Approximate travel times of waves propagating in laterally varying layered anisotropic media

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- break down in weakly anisotropic media
  and in vicinities of singularities
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standard "anisotropic" ray tracers for S waves
  - break down in weakly anisotropic media
    and in vicinities of singularities
  - do not work in isotropic media
  - generate number of multiple reflections,
    which may exceed acceptable limit
Introduction

Solution: use of common S-wave ray concept

(Bakker, 2002; Klimeš, 2006; Farra & Pšenčík, 2008)
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- common ray - artificial trajectory approximating rays of S1 and S2 waves
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- traveltimes of S1 and S2 waves evaluated by quadratures along the common ray
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- common ray - artificial trajectory approximating rays of S1 and S2 waves
- traveltimes of S1 and S2 waves evaluated by quadratures along the common ray
- at interfaces, slowness vectors of generated waves determined by solving 4th-degree polynomial equation
First-order common S-wave ray tracing

\( \frac{dx_i}{d\tau} = \frac{1}{2} \frac{\partial G^{[M]}}{\partial p_i}, \quad \frac{dp_i}{d\tau} = -\frac{1}{2} \frac{\partial G^{[M]}}{\partial x_i} \)

- \( x_i \) - coordinates of the first-order ray \( \Omega \)
- \( p_i \) - components of the first-order slowness vector \( \mathbf{p} \)
- \( \tau \) - first-order traveltime

\( G^{[M]}(x, p) \) - S-wave first-order mean eigenvalue

\[
G^{[M]}(x, p) = \frac{1}{2} [G_{S1}^{(1)}(x, p) + G_{S2}^{(1)}(x, p)]
\]

- \( G_{SI}^{(1)}(x, p) \) - S-wave first-order eigenvalues of Christoffel matrix \( \Gamma \)

Eikonal equation: \( G^{[M]}(x, p) = 1 \)
Approximate S-wave traveltime formulae

\[ \tau_{S1,S2}(\tau, \tau_0) = \tau^{[M]}(\tau, \tau_0) + \Delta \tau^{[M]}(\tau, \tau_0) + \Delta \tau_{S1,S2}(\tau, \tau_0) \]

\[ \tau^{[M]}(\tau, \tau_0) \quad \text{- first-order traveltime between } \tau_0 \text{ and } \tau \text{ on common ray} \]

\[ \Delta \tau^{[M]}(\tau, \tau_0) \quad \text{”averaging” correction of first-order traveltime} \]

\[ \Delta \tau^{[M]} = \frac{1}{4} \int_{\tau_0}^{\tau} \left( B_{13}^2 + B_{23}^2 \right) / (B_{33} - 1) d\tau \]

\[ \Delta \tau_{S1,S2}(\tau, \tau_0) \quad \text{”separation” traveltime correction} \]

\[ \Delta \tau_{S1,S2} = \mp \frac{1}{4} \int_{\tau_0}^{\tau} \sqrt{(M_{11} - M_{22})^2 + 4M_{12}^2} d\tau \]
Approximate S-wave traveltime formulae

\[ B_{mn} = B_{mn}(\mathbf{x}, \mathbf{p}) = \Gamma_{ik}(\mathbf{x}, \mathbf{p})e_{i}^{[m]}(\mathbf{x})e_{k}^{[n]}(\mathbf{x}) \]

\( \Gamma_{ik} \) - elements of Christoffel matrix \( \mathbf{\Gamma} \)

\( e^{[m]} \) - triplet of orthonormal vectors

\[ e^{[3]} = c^{[M]}\mathbf{p}, \quad e^{[1]}, e^{[2]} \text{ arbitrarily in the plane } \perp e^{[3]} \]

\( c^{[M]} \) - common S-wave phase velocity

\[ M_{MN} = M_{MN}(\mathbf{x}, \mathbf{p}) = B_{MN} - B_{M3}B_{N3}/(B_{33} - 1) \]
Transformation of first-order slowness vectors at an interface (Snell’s law)

\[ p_i^G = p_i - (p_k N_k) N_i + \xi^G N_i \]

