for inhomogeneous plane waves in dissipative media

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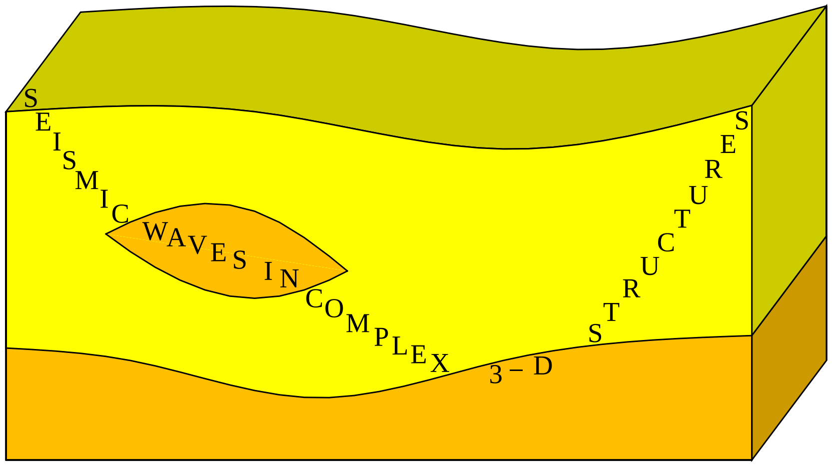

Outline:

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BASIC FORMULAE

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NUMERICAL EXAMPLES

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BASIC FORMULAE

NUMERICAL EXAMPLES

ISOTROPIC MEDIA

Outline:

BASIC FORMULAE

NUMERICAL EXAMPLES

- ISOTROPIC MEDIA

- ANISOTROPIC MEDIA

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- BASIC FORMULAE
- NUMERICAL EXAMPLES
  - ISOTROPIC MEDIA
  - ANISOTROPIC MEDIA
- CONCLUSIONS

BASIC FORMULAE

Slowness vector

$$\mathbf{p} = \mathbf{P} + \mathrm{i}\mathbf{A}$$

$\mathbf{p}$ - slowness vector, $\mathbf{P}$ - propagation vector, $\mathbf{A}$ - attenuation vector

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Attenuation (inhomogeneity) angle γ $\quad \gamma \geq 0^0$

$$\cos\gamma = \mathbf{P} \cdot \mathbf{A} / |\mathbf{P}||\mathbf{A}|$$

$\gamma = 0^0$ - homogeneous wave

$\gamma > 0^0$ - inhomogeneous wave

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Boundary attenuation angle γ^*

$$\gamma \in < 0^0, \gamma^*) \qquad \gamma^* < 180^0$$

BASIC FORMULAE

Directional specification of slowness vector

$$\mathbf{p} = \mathcal{C}^{-1}(\mathbf{N} + \mathrm{i}\delta\mathbf{M}), \qquad \mathbf{N} \parallel \mathbf{P}, \qquad \mathbf{M} \parallel \mathbf{A}$$

$\mathbf{N}$, $\mathbf{M}$ - unit vectors (given; real valued) $\Rightarrow$ γ **given**

$\mathcal{C}$ - phase velocity (sought; real valued)

δ - attenuation-propagation ratio (sought; real valued)

$\mathcal{C}$ and δ solutions of coupled polynomial equations

of 3rd and 6th degree

BASIC FORMULAE

Mixed specification of slowness vector

$$\mathbf{p} = \sigma\mathbf{n} + \mathrm{i}D\mathbf{m}, \qquad \mathbf{n}\cdot\mathbf{m} = 0, \qquad \mathbf{n} \parallel \mathbf{P}$$

$\mathbf{n}$, $\mathbf{m}$ - unit vectors (given; real valued)

D - inhomogeneity parameter (given, real valued,

non-negative, $0 \leq D < \infty$, dimension s/km)

σ - sought; complex-valued; solution of the polynomial equation

of the 6th degree; $\gamma = \gamma(\mathrm{Im}\sigma) \quad \Rightarrow \quad \gamma$ **sought**

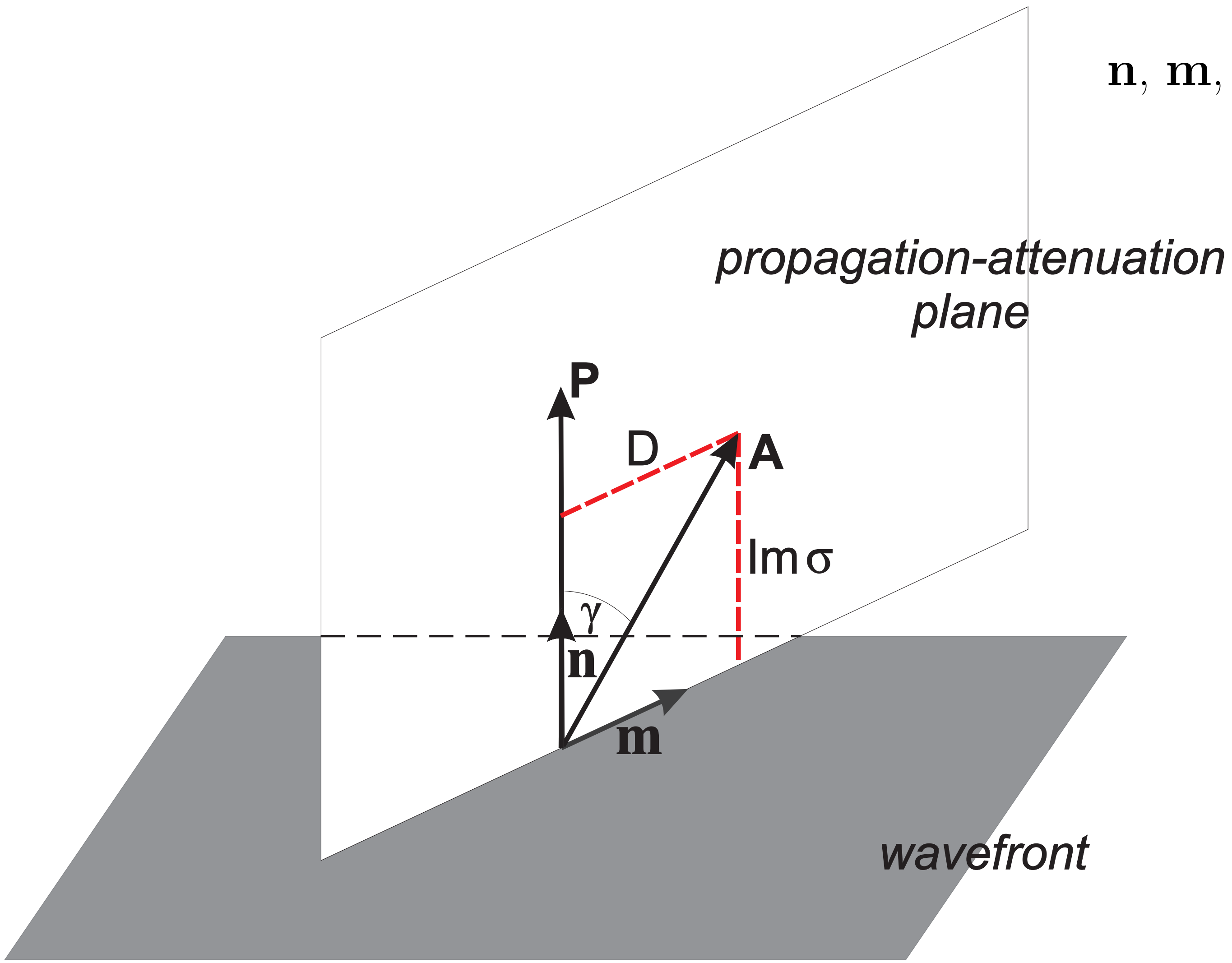
$D = 0$ - homogeneous wave, $\qquad D \neq 0$ - inhomogeneous wave

