Approximate traveltimes of waves propagating in laterally varying layered anisotropic media

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standard "anisotropic" ray tracers for S waves

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- do not work in isotropic media

- generate number of multiple reflections, which may exceed acceptable limit

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Solution: use of common S-wave ray concept

(Bakker, 2002; Klimeš, 2006; Farra & Pšenčík, 2008)

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- common ray - artificial trajectory approximating rays of S1 and S2 waves

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- common ray - artificial trajectory approximating rays
  of S1 and S2 waves

- traveltimes of S1 and S2 waves evaluated by quadratures
  along the common ray

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- common ray - artificial trajectory approximating rays of S1 and S2 waves
- traveltimes of S1 and S2 waves evaluated by quadratures along the common ray
- at interfaces, slowness vectors of generated waves determined by solving 4th-degree polynomial equation

First-order common S-wave ray tracing

$$dx_i/d\tau = \tfrac{1}{2}\partial G^{[\mathcal{M}]}/\partial p_i \ , \qquad dp_i/d\tau = -\tfrac{1}{2}\partial G^{[\mathcal{M}]}/\partial x_i$$

x_i - coordinates of the first-order ray Ω

p_i - components of the first-order slowness vector $\mathbf{p}$

τ - first-order traveltime

$G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p})$ - S-wave first-order mean eigenvalue

$$G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p}) = \tfrac{1}{2}[G^{(1)}_{S1}(\mathbf{x},\mathbf{p}) + G^{(1)}_{S2}(\mathbf{x},\mathbf{p})]$$

$G^{(1)}_{SI}(\mathbf{x},\mathbf{p})$ - S-wave first-order eigenvalues of Christoffel matrix $\mathbf{\Gamma}$

Eikonal equation: $G^{[\mathcal{M}]}(\mathbf{x},\mathbf{p}) = 1$

Approximate S-wave traveltime formulae

$$\tau_{S1,S2}(\tau,\tau_0) = \tau^{[\mathcal{M}]}(\tau,\tau_0) + \Delta\tau^{[\mathcal{M}]}(\tau,\tau_0) + \Delta\tau_{S1,S2}(\tau,\tau_0)$$

$\tau^{[\mathcal{M}]}(\tau,\tau_0)$ - first-order traveltime between τ_0 and τ on common ray

$\Delta\tau^{[\mathcal{M}]}(\tau,\tau_0)$ - "averaging" correction of first-order traveltime

$$\Delta\tau^{[\mathcal{M}]} = \tfrac{1}{4}\int_{\tau_0}^{\tau}(B_{13}^2 + B_{23}^2)/(B_{33}-1)d\tau$$

$\Delta\tau_{S1,S2}(\tau,\tau_0)$ - "separation" traveltime correction

$$\Delta\tau_{S1,S2} = \mp\tfrac{1}{4}\int_{\tau_0}^{\tau}\sqrt{(M_{11}-M_{22})^2 + 4M_{12}^2}d\tau$$

Approximate S-wave traveltime formulae

$$B_{mn} = B_{mn}(\mathbf{x},\mathbf{p}) = \Gamma_{ik}(\mathbf{x},\mathbf{p}) e_i^{[m]}(\mathbf{x}) e_k^{[n]}(\mathbf{x})$$

Γ_{ik} - elements of Christoffel matrix $\mathbf{\Gamma}$

$\mathbf{e}^{[m]}$ - triplet of orthonormal vectors

$\mathbf{e}^{[3]} = c^{[\mathcal{M}]}\mathbf{p},\qquad \mathbf{e}^{[1]},\, \mathbf{e}^{[2]}$ arbitrarily in the plane $\perp \mathbf{e}^{[3]}$

$c^{[\mathcal{M}]}$ - common S-wave phase velocity

$$M_{MN} = M_{MN}(\mathbf{x},\mathbf{p}) = B_{MN} - B_{M3}B_{N3}/(B_{33}-1)$$

Transformation of first-order slowness vectors at an interface (Snell's law)

$$p_i^G = p_i - (p_k N_k) N_i + \xi^G N_i$$

$\mathbf{p}$ - first-order common S-wave slowness vector of incident wave

$\mathbf{p}^G$ - first-order common S-wave slowness vector of generated wave

$\mathbf{N}$ - normal to the interface, $\xi^G = p_k^G N_k$

$p_i^G - (p_k^G N_k) N_i = p_i - (p_k N_k) N_i$ - Snell's law

Transformation of first-order slowness vectors at an interface (Snell's law)

$p_i^G = p_i - (p_k N_k) N_i + \xi N_i$

$G^{[\mathcal{M}]}(\mathbf{x}, \mathbf{p^G}) = 1$ - eikonal equation

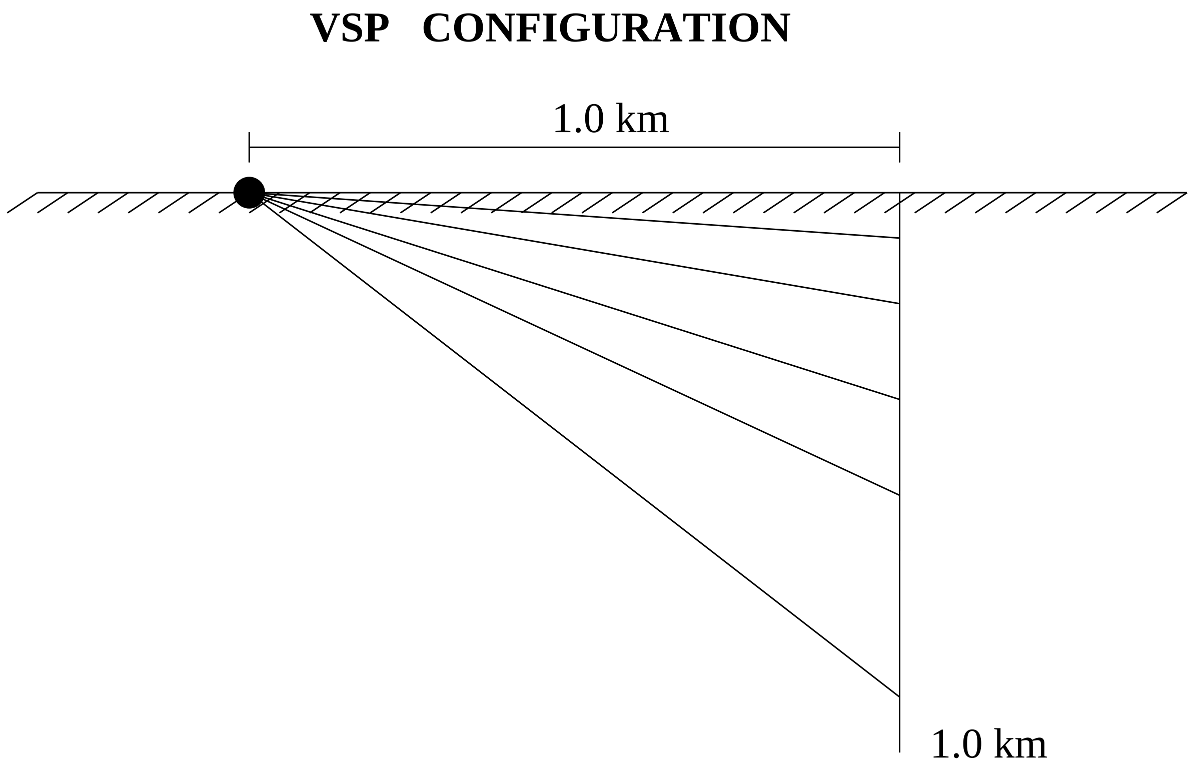
$G^{[\mathcal{M}]}(\xi) = 1$ - polynomial equation of 4th degree for ξ

