# RESEARCH PROGRAMME ON SEISMIC WAVES IN COMPLEX 3–D STRUCTURES (SW3D)

The research is focused primarily on the fundamental issues of high–frequency seismic wave propagation in complex 3–D isotropic and anisotropic structures, which go beyond the traditional approaches. The ray method and its extensions, as well as its combination with other methods are mainly applied and investigated. The emphasis is put on new, stable, more efficient and flexible algorithms for both forward numerical modelling and inversion of seismic wave fields in 3–D inhomogeneous, isotropic or anisotropic, elastic or attenuating structures. Considerable attention is also devoted to applications involving S waves, converted waves, S–wave splitting and coupling in anisotropic media, particle ground motions, etc. Much more detailed information can be obtained at "http://sw3d.cz". The research programme was begun on October 1, 1993.

## SW3D PROGRAM PACKAGES

Package CRT:

*Model*: Using package MODEL (see below).

*Type of waves*: Arbitrary type of elementary seismic body wave corresponding to the zero–order ray theory (P, S, converted, coupled S waves).

*Computations*: Arbitrary position and shape of the source, initial–value ray tracing by numerical integration of ray equations, general isotropic–ray–theory rays, anisotropic–ray–theory P–wave rays and anisotropic common S–wave rays in smooth models, two–point ray tracing by the shooting method based on ray histories, travel–time computation, dynamic ray tracing, paraxial–ray propagator matrix, geometrical spreading, vectorial amplitudes, polarization vectors. The package may be applied to the evaluation of the elastodynamic ray–theory Green function, and to the computation of synthetic seismograms, including the coupling ray theory along isotropic or anisotropic common S–wave rays and the response of fine layers at receiver sites (program package RMATRIX by C.J. Thomson, linked to the CRT package). Least–square travel–time tomography with smoothing using Sobolev scalar products.

*Acquisition schemes*: Surface seismics (land and marine), vertical seismic profiling, cross–hole, ocean bottom.

*Planned innovations*: Extension of the anisotropic–ray–theory P–wave and anisotropic common S–wave ray tracing towards general initial conditions and models with structural interfaces. Tracing anisotropic–ray–theory S–wave rays in smooth models without interfaces. Coupling ray theory along the anisotropic–ray–theory rays in smooth models without interfaces. The package will be extended to solve various inverse problems, stochastic travel–time tomography in particular.

Package ANRAY:

*Model*: 3–D laterally varying structures containing isotropic and anisotropic non–vanishing layers. Specification of elastic parameters inside individual layers either by linear interpolation between isosurfaces of elastic parameters, or by B–spline interpolation within a 3–D rectangular grid of elastic parameters. VRML and GOCAD visualization.

*Types of waves*: Arbitrary type of elementary seismic body wave (P, S, any converted wave, coupled S waves).

*Computations*: Numerical integration of ray tracing and dynamic ray tracing equations, calculation of ray vectorial amplitudes, ray–theory Green function including the Green function in the quasi–isotropic approximation for S waves, ray synthetic seismograms, particle ground motions.
*Acquisition schemes*: Surface seismics (land and marine), vertical seismic profiling, cross–hole, ocean bottom.
*Planned innovations*: (a) Incorporation of effects of weak attenuation. (b) Ray tracing and dynamic ray tracing in media with rotated higher-symmetry anisotropy (transverse isotropy, orthorhombic symmetry). (c) First–order S–wave common–ray computations for inhomogeneous weakly anisotropic media. (d) First–order S–wave common–ray computations for layered media. (e) Computation of the complete propagator matrix in Cartesian coordinates. (f) Calculation of KMAH index in anisotropic media. (g) Stabilization of ray tracing in a vicinity of S–wave singularities. (h) Removal of problems of P–wave reflections/transmissions in a vicinity of S–wave singularities. (i) Further debugging, completion, and removal of inconsistencies in the description of the package.

Package SEIS:
*Model*: 2–D laterally varying isotropic structures composed of layers separated by curved interfaces. Any interface may form edges. It may also coincide with a neighbouring interface(s) in some region. Thus, the models with isolated bodies and pinchouts can be considered. Inside the layers, the velocities of P and S waves may vary in two directions. Weak dissipation and non–planar topography can be considered.
*Types of waves*: Arbitrary type of elementary seismic body wave (P,S, any converted or multiply reflected wave).
*Computations*: Arbitrary position of a point source, numerical integration of 2–D ray tracing and dynamic ray tracing equations, computation of ray vectorial amplitudes or Green functions of individual elementary waves, ray synthetic seismograms, particle ground motions.
*Acquisition schemes*: Surface seismics (land and marine), vertical seismic profiling, cross-hole.
*Planned innovations*: Alternative computation of synthetic seismograms in the frequency domain. Ocean bottom configuration. Extended visualization. Extension of test examples.