BASIC FORMULAE

Mixed specification of slowness vector

$\mathbf{n}$, $\mathbf{m}$, D - given

γ - sought



NUMERICAL EXAMPLES - SH WAVES

ISOTROPIC CASE

$\bar{\mu} = \bar{\mu}^R - \mathrm{i}\bar{\mu}^I$ - density-normalized viscoelastic shear modulus

$\bar{\mu}^R = \mathrm{Re}\bar{\mu}, \quad \bar{\mu}^I = \mathrm{Im}\bar{\mu}$

ANISOTROPIC CASE

$\mathbf{A} = \mathbf{A}^R - \mathrm{i}\mathbf{A}^I$ - 6×6 matrix of density-normalized viscoelastic moduli

$\mathbf{A}^R = \mathrm{Re}\mathbf{A}, \quad \mathbf{A}^I = \mathrm{Im}\mathbf{A}, \qquad \mathbf{A}^R, \mathbf{A}^I$ - positive definite

$$\begin{pmatrix} A_{11} & A_{12} & A_{13} & 0 & A_{15} & 0 \\ & A_{22} & A_{23} & 0 & A_{25} & 0 \\ & & A_{33} & 0 & A_{35} & 0 \\ & & & \mathbf{A_{44}} & 0 & \mathbf{A_{46}} \\ & & & & A_{55} & 0 \\ & & & & & \mathbf{A_{66}} \end{pmatrix}$$

ISOTROPIC CASE

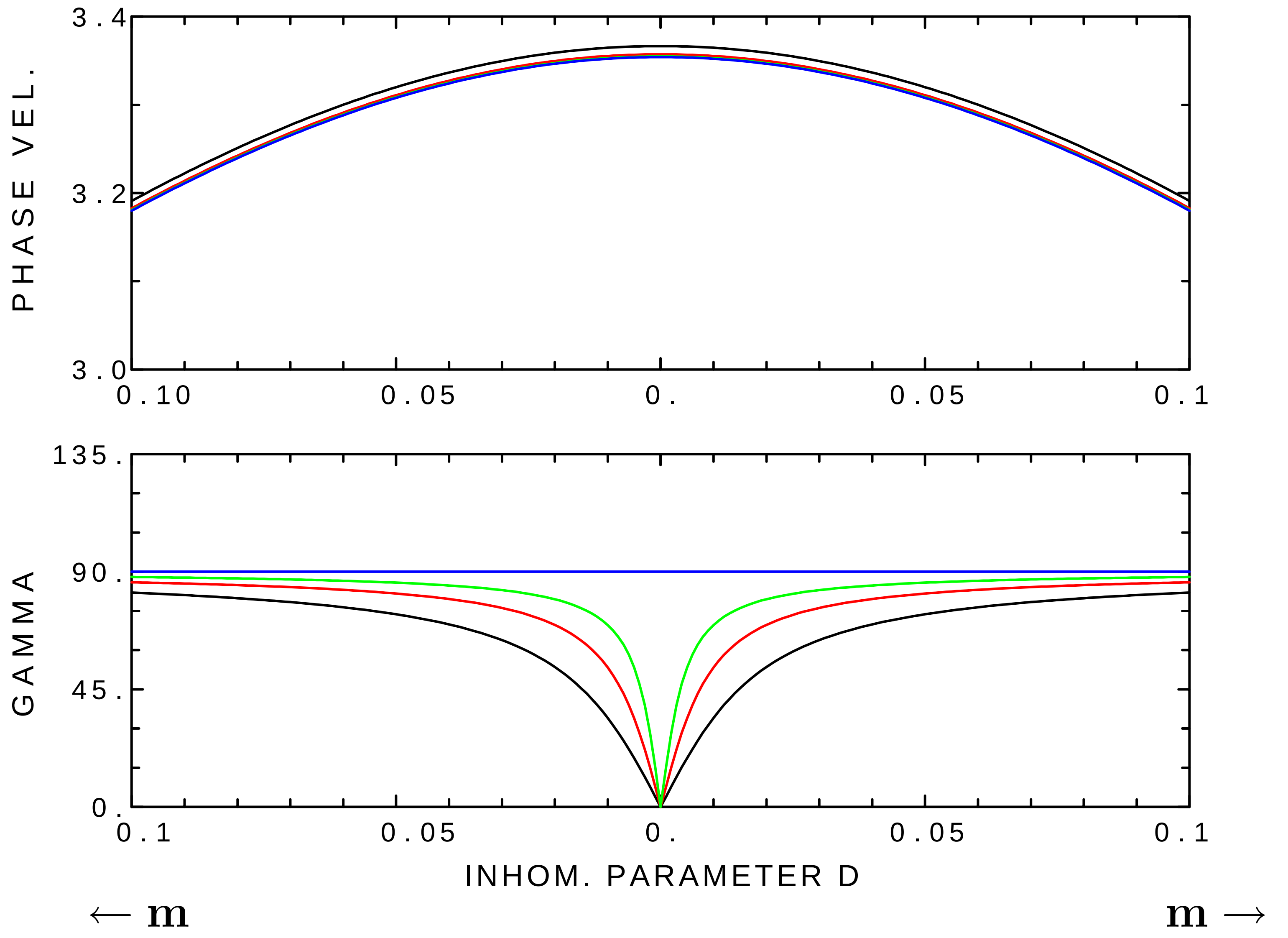
Parameterization by density-normalized shear modulus $\bar{\mu}^I$