Iterative solution **(Dehghan et al., 2007)**

$\xi^{\{j\}} = \xi^{\{j-1\}} - [G^{[\mathcal{M}]}(p_m^{\{j-1\}}) - 1]/[N_k \frac{\partial G^{[\mathcal{M}]}}{\partial p_k}(p_m^{\{j-1\}})]$

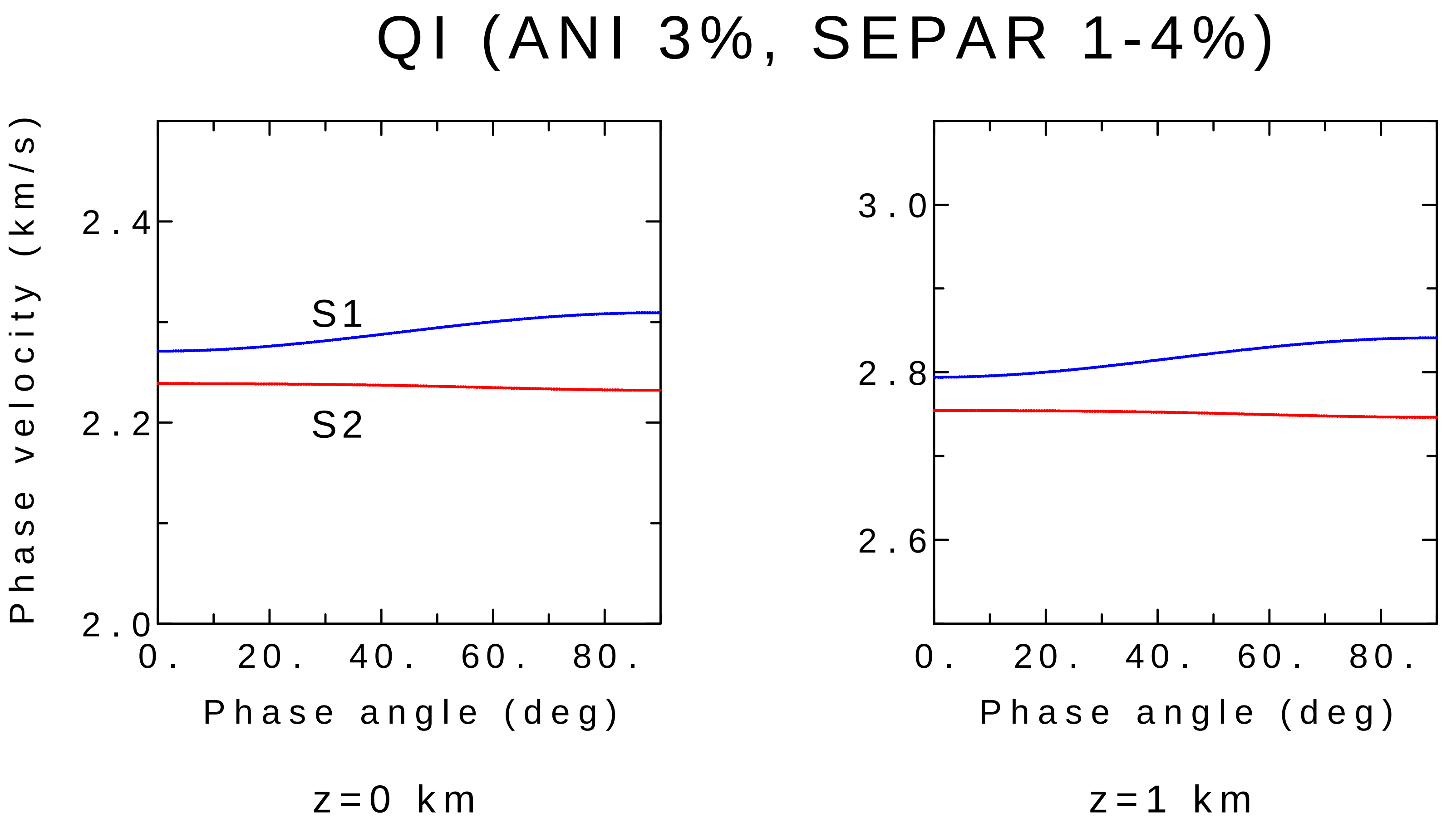
$\xi^{\{0\}}$ - in reference isotropic medium

Numerical examples



Numerical examples (Klimeš & Bulant, 2004)