- \( p \) - first-order common S-wave slowness vector of incident wave
- \( p^G \) - first-order common S-wave slowness vector of generated wave
- \( N \) - normal to the interface, \( \xi^G = p_k^G N_k \)

\[ p_i^G - (p_k^G N_k) N_i = p_i - (p_k N_k) N_i \] - Snell’s law
Transformation of first-order slownessness vectors at an interface (Snell’s law)

\[ p_i^G = p_i - (p_k N_k) N_i + \xi N_i \]

\[ G^{[\mathcal{M}]}(x, p^G) = 1 \quad \text{- eikonal equation} \]

\[ G^{[\mathcal{M}]}(\xi) = 1 \quad \text{- polynomial equation of 4th degree for } \xi \]

Iterative solution (Dehghan et al., 2007)

\[ \xi^{\{j\}} = \xi^{\{j-1\}} - \left[ G^{[\mathcal{M}]}(p_{m}^{\{j-1\}}) - 1 \right] / \left[ N_k \frac{\partial G^{[\mathcal{M}]}(p_{m}^{\{j-1\}})}{\partial p_k} \right] \]

\[ \xi^{\{0\}} \quad \text{- in reference isotropic medium} \]
Numerical examples

VSP CONFIGURATION

1.0 km

1.0 km
Numerical examples  (Klimeš & Bulant, 2004)

HTI rotated by 45 degrees

QI (ANI 3%, SEPAR 1-4%)
Numerical examples

![Graph showing the relationship between S1 and S2 waves with depth (km)].

QI

Rel tt dif. (%) vs Depth (km)

S1 AND S2 WAVES
Numerical examples  (Klimeš & Bulant, 2004)

HTI rotated by 45 degrees

QI4 (ANI 6%, SEPAR 11-13%)
Numerical examples

\begin{figure}
\centering
\begin{tikzpicture}
\begin{axis}[
    title={Q14},
    xlabel={Depth (km)},
    ylabel={Relative diff. (\%)},
    xmin=0, xmax=0.6,
    ymin=-0.8, ymax=0.4,
    xtick={0,0.2,0.4,0.6},
    ytick={-0.8,-0.4,0,0.4},
    x tick label style={align=center},
    y tick label style={align=center},
]
\addplot[blue,mark=+] coordinates {
    (0,0.4)
    (0.2,0.3)
    (0.4,0.2)
    (0.6,0.1)
};
\addplot[red,mark=+] coordinates {
    (0,0)
    (0.2,-0.1)
    (0.4,-0.2)
    (0.6,-0.3)
};
\end{axis}
\end{tikzpicture}
\end{figure}

S1 AND S2 WAVES
Numerical examples  (Shearer & Chapman, 1989)

VTI: MODEL SC1 (ANI 11%, SEPAR 0-11%)

![Graph showing phase velocity vs phase angle for SV and SH waves.](image)
Numerical examples

![Graph showing travel time vs. depth]

**SC1 - HOM., EXACT,**
Numerical examples

SC1 - HOM., EXACT, APPR.
Numerical examples
Numerical examples

SCG1 - INHOM., EXACT, APPR.
Numerical examples  (Shearer & Chapman, 1989)

VTI: MODEL SC4 (ANI 30%, SEPAR 0-30%)
Numerical examples
Numerical examples
Conclusions

- applicable to S waves in inhomogeneous isotropic, weakly anisotropic and moderately anisotropic media
- in isotropic media exact, in anisotropic media approximate
- single common S-wave ray necessary for computation of traveltimes of $S_1$ and $S_2$ waves
- common S-wave ray tracing stable, does not collapse anywhere
- computer time savings in computing traveltimes of reflected/transmitted waves
- performs better in inhomogeneous media; smoothes loops in traveltime curves
Seismic Waves in Complex 3-D Structures