$\bar{\mu}^I = 0$

$\bar{\mu}^I = 0.28125$

$\bar{\mu}^I = 0.5625$

$\bar{\mu}^I = 1.125$



ISOTROPIC CASE

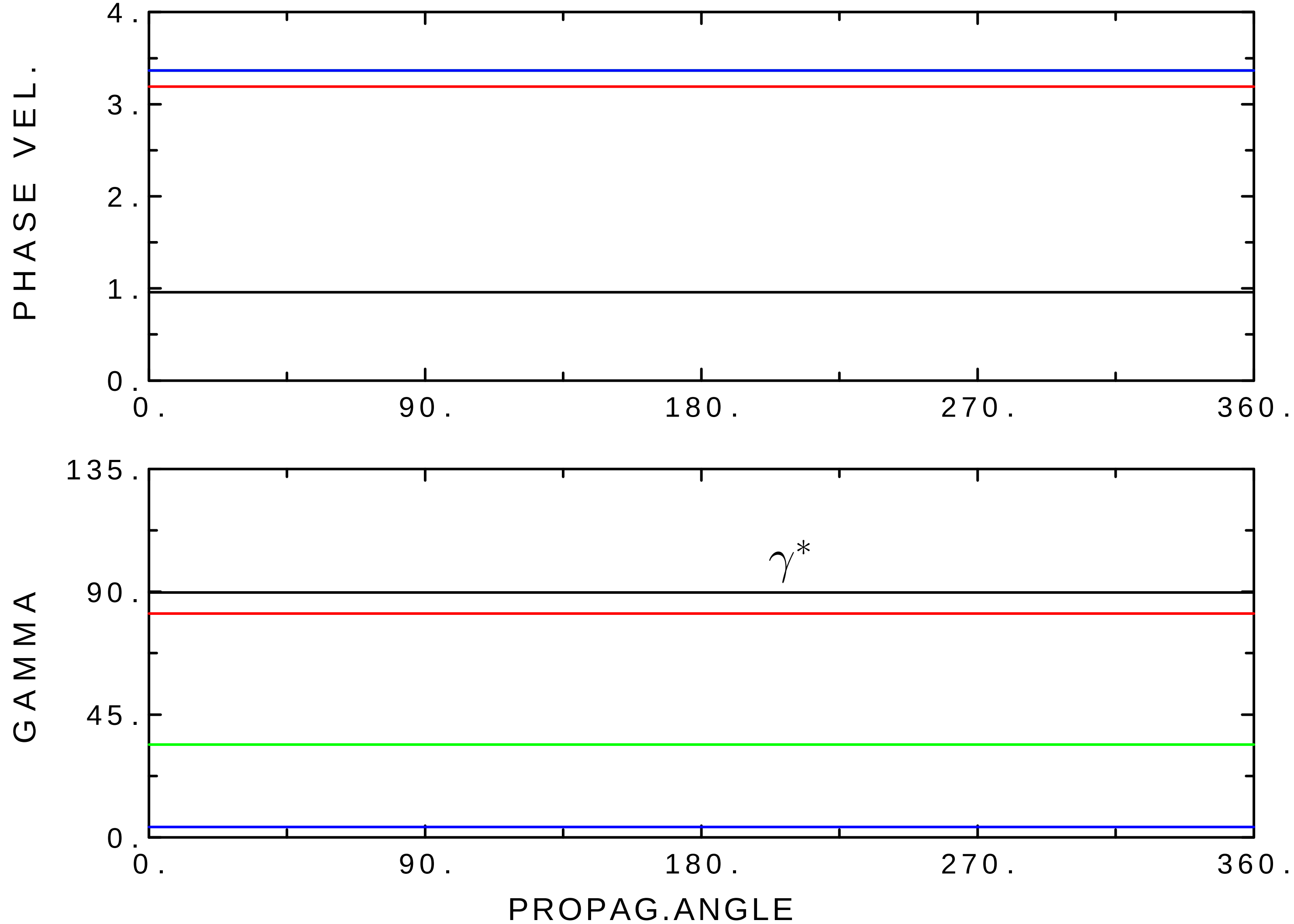
Dissipative medium; parameterization by D

$D = 0.001$

$D = 0.01$

$D = 0.1$

$D = 1$



$\uparrow$ **n** $\quad \rightarrow$ **m**

ANISOTROPIC CASE

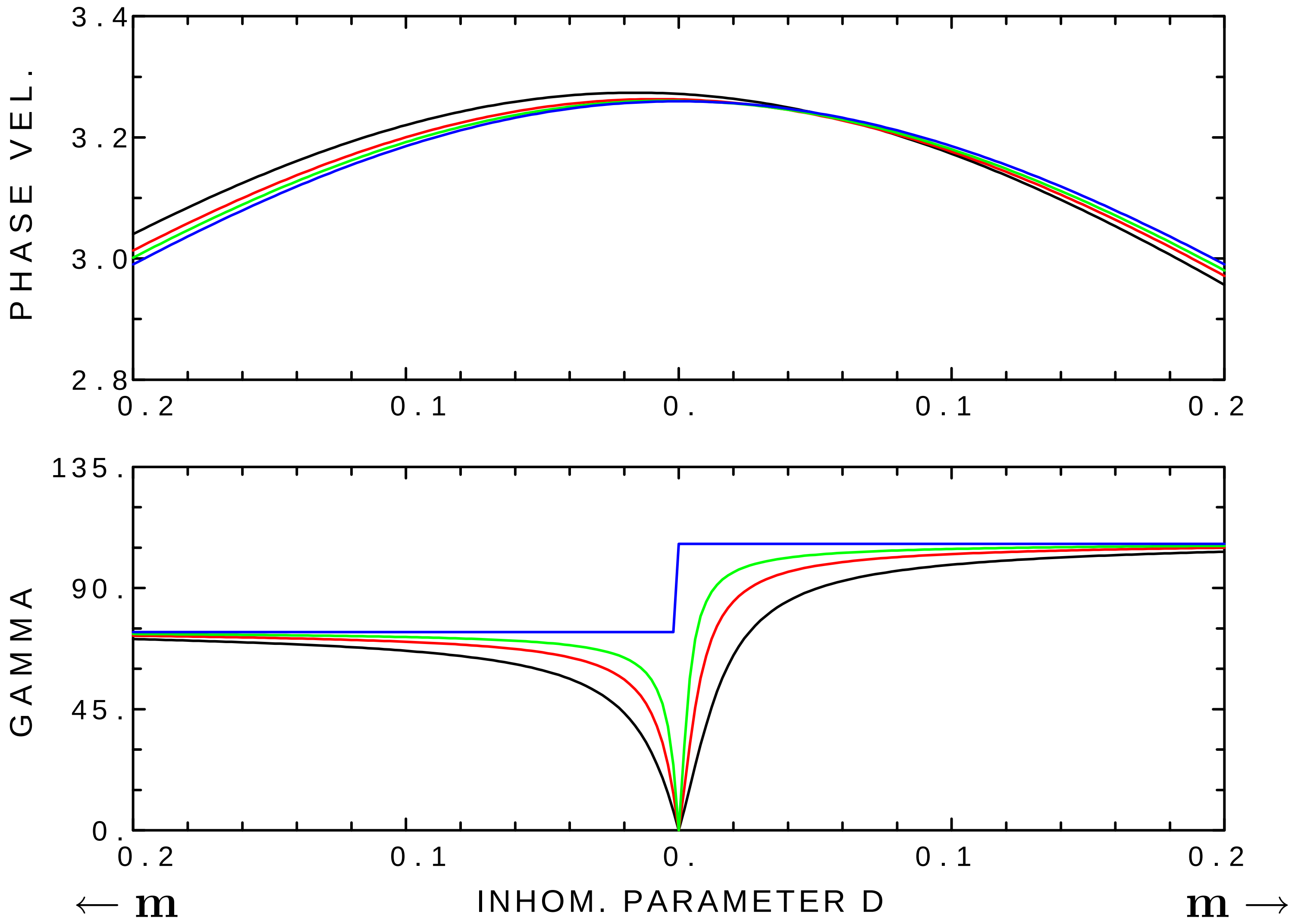
Parameterization by $A^I_{\alpha\beta}$; propagation angle 45^0