HTI rotated by 45 degrees



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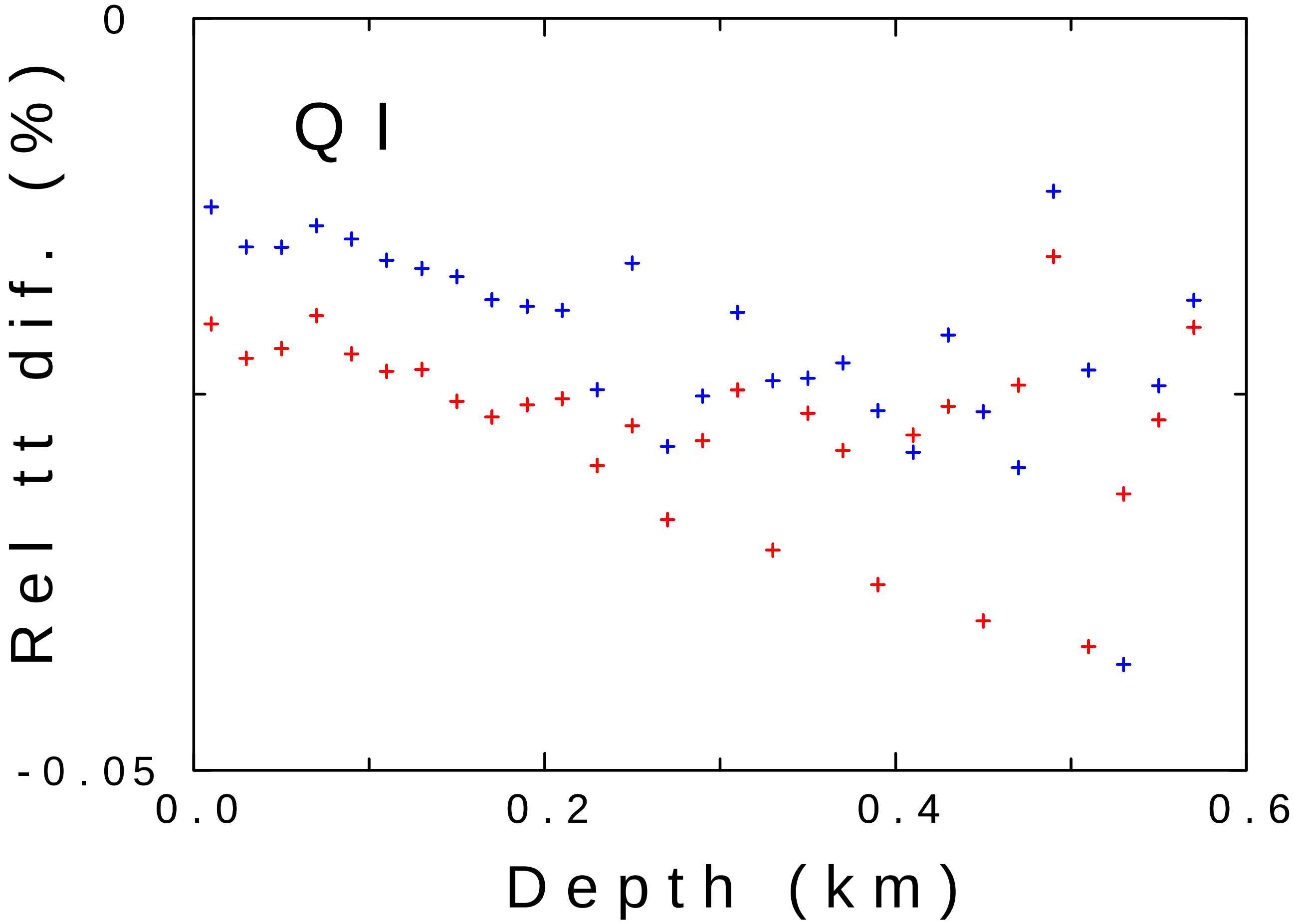

S1 AND S2 WAVES

Numerical examples **(Klimeš & Bulant, 2004)**

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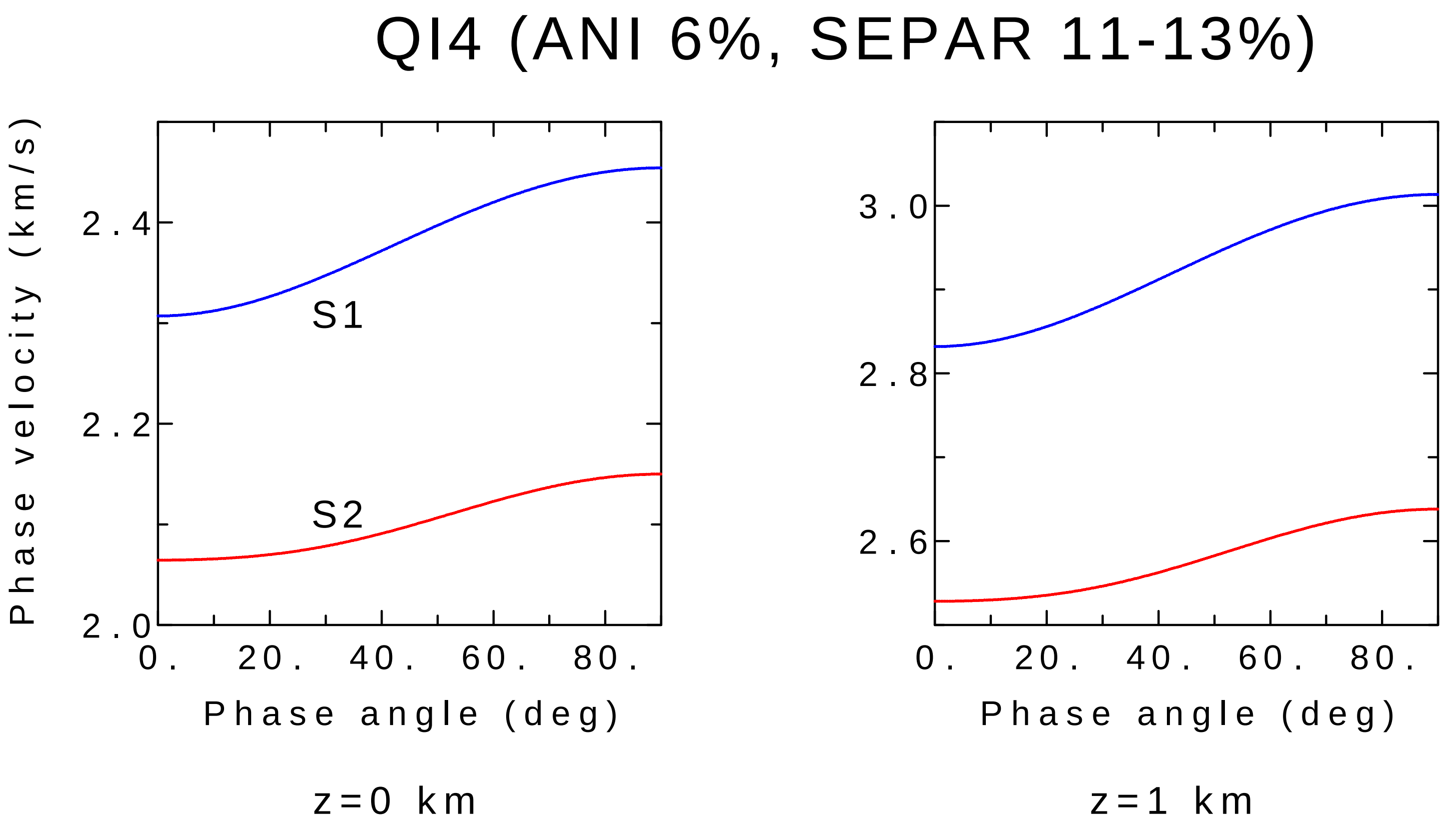