Package MODEL:
*Model*: General 3–D layered and block isotropic or anisotropic structures, containing isolated bodies, pinchouts, etc. Inside the layers and blocks, the elastic parameters may vary in all three dimensions. Dissipation and non–planar topography can be considered. Possibility of model smoothing, data fitting by inversion including fitting and smoothing GOCAD models, conversion of model parametrization, triangulation of structural interfaces, VRML and GOCAD visualization.

Package NET:
*Model*: Using package MODEL or using gridded velocities.
*Types of waves*: First arrivals, constrained first arrivals.
*Computations*: Arbitrary position and shape of the source. First–arrival travel times in the whole model are computed. The algorithm of computation is independent of the model's complexity.
*Acquisition schemes*: Surface seismics (land and marine), vertical seismic profiling,

cross–hole, ocean bottom.

Package FORMS:
*Computations*: Subroutines used by other program packages including data input and output subroutines, management and plotting of synthetic seismograms, 2–D and 3–D graphics including 3–D virtual reality with VRML and GOCAD visualization, manipulation and calculation with gridded data (data cubes), programs for matrix and vector operations necessary for inversion, other general–purpose seismic software.
*Planned innovations*: Program for computation of plane–wave reflection/transmission coefficients at planar interfaces separating arbitrary anisotropic media.

## RESEARCH PROGRAMME FOR THE SEVENTEENTH YEAR

October 1, 2009 — September 30, 2010

1. *Sample data for the program packages*

Examples of the input data describing or approximating models delivered by the consortium members or other typical models will be prepared. Examples of the input data to perform calculations in such models will also be prepared.

2. *Ray histories and two–point ray tracing in complex 3–D structures*

Ray histories are of principal importance not only for two–point ray tracing and for wavefront tracing, but also for the summation of Gaussian beams. Properties of the projection of ray coordinates onto Cartesian coordinates within individual ray histories will be studied. New applications of ray histories to numerical algorithms will be proposed. The two–point ray tracing code will further be tested and applied to various models.

3. *Paraxial Fresnel edge waves*

Contributions of single diffractions to the Green function may be approximated by paraxial approximations of Fresnel edge waves in vicinities of boundary rays between ray histories.

4. *Synthetic seismograms in 3–D isotropic and anisotropic complex structures*

Methods to calculate synthetic seismograms in complex structures will be studied, mutually compared and combined. The synthetic seismograms will also be compared with synthetic seismograms generated by non–ray methods. For models suggested by the consortium members, we are ready to perform ray–synthetic studies illustrating how the wave responses differ in heterogeneous and anisotropic media from homogeneous or isotropic media. Calculation of synthetic seismograms by coupling ray theory in layered models will further be tested. Emphasis will be put on numerical implementation of Gaussian beams.

Gaussian–beam synthetic seismograms: Deriving the discretization error of the uneven summation of Gaussian beams. Developing an algorithm for sampling the ray–parameter domain for Gaussian beams.

5. *Gaussian beams in inhomogeneous anisotropic media*

Paraxial Gaussian beams in smoothly varying anisotropic media. Paraxial Gaussian beams in laterally varying layered anisotropic structures. Paraxial Gaussian beams in smoothly varying anisotropic dissipative media.

6. *Seismic wave propagation in inhomogeneous weakly anisotropic media*

Anisotropic common ray approximation of the coupling ray theory: extension of the anisotropic–ray–theory P–wave and anisotropic common S–wave ray tracing towards general initial conditions and models with structural interfaces.

Coupling ray theory along the anisotropic–ray–theory rays in smooth models without interfaces.

Derivation of coupling ray theory from the elastodynamic equation concentrated on the study of errors due to neglected terms in order to estimate the accuracy of coupling ray theory. Study of coupling ray series. Theory of coupling S–wave Gaussian packets. Definition of coupling ray theory travel times and amplitudes, and study of frequency dispersion of S–wave coupling. Quantification of the relevance of anisotropic S–wave coupling to velocity analysis and imaging.

Continuing development of a code for modelling P– and S–wave propagation (including coupling) based on the first–order ray tracing (FORT) and dynamic ray tracing (FODRT) in smooth laterally varying weakly anisotropic media with varying axes or planes of symmetry. Generalization to layered media.

7. *Applicability of the high-frequency asymptotics in the vicinity of S–wave singularities*

Anisotropic S–wave ray tracing in a vicinity of singularities in inhomogeneous anisotropic media. Investigation of accuracy of various high–frequency approximations (zeroth–order anisotropic ray approximation, coupling ray theory, higher–order anisotropic ray approximations, Gaussian beams, etc.) for S waves, propagating in strongly or weakly anisotropic homogeneous or inhomogeneous media in the vicinity of singularities, by comparison with more precise methods (exact solutions, finite differences, etc.).

8. *Seismic wave propagation in anisotropic dissipative media*

Investigation of plane waves propagating in anisotropic viscoelastic media. Both homogeneous and inhomogeneous waves will be considered. For weakly dissipative media, the perturbation methods will be used. Special attention will be devoted to the attenuation vector and to the quality factor Q, particularly to its directional dependence.

The results obtained for plane waves propagating in homogeneous media will be generalized to non–planar waves propagating in smoothly varying anisotropic weakly attenuating media using perturbation methods based on the ray theory. A special attention will be devoted to waves generated by point sources.