$A^I_{44} = 0$
$A^I_{66} = 0$
$A^I_{46} = 0$

$A^I_{44} = 0.25$
$A^I_{66} = 0.28125$
$A^I_{46} = 0$

$A^I_{44} = 0.5$
$A^I_{66} = 0.5625$
$A^I_{46} = 0$

$A^I_{44} = 1$
$A^I_{66} = 1.125$
$A^I_{46} = 0$



ANISOTROPIC CASE

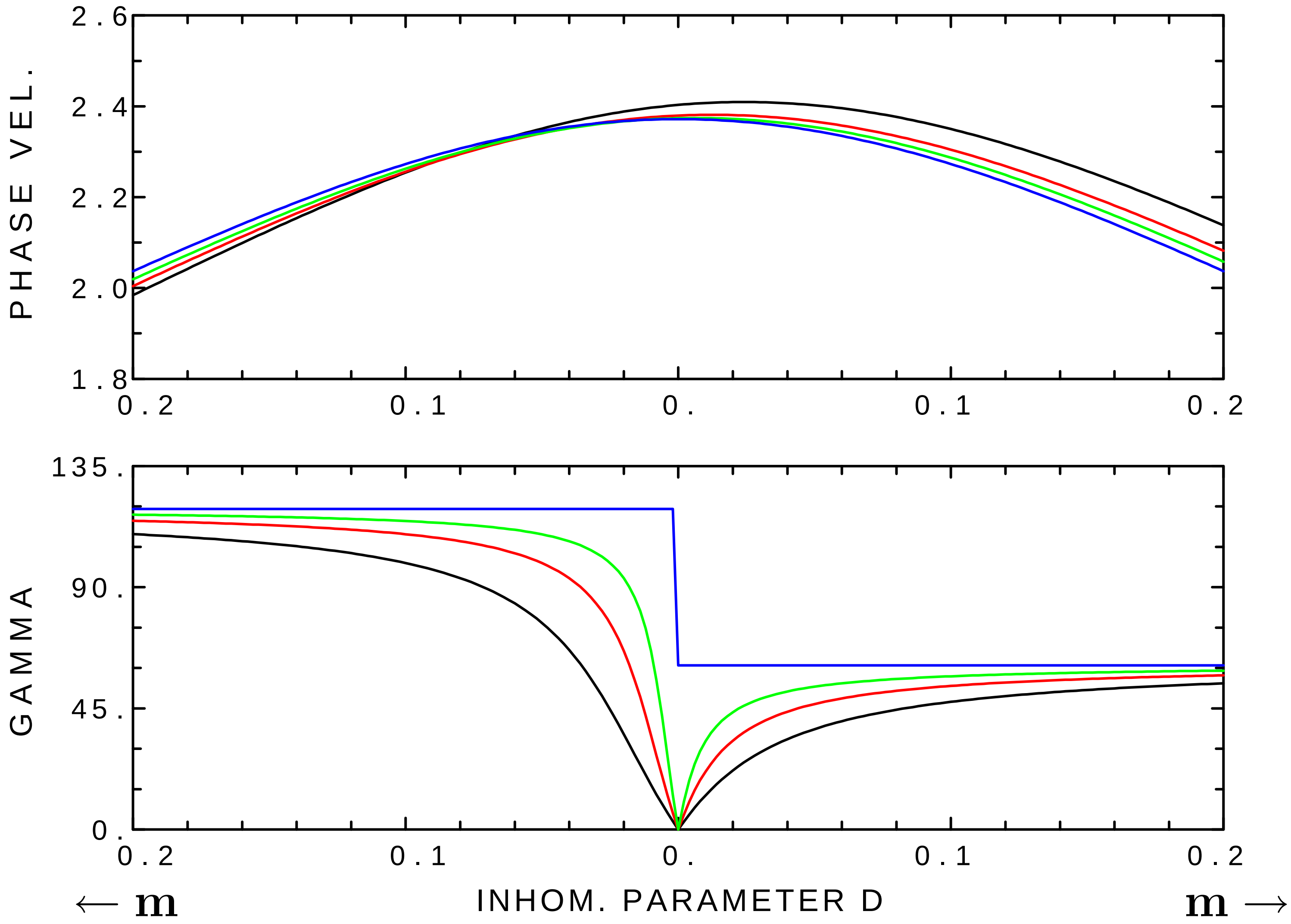
Parameterization by $A^I_{\alpha\beta}$; propagation angle 135^0