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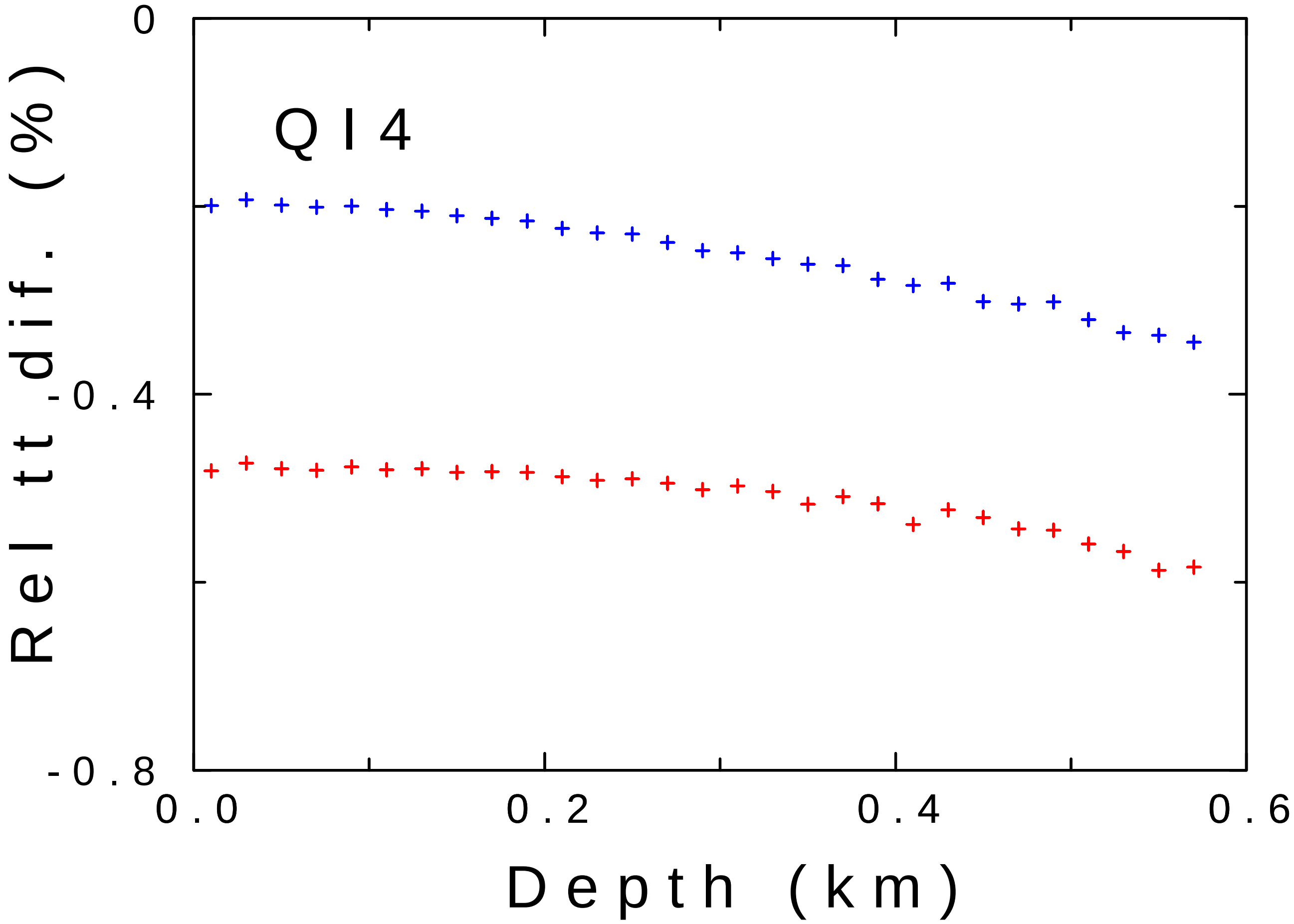
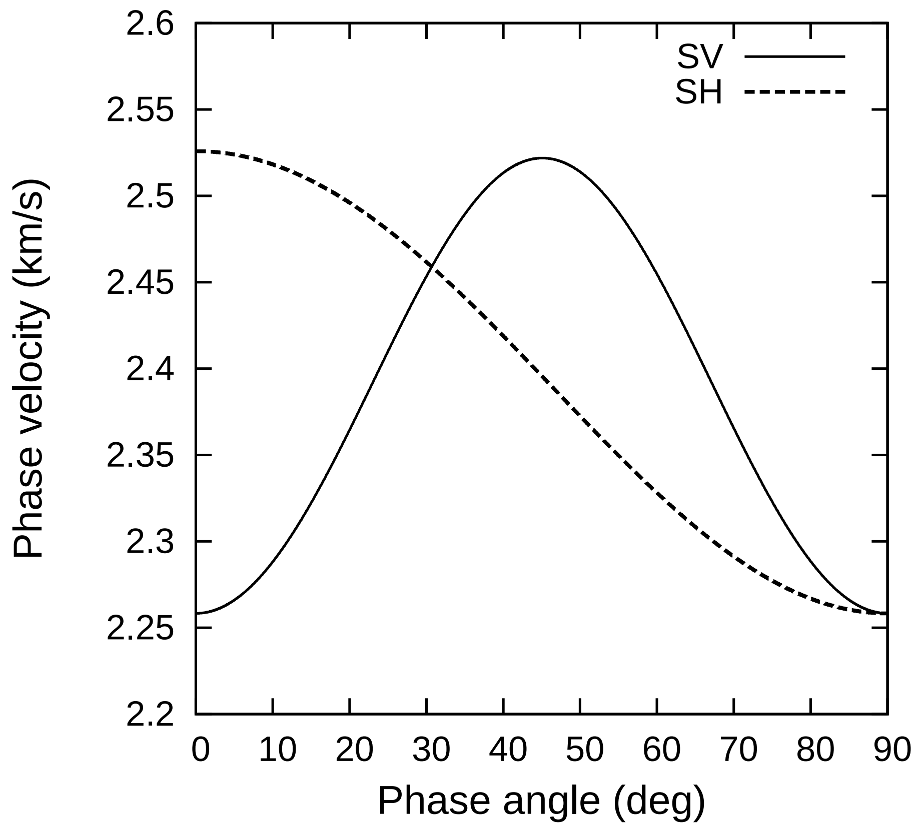


S1 AND S2 WAVES

Numerical examples (Shearer & Chapman, 1989)

VTI: MODEL SC1 (ANI 11%, SEPAR 0-11%)