9. *Reflection/transmission coefficients*

Effective reflection coefficients for spherical waves incident at a planar interface between two homogeneous isotropic dissipative media.

Perturbation of reflection/transmission coefficients with respect to elastic moduli $c_{ijkl}$. Estimating influence of attenuation on the reflection/transmission coefficients.

10. *Computation of ray–theory travel times, amplitudes and other quantities at the nodes of 3–D grids*

Algorithms of fast calculation of ray–theory travel times in dense rectangular grids will be investigated further. The accuracy and efficiency of the interpolation of ray–theory travel times within ray cells in 3–D models will be studied, and the relevant numerical algorithms will be improved, or new ones will be proposed. Attention will also be devoted to the interpolation between different shot and receiver positions.

11. *Perturbations of travel time*

Generalization of the equations for the second–order and higher–order perturbations of travel time to models with structural interfaces. Generalization of spatial and perturbation derivatives of travel time to Hamilton–Jacobi equation with right–hand sides dependent on ray parameters. Spatial and perturbation derivatives of two–point travel time (characteristic function).

12. *Accuracy of seismic modelling*

The research will be concentrated mainly on the accuracy of travel–time calculations, on the accuracy of finite–difference modelling of seismic wave fields, and on the accuracy of other modelling methods designed or studied within the framework of the project. The main attention will be devoted to the estimation of the feasibility and costs of ray tracing, and to the definition and high–frequency validity of velocity models.

13. *Sensitivity of seismic waves to the structure*

Continuing investigation how the perturbations of a generally inhomogeneous isotropic or anisotropic structure manifest themselves in the wave field, and which perturbations can be detected within a limited aperture and a limited frequency band.

Developing the corresponding algorithm for linearized inversion based on wave–field sensitivity to structural Gabor functions.

14. *Lyapunov exponents and model smoothing*

Construction and smoothing of velocity macro models will further be studied, with emphasis on the application of Sobolev scalar products and Lyapunov exponents. Attention will be paid to a possible extension of the estimation of Lyapunov exponents to smooth 3–D models and to 2–D models with structural interfaces. Sobolev scalar products with spatially variable weights will also be studied.

15. *Seismic tomography and related problems*

Development of theory, algorithms and programs applicable in seismic travel–time tomography and inversion of the coherency panels, with emphasis on the estimation of the accuracy of the resulting model compared to the geological structure.

Stochastic travel–time tomography: Developing an algorithm for calculating geometrical covariances of travel times. Developing an algorithm for calculating geometrical covariances between rays and B–splines.

Determination of the medium correlation functions from well logs. Calculation of sonic–log travel times in anisotropic media. Estimation of uncertainty of sonic–log travel times. Estimation of attenuation from vertical–seismic–profiling travel times.

16. *Seismic sources and hydrofracture monitoring*

Studying the inaccuracy of the absolute source location and the inaccuracy of the relative source location due to the inaccurate velocity model.

Forward and inverse problems for moment tensors of seismic sources in isotropic and anisotropic media.

17. *Local anisotropy parameter estimation from vertical–seismic–profiling measurements*

Local determination of elastic parameters from the vertical–seismic–profiling measurements. Use of P–wave and, possibly, S–wave data. Inversion for parameters of TI media with tilted axis of symmetry and for the angles specifying the axis. Study of possibility to use NMO or AVO data to constrain the inversion. Applications to real data sets.

18. *Decomposition of a wave field into Gabor functions*

Decomposition of a spatial wave field at a fixed time or a time–dependent wave field along a smooth surface into Gabor functions in 2–D or 3–D. The Gabor functions may be frequency–dependent and their shape may be smoothly varying in space and time, which requires much more general decomposition than the Gabor transform. The decomposition into Gabor functions may be useful for the decomposition of a general wave field into Gaussian beams or packets.

Two alternative decompositions will be studied and compared: (a) discretized analytical integral decomposition, (b) numerical discrete decomposition. Whereas the discretized analytical integral decomposition has been applied to Gaussian–packet prestack depth migration, the numerical discrete decomposition can also be applied to linearized inversion based on wave–field sensitivity to structural Gabor functions.

19. *Migrations*

Resolution and accuracy of migrations will be studied. Attention will be paid to the physical meaning of the migrated sections and to their sensitivity to the velocity model, including its anisotropy.

Algorithm of the Gaussian–packet prestack depth migration is being developed. Gaussian packets should offer explicit correspondence between the time and depth sections. Attention is paid to the optimization of the shape of Gaussian packets.

Study of possibilities to include coupling ray theory in seismic imaging.

Amplitude preserving Kirchhoff migration in anisotropic media: Synthetic study of possibilities and limitations to recover reflection coefficients from data measured in inhomogeneous anisotropic media.

20. *Concluding remarks*

In addition to this programme, we will certainly be responsive to specific technical suggestions and recommendations of the consortium members within the general framework of the project. The research in most directions listed above will continue into the future years of the project.