$A^I_{44} = 0$
$A^I_{66} = 0$
$A^I_{46} = 0$

$A^I_{44} = 0.25$
$A^I_{66} = 0.28125$
$A^I_{46} = 0$

$A^I_{44} = 0.5$
$A^I_{66} = 0.5625$
$A^I_{46} = 0$

$A^I_{44} = 1$
$A^I_{66} = 1.125$
$A^I_{46} = 0$



ANISOTROPIC CASE

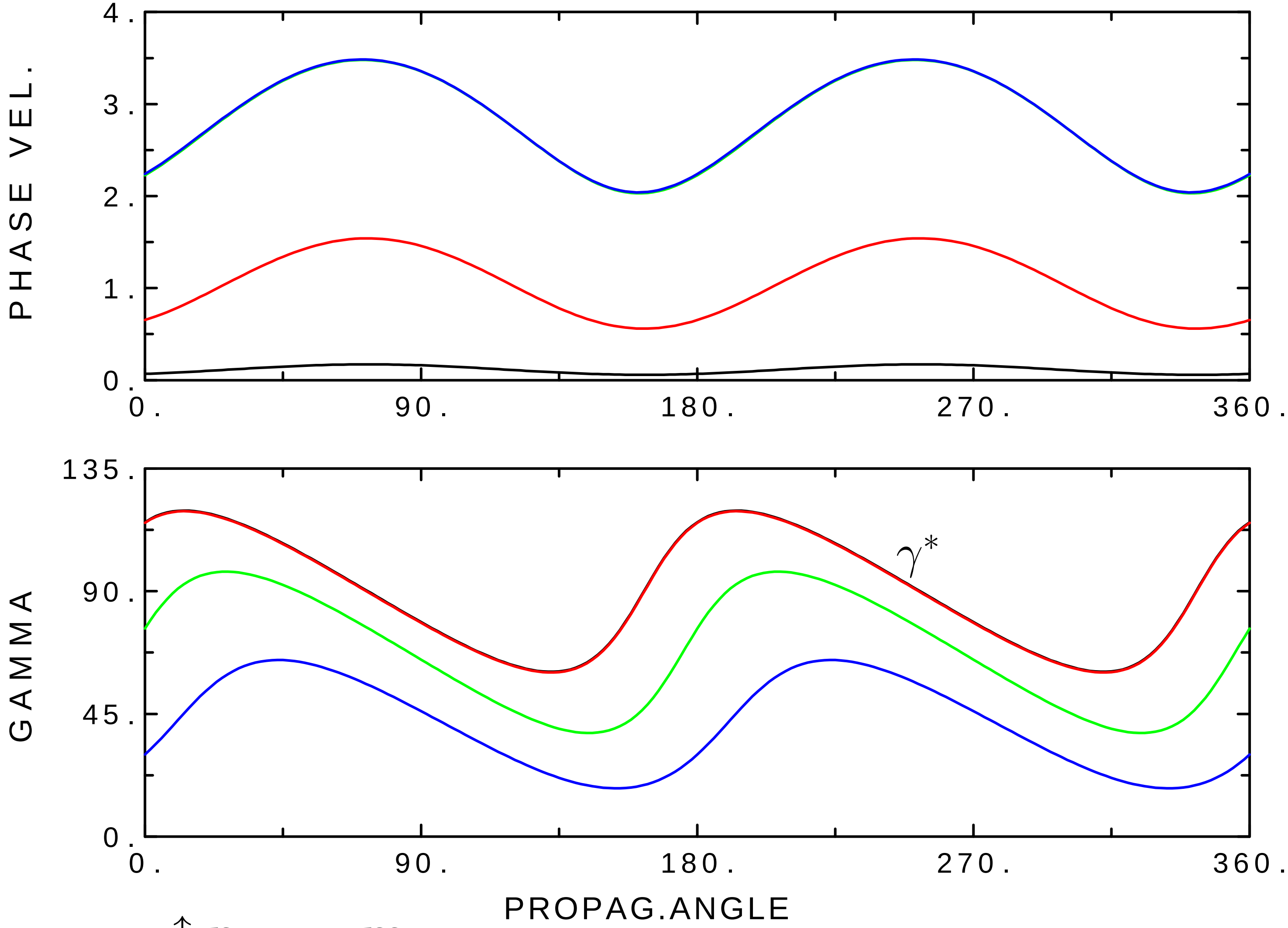
Parameterization by inhomogeneity parameter D

$D = 0.01$

$D = 0.03$

$D = 1$

$D = 10$



$\uparrow \mathbf{n} \quad \rightarrow \mathbf{m}$

ANISOTROPIC CASE

Parameterization by inhomogeneity parameter D

$D = 0.01$

$D = 0.03$

$D = 1$

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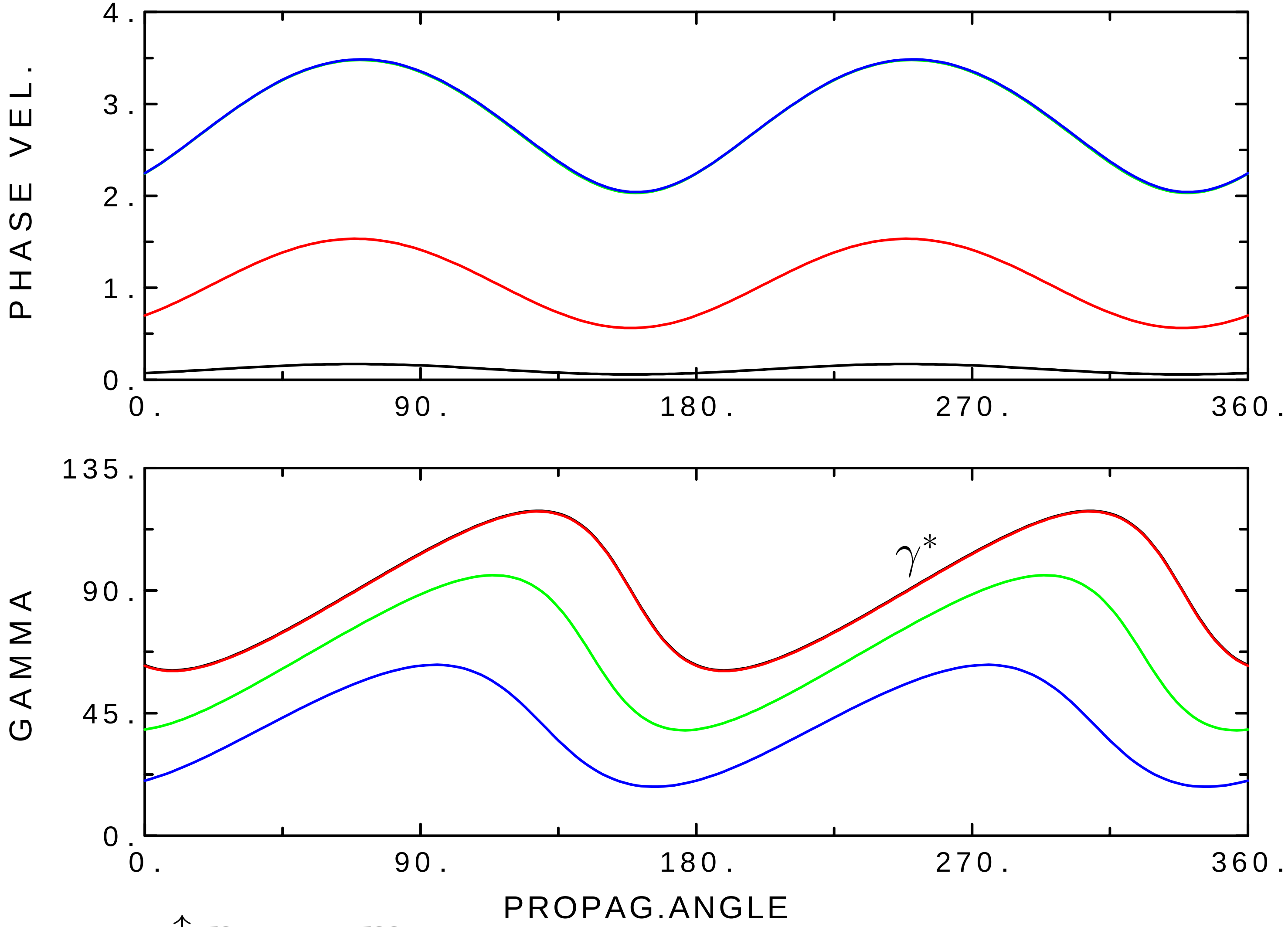