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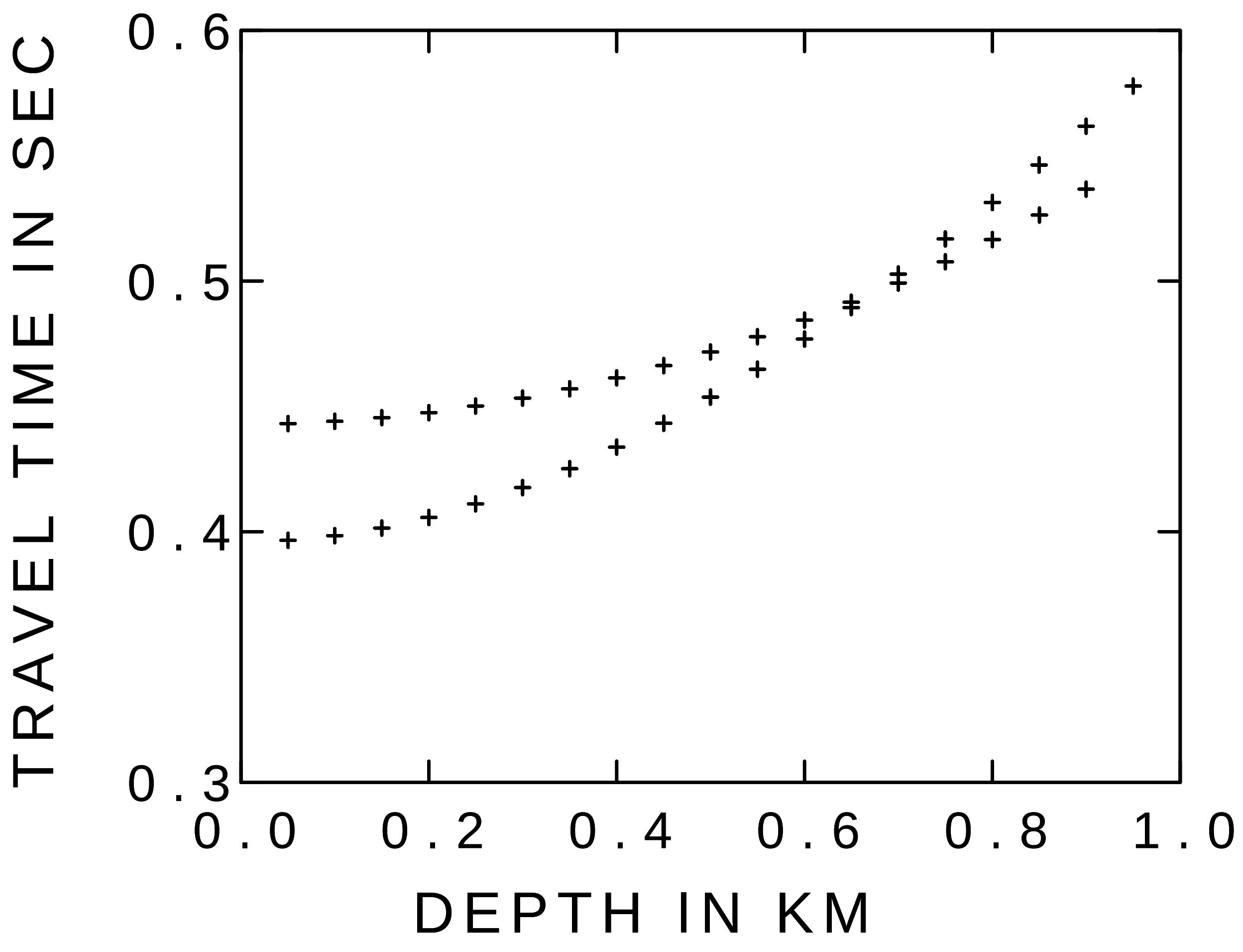
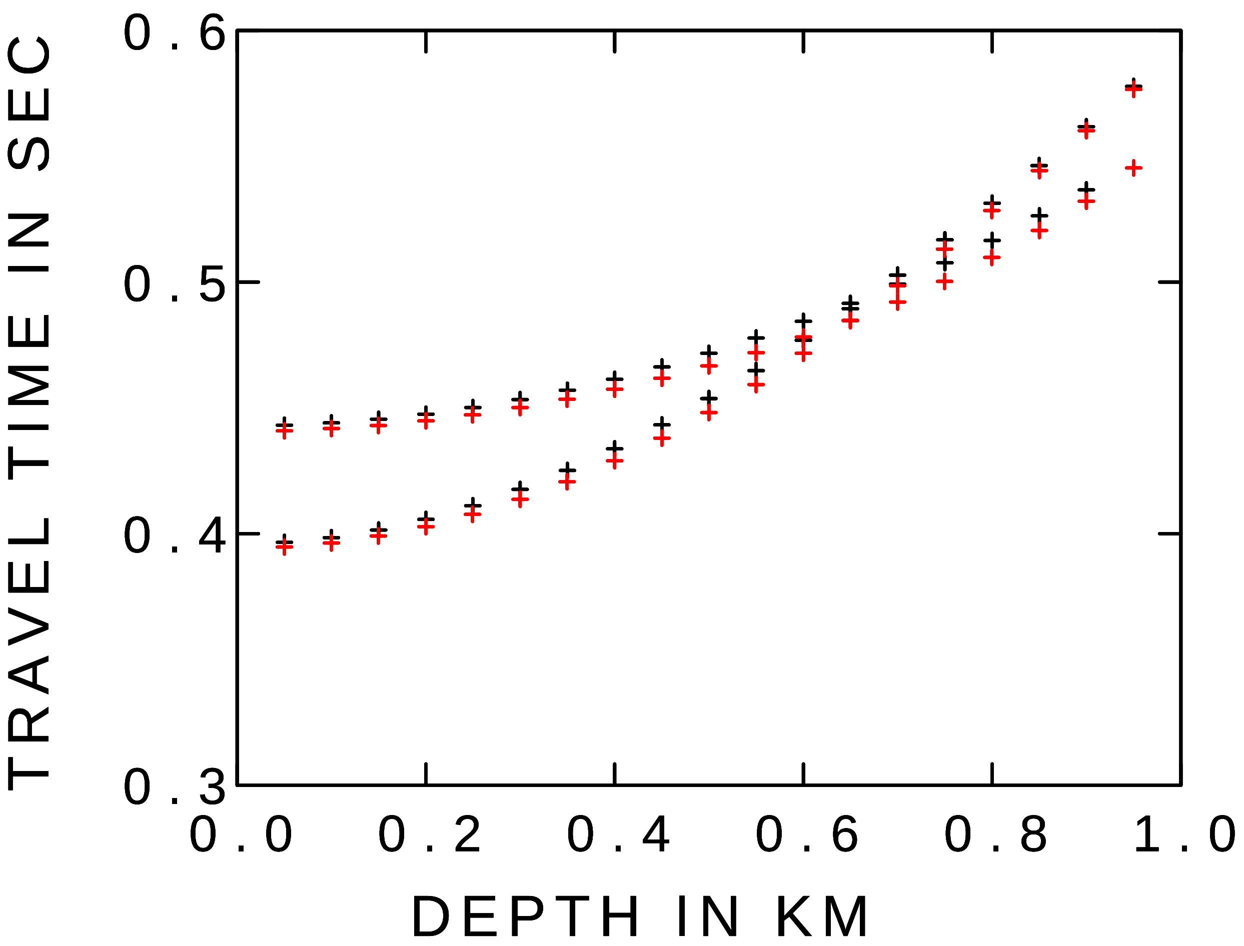


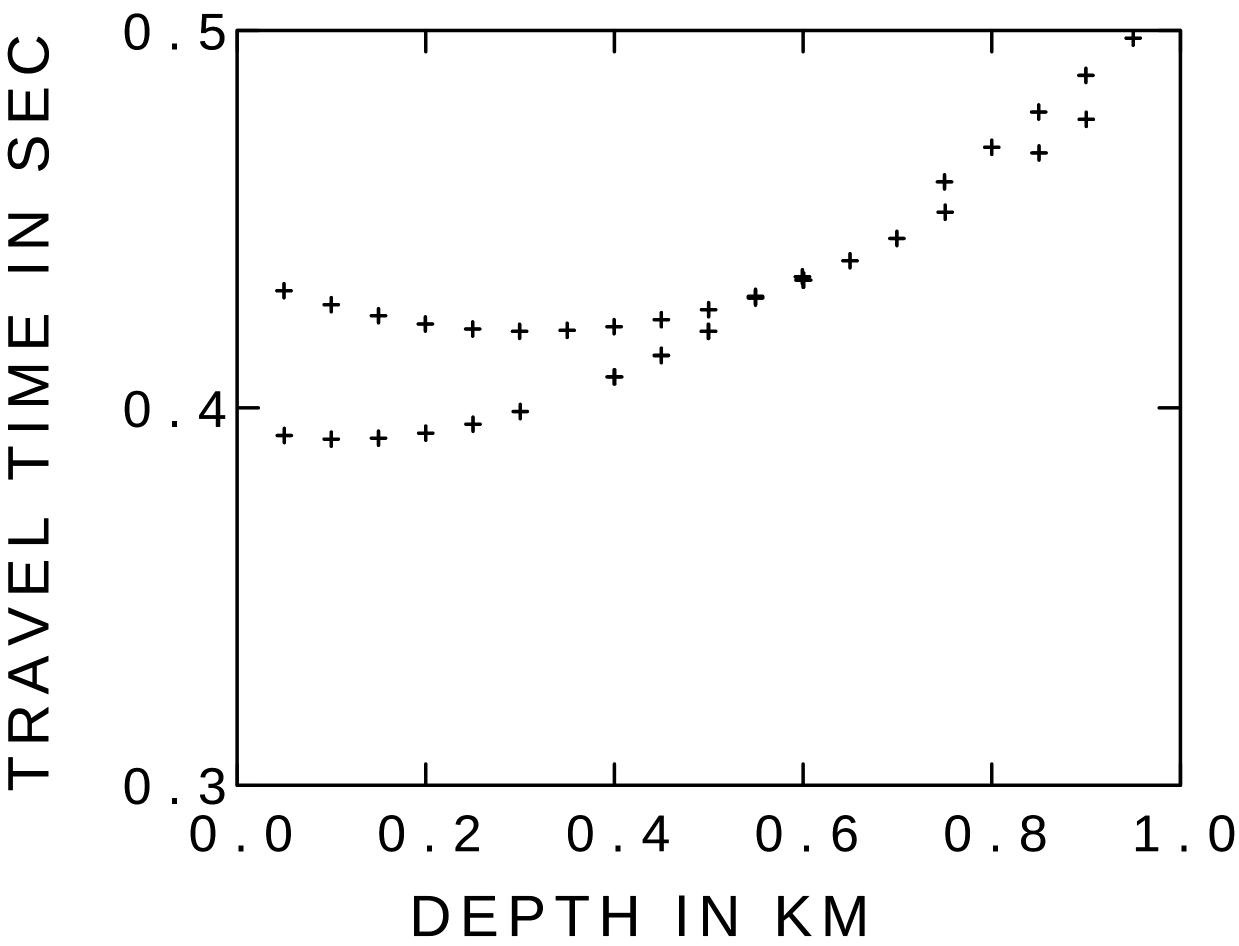
SC1 - HOM., EXACT,

Numerical examples



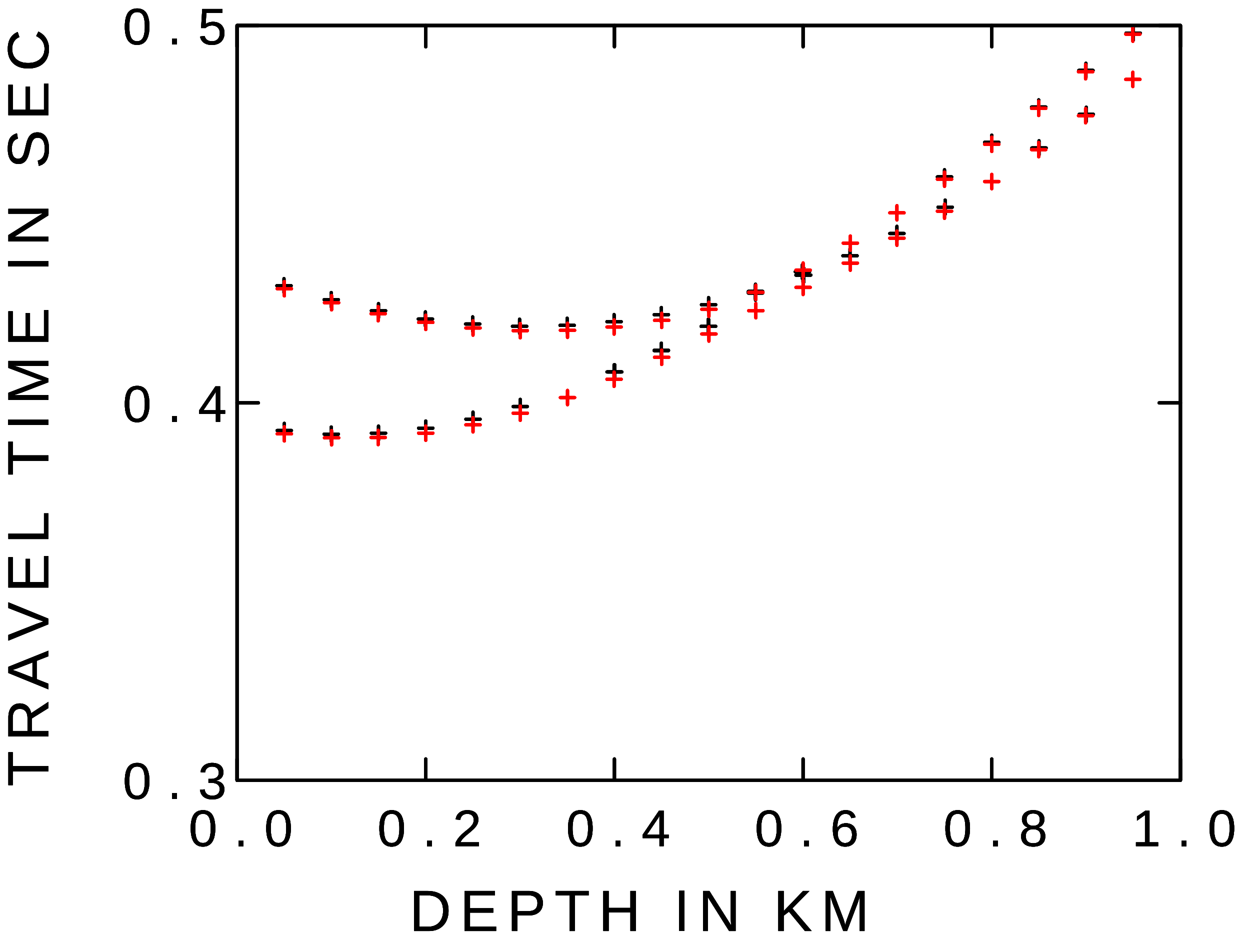
SC1 - HOM., EXACT, APPR.