$\uparrow \mathbf{n} \quad \leftarrow \mathbf{m}$

ANISOTROPIC CASE

Dissipative medium; parameterization by $A^R_{\alpha\beta}$; $D = 0.02$

$A^R_{44} = 8.25$

$A^R_{66} = 8.25$

$A^R_{46} = 0$

$A^R_{44} = 7.4375$

$A^R_{66} = 9.0625$

$A^R_{46} = 0.625$

$A^R_{44} = 6.625$

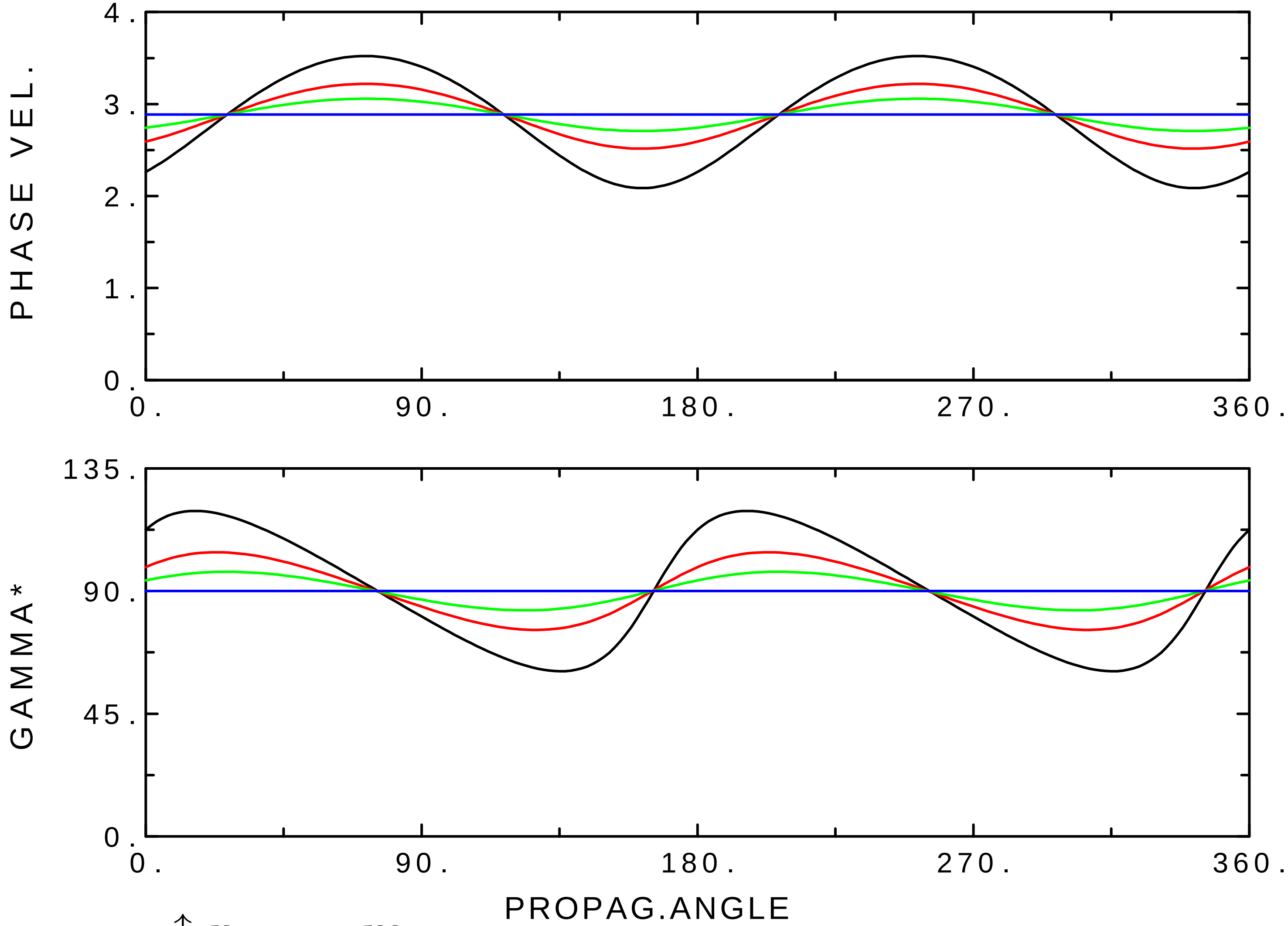
$A^R_{66} = 9.875$

$A^R_{46} = 1.25$

$A^R_{44} = 5$

$A^R_{66} = 11.5$

$A^R_{46} = 2.5$



$\uparrow$ **n** $\rightarrow$ **m**

ANISOTROPIC CASE

Parameterization by $A^I_{\alpha\beta}$; $D = 0.02$