Numerical examples



SCG1 - INHOM., EXACT,

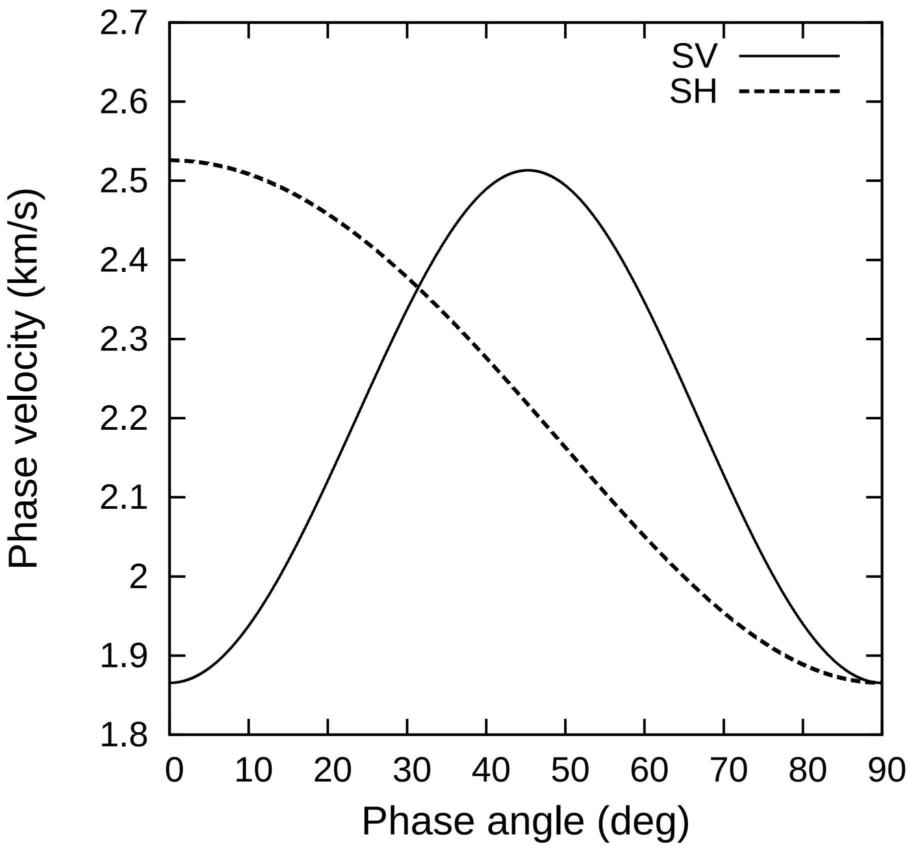
Numerical examples



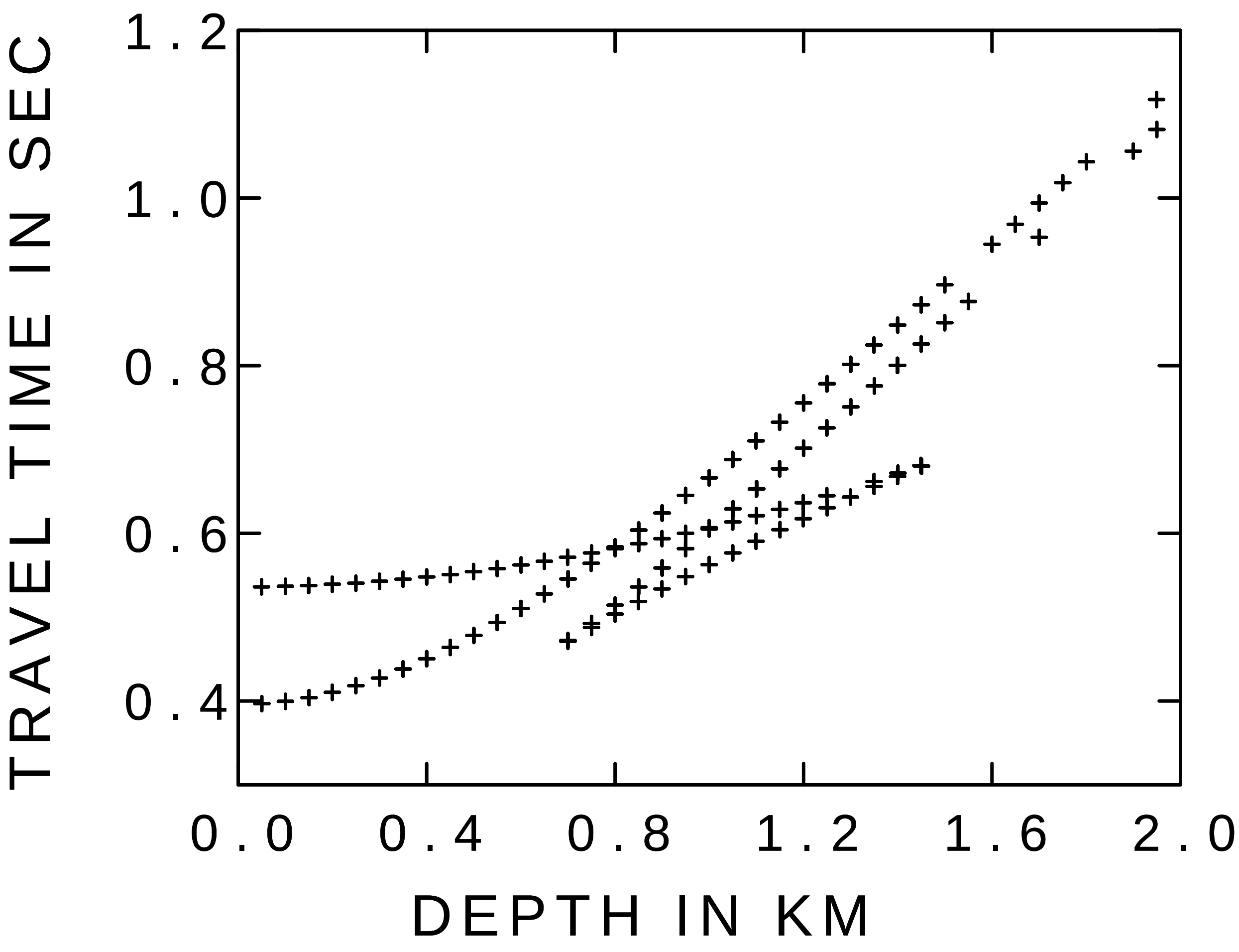
SCG1 - INHOM., EXACT, APPR.