$A^I_{44} = 0$
$A^I_{66} = 0$
$A^I_{46} = 0$

$A^I_{44} = 0.25$
$A^I_{66} = 0.28125$
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$A^I_{44} = 0.5$
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$A^I_{44} = 1$
$A^I_{66} = 1.125$
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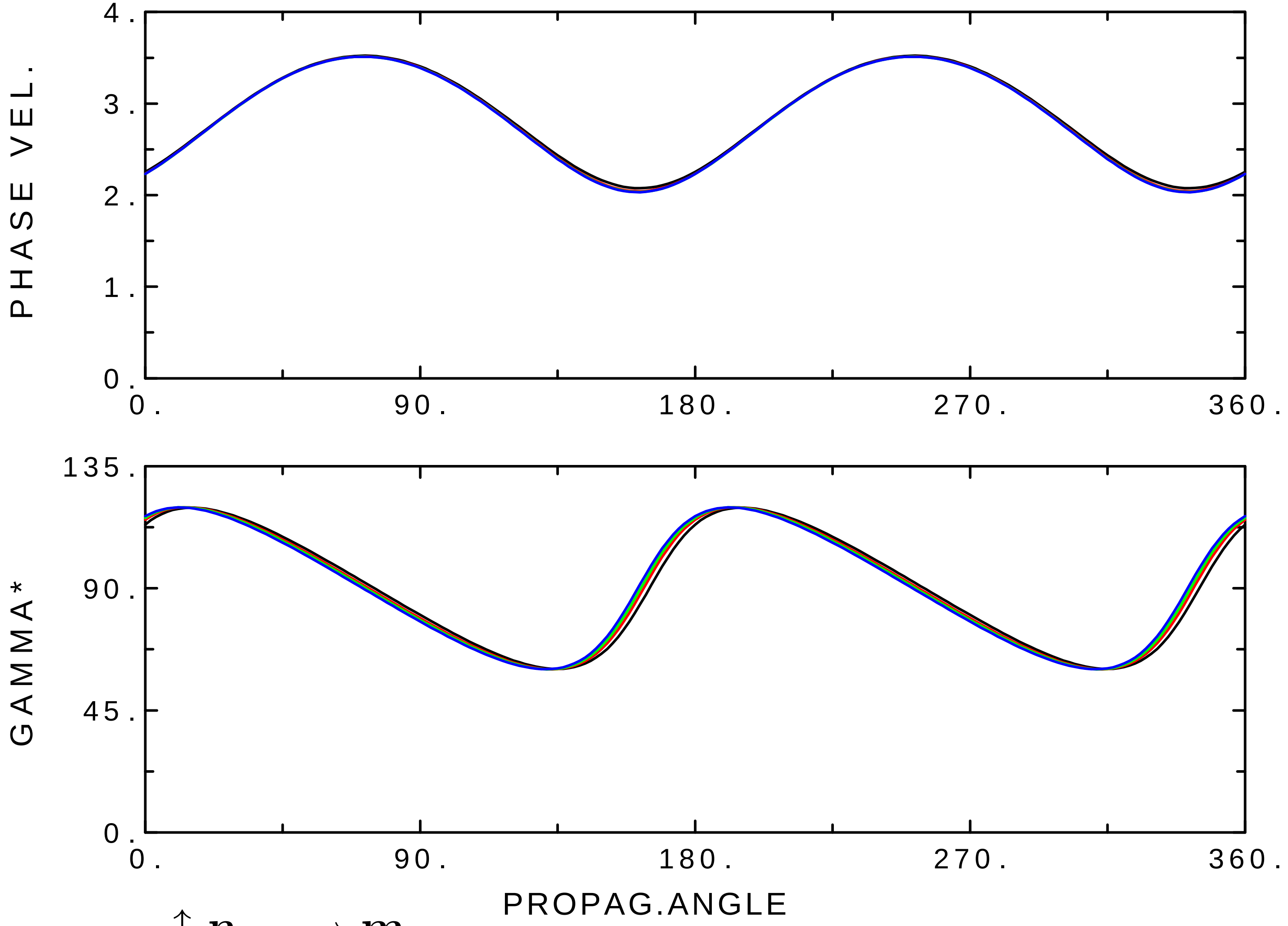


$\uparrow$ **n** $\rightarrow$ **m**

ANISOTROPIC CASE - **directional specification of p**

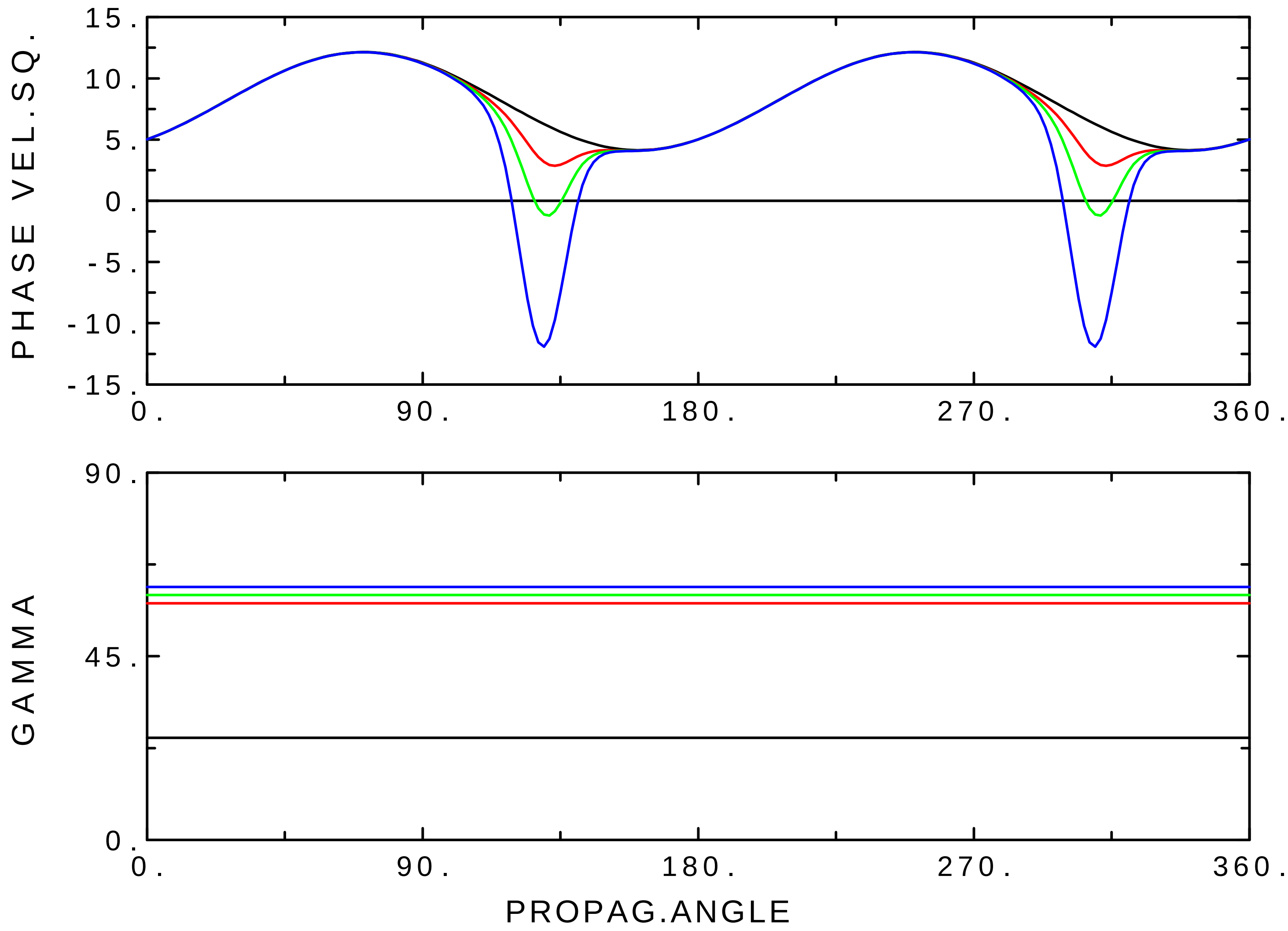
Parameterization by attenuation angle γ (data after Carcione)