Numerical examples (Shearer & Chapman, 1989)

VTI: MODEL SC4 (ANI 30%, SEPAR 0-30%)

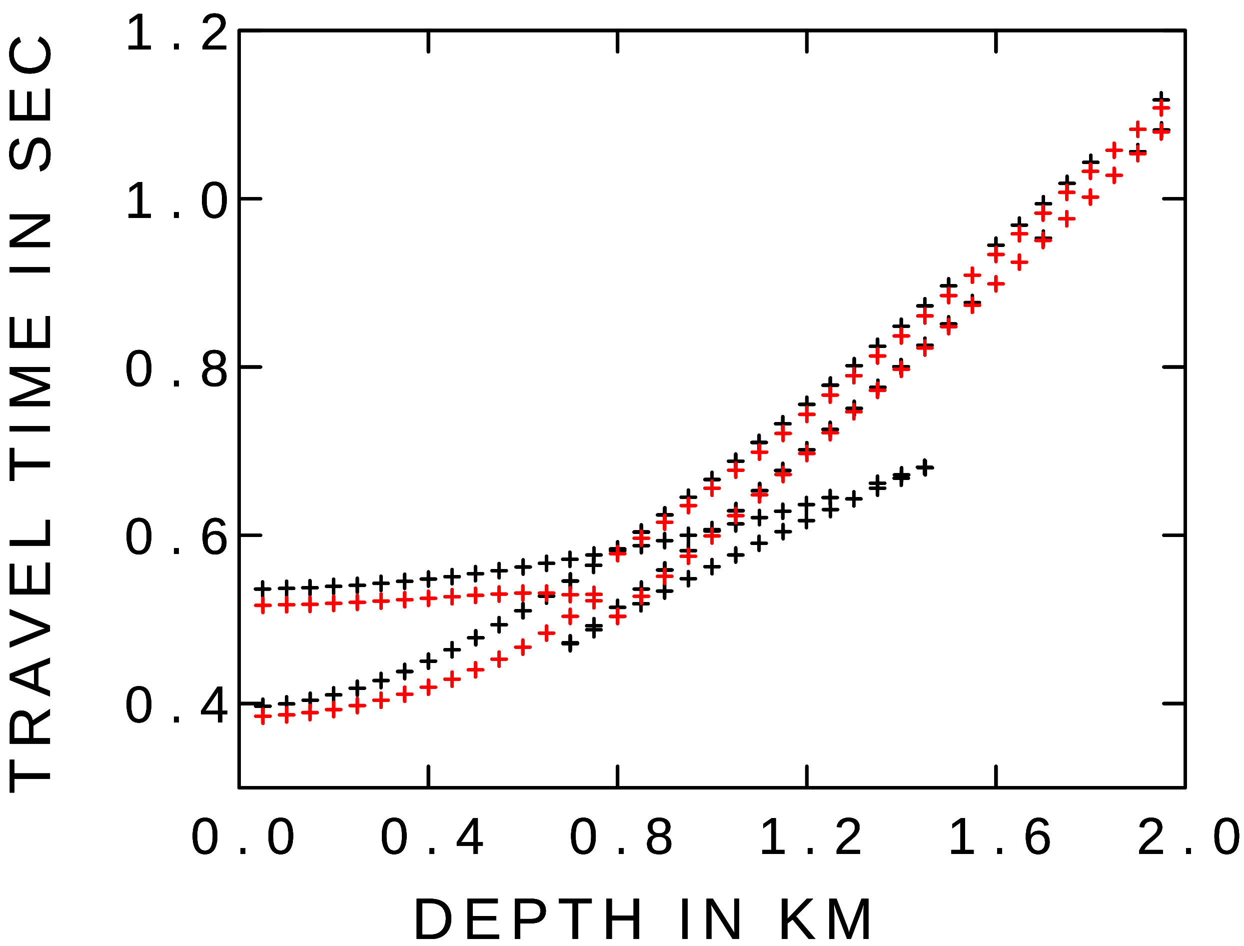


Numerical examples



SC4

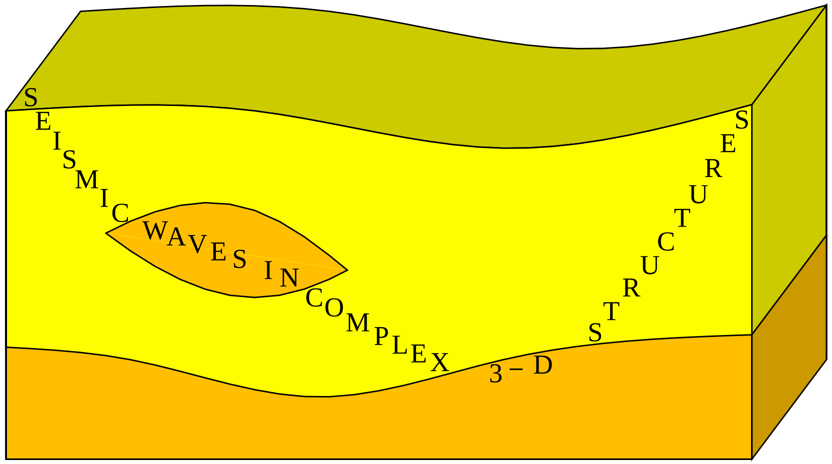
Numerical examples



SC4

Conclusions

- applicable to S waves in inhomogeneous isotropic, weakly anisotropic and moderately anisotropic media
- in isotropic media exact, in anisotropic media approximate
- single common S-wave ray necessary for computation of traveltimes of S1 and S2 waves
- common S-wave ray tracing stable, does not collapse anywhere
- computer time savings in computing traveltimes of reflected/transmitted waves
- performs better in inhomogeneous media; smoothes loops in traveltime curves


SEISMIC WAVES IN COMPLEX 3 – D STRUCTURES