$\gamma = 25^0$

$\gamma = 58^0$

$\gamma = 60^0$

$\gamma = 62^0$



$\uparrow$ **n** $\quad \rightarrow$ **m**

ANISOTROPIC CASE - directional specification of p

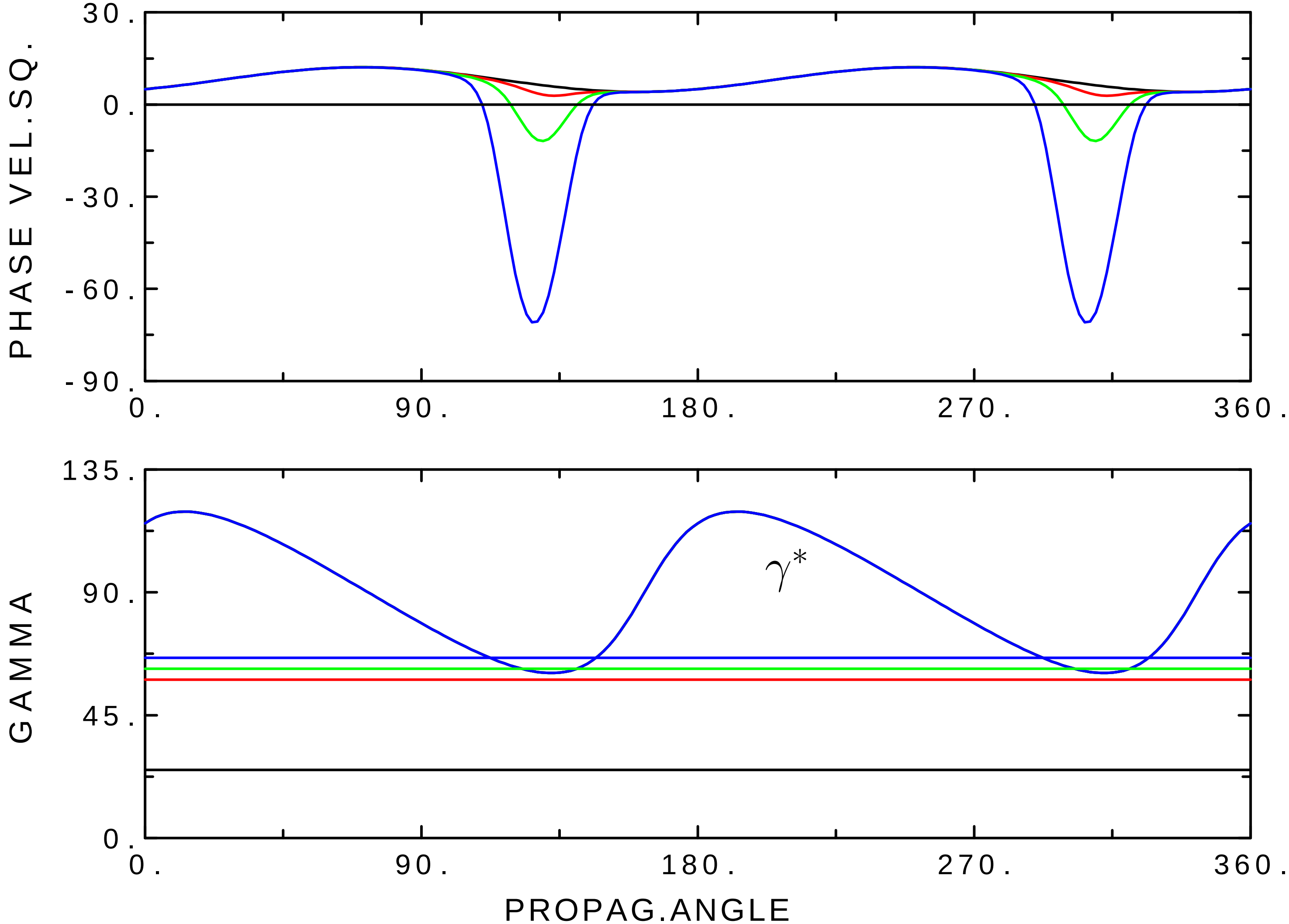
Parameterization by γ, curve γ^* calculated by mixed specification of $\mathbf{p}$

$\gamma = 25^0$

$\gamma = 58^0$

$\gamma = 62^0$

$\gamma = 66^0$



$\uparrow \mathbf{n} \quad \rightarrow \mathbf{m}$

CONCLUSIONS

mixed specification of the slowness vector

- makes possible analytic calculation of γ^*
- removes problems with forbidden directions
- removes problems of $Q \cdot \cos\gamma$ for $Q \to \infty$, $\cos\gamma \to 0$
- indicates non-physical results in a broader region

any set of $\mathbf{n}$, $\mathbf{m}$ and D

specifies just one inhomogeneous wave

each inhomogeneous wave specified by

just one set $\mathbf{n}$, $\mathbf{m}$ and D

CONCLUSIONS

ISOTROPIC CASE

γ^* independent of propagation angle

γ^* independent of D

$\gamma^* = 90^0$ for $\mathbf{m} \rightarrow$ and $\mathbf{m} \leftarrow$

CONCLUSIONS

ANISOTROPIC CASE

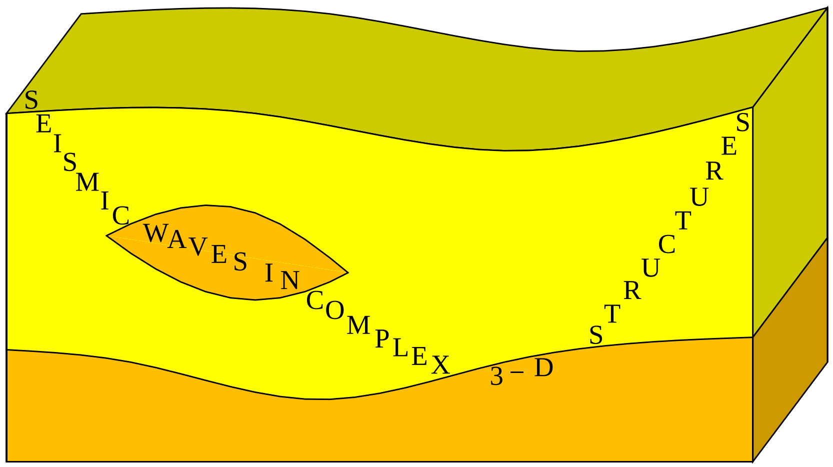
γ^* depends on propagation angle

γ^* independent of D, $\gamma^*(\mathbf{m} \rightarrow) \neq \gamma^*(\mathbf{m} \leftarrow)$

for $\gamma > \gamma^*$ and $(\gamma \sim \gamma^*)$ non-physical results

- directional specification: γ specified; γ^* a priori uknown
- mixed specification: γ computed; automatically $\gamma < \gamma^*$

Inhomogeneous wave in a forbidden direction - consequence of inappropriate specification of slowness vector (directional specification)


SEISMIC WAVES IN COMPLEX 3-D STRUCTURES